$\frac{R_1}{N_2} : \frac{R_2}{N_1}$

Francis J. Piccioto.

SHOP MATHEMATICS

PART I

SHOP ARITHMETIC

INDUSTRIAL EDUCATION SERIES

SHOP MATHEMATICS

PART I

SHOP ARITHMETIC

PREPARED IN THE
EXTENSION DIVISION OF
THE UNIVERSITY OF WISCONSIN

BY

EARLE B. NORRIS, M. E.
Dean of Engineering, Virginia Polytechnic Institute; Formerly Professor of Mechanical Engineering, in charge of Mechanical Engineering Courses in the Extension Division, University of Wisconsin

AND

KENNETH G. SMITH, A. B., B. S.
State Supervisor of Vocational Industrial Education, Michigan; Formerly Associate Professor of Mechanical Engineering, in charge of Milwaukee District, Extension Division, University of Wisconsin

THIRD EDITION
FOURTH IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1931

COPYRIGHT, 1913, 1924, 1931, BY THE
McGRAW-HILL BOOK COMPANY, INC.

PRINTED IN THE UNITED STATES OF AMERICA

All rights reserved. This book, or parts thereof, may not be reproduced in any form without permission of the publishers.

THE MAPLE PRESS COMPANY, YORK, PA.

INTRODUCTION

This text on Shop Arithmetic was so fundamentally conceived and so meticulously elaborated by the original authors that with each of the many impressions through which it has gone only minor adjustments have been necessary from time to time. In each revision the greatest pains have been taken to make use of every suggestion for improvement and adaptability to the changing requirements of time. Through a number of succeeding years Professor Ben G. Elliott, as Head of the Mechanical Engineering Department in the University Extension Division, has maintained a continuously close and sympathetic touch with the practices of industry and the requirements of students which this text serves, and every constructive suggestion from every source has been noted and incorporated by him in the present revision. Professor Elliott makes particular acknowledgment of the suggestions and contributions made by Professor I. N. Warner of the Platteville State Teachers College, and of the cooperation of Edward M. Tyler in the present revision.

Grateful acknowledgment is made to the original authors, Earle B. Norris, Dean of Engineering in the Virginia Polytechnic Institute, and Kenneth G. Smith, Director of Vocational Education for the State of Michigan.

W. H. LIGHTY
Director of Extension Teaching
University Extension Division
University of Wisconsin

MADISON, WISCONSIN,
July, 1931.

PREFACE TO THE SECOND EDITION

In bringing out a second edition of "Shop Arithmetic," sincere gratification is expressed for the reception accorded the first edition which ran through seventeen impressions. Twelve years ago there was great need for a plain, practical, interesting text of this kind. The University Extension Division of the University of Wisconsin, for lack of such a book, was obliged to produce this volume in connection with its work. Immediately upon its publication many other Extension Divisions adopted and used the book both for correspondence and class instruction. It has likewise been widely used in trade, continuation, shop, apprentice, and other schools of like character. This extensive use with extraordinary variations in student types has resulted in practical suggestions for change and improvement that have been incorporated in the present revision.

Two entirely new chapters have been added to the book, one on Logarithms and one on the Slide Rule. This additional text material has been found to be desirable particularly for those students who plan to go forward with subsequent studies in industrial or engineering courses.

Mr. Herbert E. Schrader, while instructor in Mechanical Engineering in the University Extension Division, prepared both the new material and the text of the revision. He is to be commended upon the success he achieved in incorporating the newer ideas and later practices in the spirit of the original work. Much valuable assistance and counsel were given by Mr. W. E. Wines and Mr. Herbert B. Byrn of our present staff. Due acknowledgment is also made to the many users of the first edition who made valuable suggestions and contributions towards this revision. It is hoped that this new edition will commend itself to old friends and win new ones.

BEN G. ELLIOTT
Professor of Mechanical Engineering
In charge of Mechanical Engineering Courses
University Extension Division

MADISON, WISCONSIN,
June, 1924.

PREFACE TO THE FIRST EDITION

The aim of this book is to teach the fundamental principles of mathematics to shop men, using familiar terms and processes, and giving such applications to shop problems as will maintain the interest of the student and develop in him an ability to apply the mathematical and scientific principles to his everyday problems of the shop. The problems and applications relate largely to the metal working trades. It has, however, been the aim in preparing this volume not to apply the work to these particular trades so closely but that it shall be of interest and value to men in other lines of industry.

This volume presents the first half of the instruction papers in Shop Mathematics as developed and used by the Extension Division of the University of Wisconsin. As here offered, it embodies the point of view obtained through apprenticeship and shop experiences as well as the experience gained through its use during the past four years as a text for both correspondence and classroom instruction. It is believed that the book will be found suitable for home study and for use as a text in trade, industrial, and continuation schools.

The instruction in arithmetic ends with Chapter XII. The remaining chapters are introduced to give further practice in calculation and to develop an ability to handle simple formulas, as well as to impart a knowledge of the principles of machines. The second volume will take up more fully the use of formulas and will teach the principles of geometry and trigonometry as applied to shop work.

The authors are indebted to Mr. F. D. Crawshaw, Professor of Manual Arts in The University of Wisconsin, for a careful reading of the proof and for valuable criticisms and suggestions.

E. B. N.

MADISON, WISCONSIN,
June 1, 1912.

CONTENTS

PAGE

INTRODUCTION v

PREFACE TO THE SECOND EDITION vii

PREFACE TO THE FIRST EDITION ix

CHAPTER I

COMMON FRACTIONS

ARTICLE

1. Why we use fractions 1
2. Definition of a fraction 2
3. The denominator 2
4. The numerator 2
5. Writing and reading fractions 2
6. Proper fractions 3
7. Improper fractions 3
8. Mixed numbers 3
9. Reduction of fractions 3
10. Reduction of improper fractions to mixed numbers 4
11. Reduction of mixed numbers 5
12. Changing to higher terms 5
13. Reduction to lower terms 6
14. Use of signs 6

CHAPTER II

ADDITION AND SUBTRACTION OF FRACTIONS

15. Common denominators 13
16. To find the L. C. D 14
17. To reduce to the L. C. D 15
18. Addition of fractions 15
19. Subtraction of fractions 16
20. Fractional dimensions on drawings 18

CHAPTER III

MULTIPLICATION AND DIVISION OF FRACTIONS

21. A whole number times a fraction 23
22. "Of" means "times" 23
23. A fraction times a fraction 24
24. Multiplying mixed numbers 25
25. Cancellation 26

ARTICLE PAGE
26. Division—the reverse of multiplication 27
27. Compound fractions 28
28. How to analyze practical problems 28

CHAPTER IV

MONEY AND WAGES

29. United States money 33
30. Addition 34
31. Subtraction 34
32. Multiplication 35
33. Division 35
34. Reducing dollars to cents 36
35. Reducing cents to dollars 37
36. The mill 37
37. Wage calculations 38

CHAPTER V

DECIMAL FRACTIONS

38. What are decimals? 43
39. Addition and subtraction 45
40. Multiplication 46
41. Short cuts 47
42. Division 47
43. Reducing common fractions to decimals 49
44. Complex decimals 50
45. The micrometer 50
46. The decimal fraction in machine work 53
46*a*. The metric system 54
46*b*. Changing from one metric unit to another 57

CHAPTER VI

PERCENTAGE

47. Explanation 63
48. The uses of percentage 65
49. Efficiencies 67
50. Discount 67
51. Classes of problems 68

CHAPTER VII

CIRCUMFERENCES OF CIRCLES; CUTTING AND GRINDING SPEEDS

52. Shop uses 73
53. Circles 73
54. Formulas 74
55. Circumferential speeds 76
56. Grindstones and emery wheels 78

ARTICLE PAGE
57. Cutting speeds 80
58. Pulleys and belts 82

CHAPTER VIII

RATIO AND PROPORTION

59. Ratios 85
60. Proportion 86
61. Inverse proportion 88
62. Speeds and diameters of pulleys 89

CHAPTER IX

GEAR RATIOS; GEAR TRAINS; CHANGE GEARS

63. Gear ratios 93
64. Gear and pulley trains 94
65. Compound gear and pulley trains 95
66. Screw cutting 97

CHAPTER X

SQUARE ROOTS AND TABLES

67. Powers 103
68. Roots 103
69. Extracting the square root 104
70. Square roots of mixed numbers 105
71. Square roots of decimals 106
72. Rules for square root 107
73. The law of right triangles 108
74. The value of tables 110
75. How to use the tables 110

CHAPTER XI

AREAS AND VOLUMES OF SIMPLE FIGURES

76. Square measure 139
77. Area of a circle 141
78. Measure of volume 144
79. Finding the volume 145
80. Weights of metals 148
81. Short rule for plates 149
82. Weight of casting from pattern 149
83. Finding dimensions from weights or volumes 150

CHAPTER XII

LEVERS

84. Types of machines 153
85. The lever 153

ARTICLE PAGE
86. Three classes of levers 156
87. Compound levers 157
88. Mechanical advantage 158
89. The wheel and axle 159
90. Efficiency 162

CHAPTER XIII

TACKLE BLOCK; THE INCLINED PLANE AND SCREW

91. Types of block 167
92. Differential pulleys 170
93. The use of inclined planes 171
94. Theory of the inclined plane 171
95. The wedge 173
96. The jack screw 174

CHAPTER XIV

WORK; POWER AND ENERGY; HORSEPOWER OF BELTING

97. Work 181
98. Unit of work 181
99. Power 182
100. Horsepower of belting 183
101. Widths of belts 185
102. Rules for belting 186

CHAPTER XV

HORSEPOWER OF ENGINES

103. Steam engines 191
104. Gas engines 194
105. Air compressors 195
106. Brake horsepower 196
107. Frictional horsepower 197
108. Mechanical efficiency 198

CHAPTER XVI

MECHANICS OF FLUIDS

109. Fluids 201
110. Specific gravity 201
111. Transmission of pressure through fluids 203
112. The hydraulic jack 204
113. Hydraulic machinery 207
114. Hydraulic heads 207
115. Steam and air 208

CHAPTER XVII

HEAT

116. Nature of heat 211
117. Temperatures 212

ARTICLE PAGE
118. Expansion and contraction 216
119. Allowances for shrink fits 219
120. Unit of heat 220
121. Efficiencies of boilers and engines 220

CHAPTER XVIII

STRENGTH OF MATERIALS

122. Stress 223
123. Unit stress 224
124. Ultimate strength 225
125. Safe working stresses 225
126. Strength of bolts 226
127. Strength of hemp ropes 228
128. Wire ropes and cables 228
129. Strength of chains 228
130. Columns 229

CHAPTER XIX

LOGARITHMS

131. Common logarithms 233
132. Mantissas and characteristics 233
133. Use of logarithmic tables 234
134. Interpolation 235
135. Anti-logarithms 236
136. Multiplication by logarithms 237
137. Division by logarithms 238
138. Roots and powers 238
139. Logarithms of decimals 239

CHAPTER XX

THE SLIDE RULE

140. Use of the slide rule 245
141. Logarithmic scale 245
142. Calculation by logarithmic scale 247
143. The simple slide rule 250
144. Description of a slide rule 254
145. Slide rule settings 254
146. Roots and powers 260
147. Decimal points 263

INDEX 265

SHOP ARITHMETIC

CHAPTER I

COMMON FRACTIONS

ıy We Use Fractions.—When we find it necessary to deal ings that are less than one unit, we must use fractions. inist cannot do all his work in full inches because it is y impossible to have all measurements in exact inches. ıently, for measurements less than 1 in., he uses fractions ıch; he also makes use of fractions for measurements . one whole number of inches and the next whole number. t is wanted longer than 4 in. but shorter than 5 in., it ›e 4 in. and a fraction of an inch. This fraction of an ght be nearly a whole inch or it might be a very small part

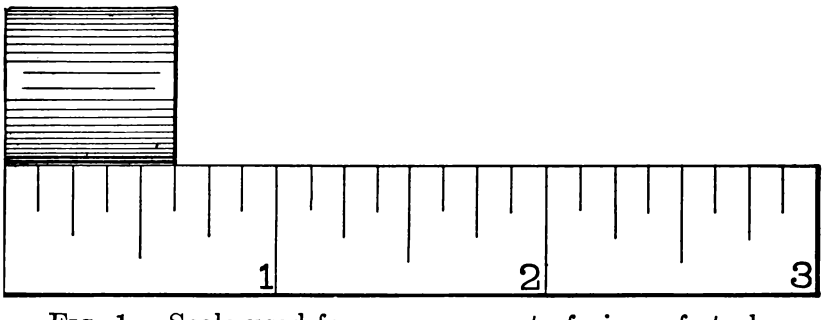


FIG. 1.—Scale used for measurement of piece of steel.

ıch. The system used to designate parts of a unit is ɛen by looking at a machinist's scale or at a foot rule of t. Each inch on the scale is divided into a number of arts. A wooden foot rule usually has 8 or 16 parts to ɔh, while a machinist's steel scale has much finer divisions. we want to measure a piece of steel which is not an inch e hold a scale against it, as in Fig. 1, and find out how f these divisions of an inch it takes to equal the length ›iece. The scale in Fig. 1 is 3 in. long, and each inch is into 8 parts. We see that this piece of steel is as long ›f these *eight* parts of an inch, or we say that it is "*five-*
' of an inch long

2. Definition of a Fraction.—*A Fraction is one or more of the equal parts into which anything may be divided.* Every fraction must contain two numbers, a denominator and a numerator. These are called the *terms* of a fraction.

3. The Denominator.—The *Denominator* tells into how many equal parts the unit is divided. In the case shown in Fig. 1, 1 in. was the unit, and it was divided into 8 equal parts. The denominator in this case was 8.

4. The Numerator.—The *Numerator* shows how many of these parts are taken. In giving the length of the piece of steel in Fig. 1, we divided the inch into 8 parts and took 5 of them for the length. Five (5) is the numerator, and 8 is the denominator.

5. Writing and Reading Fractions.—In writing fractions, the numerator is placed over the denominator and either a slanting line, as in 5/8, or a horizontal line, as in $\frac{5}{8}$, drawn between them. The horizontal line is the better form to use, as mistakes are easily made when a whole number and a fraction with a slanting line are written close together.

$\frac{1}{4}$ is read *one-fourth* or *one-quarter.*

$\frac{1}{2}$ is read *one-half.*

$\frac{3}{4}$ is read *three-fourths* or *three-quarters.*

$\frac{5}{8}$ is read *five-eighths.*

$\frac{3}{7}$ is read *three-sevenths.*

We can have fractions of many things. An *hour* of time is divided into 60 equal parts called *minutes.* A minute is thus $\frac{1}{60}$ of an hour. Likewise, 20 minutes is $\frac{20}{60}$ of an hour. In the same way, 1 second is $\frac{1}{60}$ of a minute.

In the early days, before the unit called the inch was in use, the foot was the common unit for measuring lengths. When it was necessary to measure lengths less than 1 ft., fractions of a foot were used. This became too troublesome, so $\frac{1}{12}$ of a foot was given the name of *inch* to avoid using so many fractions. For instance, where formerly the term $\frac{5}{12}$ of a foot was used, we now use 5 in. This shows how the use of a smaller unit reduces the use of fractions. Another small unit, the one-thousandth of an inch, is used by electricians, and is called the *mil.* The diameter of wire, for example, is expressed more conveniently in mils than in thousandths of an inch.

6. Proper Fractions.—If the numerator of a fraction is less than the denominator, the value of the fraction is less than 1, for the reason that the numerator shows that less than the full number of parts into which the unit has been divided have been taken.

A Proper Fraction is one whose numerator is less than the denominator. The value of a proper fraction, therefore, is always less than 1.

$\frac{3}{4}, \frac{5}{16}, \frac{7}{8}, \frac{27}{32}$ are all proper fractions.

7. Improper Fractions.—If the numerator and denominator of a fraction are equal, the value of the fraction is 1. If the numerator is larger than the denominator, the value of the fraction is greater than 1.

An Improper Fraction is one whose numerator is equal to or larger than the denominator. An improper fraction, therefore, is just equal to or greater than 1.

$\frac{4}{4}, \frac{8}{8}, \frac{10}{10}$ are equal to 1 and are improper fractions.

$\frac{24}{12}, \frac{14}{8}, \frac{17}{16}$ are greater in value than 1, and are improper fractions.

Referring again to the scale, $\frac{11}{8}$ in. is greater than 1, as all of the 8 divisions into which 1 in. is divided have been taken and also 3 more of the 8 divisions in the next inch.

This form of improper fraction leads to a consideration of mixed numbers.

8. Mixed Numbers.—*A Mixed Number is a whole number and a fraction written together:* for example, $4\frac{1}{2}$ is a mixed number; $4\frac{1}{2}$ is read *four and one-half* and means 4 whole units and $\frac{1}{2}$ a unit more.

9. Reduction of Fractions.—Quite often we find it desirable to change the form of a fraction in order to make certain calcula-

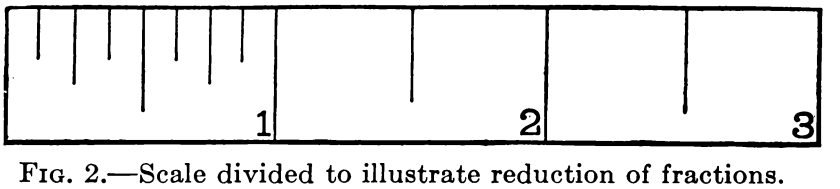


FIG. 2.—Scale divided to illustrate reduction of fractions.

tions; but, of course, the real value of the fraction must not be changed. The operation of changing a fraction from one form to another without changing its value is called *Reduction.*

By referring to the scale in Fig. 2 it will be seen that if we take the first inch and divide it into 8 parts, each $\frac{1}{2}$ in. will contain 4 of these parts. Hence, $\frac{1}{2}$ in. $= \frac{4}{8}$ in. In this case, we make the denominator of the fraction 4 times as large, by making 4 times as many parts in the whole. It then takes a numerator 4 times as large to represent the same fractional part of an inch. This relation holds whether we are dealing with inches or with any other thing as a unit.

10. Reduction of Improper Fractions to Mixed Numbers.— In the reference to the scale, Paragraph 7, we had one whole number of units, or 8 divisions, and in addition a fractional number of units, or 3 divisions. The $\frac{8}{8}$ is the whole number and equals 1; $\frac{3}{8}$ is the fraction. We see, therefore, that the improper fraction $\frac{11}{8}$ may be written as a mixed number, $1\frac{3}{8}$, or $\frac{11}{8} = 1\frac{3}{8}$. This is called reducing an Improper Fraction to a Mixed Number.

When the numerator is greater than the denominator, the value of the fraction is one or more units, plus a proper fraction, or a whole plus some part of a whole.

Examples:

$$\frac{12}{8} = \frac{8}{8} + \frac{4}{8} = 1\frac{4}{8} \text{ or } 1\frac{1}{2}$$

$$\frac{47}{12} = \frac{36}{12} + \frac{11}{12} = 3\frac{11}{12}$$

From these examples we may see that to reduce an improper fraction to a whole or mixed number the simplest way is as follows:

Divide the numerator by the denominator. The quotient will be the number of whole units. If there is anything left over, or a remainder, write this remainder over the denominator, since it represents the number of parts left in addition to the whole units. We now have a mixed number, or an exact whole number, in place of the improper fraction.

Examples:

$$\frac{27}{7} = 27 \div 7 = 3\frac{6}{7} \qquad \begin{array}{r} 7)27(3 \\ \underline{21} \\ 6 \end{array}$$

$$\frac{45}{6} = 45 \div 6 = 7\frac{3}{6} = 7\frac{1}{2} \qquad \begin{array}{r} 6)45(7 \\ \underline{42} \\ 3 \end{array}$$

These show that a fraction represents unperformed division. In fact, division is often indicated in the form of a fraction. The numerator is the dividend and the denominator is the divisor.

$24 \div 3$ can be written $\frac{24}{3}$

$2 \div 8$ can be written $\frac{2}{8}$

11. Reduction of Mixed Numbers.—It is often necessary or desirable to change mixed numbers to improper fractions. The method of doing this may be seen from the following examples.

Examples:

Reduce $5\frac{1}{2}$ to an improper fraction.

$$5\frac{1}{2} = 5 + \frac{1}{2}$$

In one unit there are two halves. Therefore,

$$5 = \frac{5 \times 2}{2} = \frac{10}{2}$$

$$5\frac{1}{2} = \frac{10}{2} + \frac{1}{2} = \frac{11}{2}$$

If $7\frac{1}{4}$ were to be reduced to an improper fraction, we would say: "Since there are 4 fourths in 1, in 7 there are 4×7, or 28 fourths; 28 fourths plus 1 fourth equals 29 fourths."

$$7\frac{1}{4} = \frac{28}{4} + \frac{1}{4} = \frac{29}{4}$$

The rule which this gives us is very simple: Multiply the whole number by the denominator of the fraction and write the product over the denominator. This reduces the whole number to a fraction. Add to this the fractional part of the mixed number. The sum is the desired improper fraction.

In working problems like the above, the work should be arranged as in the following example.

Example:

Reduce $5\frac{1}{8}$ in. to eighths of an inch.

$$5\frac{1}{8} = \frac{40}{8} + \frac{1}{8} = \frac{41}{8} \text{ in., } \textit{Answer}$$

12. Changing to Higher Terms.—When we change a fraction to higher terms, we increase the number of parts in the whole, and this likewise increases the number of parts taken. Therefore, the numerator and denominator both become larger numbers.

Thus,

$$\frac{1}{2} \text{ in.} = \frac{4}{8} \text{ in.} \qquad \frac{5}{16} \text{ in.} = \frac{10}{32} \text{ in.}$$

A fraction is raised to higher terms by multiplying both numerator and denominator by the same number.

Examples:

$$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Similarly,

$$\frac{5}{16} = \frac{5 \times 2}{16 \times 2} = \frac{10}{32}$$

Suppose we want to change $\frac{3}{16}$ of an inch to 64ths. To get 64 for the denominator, we must multiply 16 by 4 and, therefore, must multiply 3 by the same number.

$$\frac{3}{16} = \frac{3 \times 4}{16 \times 4} = \frac{12}{64}$$

13. Reduction to Lower Terms.—When we reduce a fraction to lower terms we reduce the number of parts into which the whole unit is divided. This likewise reduces the number of parts which are taken.

Thus,

$$\frac{4}{8} \text{ in.} = \frac{1}{2} \text{ in.} \qquad \frac{2}{16} \text{ in.} = \frac{1}{8} \text{ in.}$$

A fraction is reduced to lower terms by dividing both numerator and denominator by the same number. When there is no number which will exactly divide both numerator and denominator, the fraction is already in its *lowest* terms.

Example:

Reduce $\frac{36}{128}$ to its lowest terms.

$$\frac{36}{128} = \frac{36 \div 2}{128 \div 2} = \frac{18 \div 2}{64 \div 2} = \frac{9}{32}$$

There is no number that will evenly divide both 9 and 32. Therefore the fraction is reduced to its lowest terms.

14. Use of Signs.—Throughout the preceding paragraphs and in all the following text material five "signs of operation" are used repeatedly: *the plus sign* (+); *the minus sign* (−); *the equals sign* (=); *the multiplication sign* (×); *and the division sign* (÷).

The plus sign (+) is a signal, or indication, of addition and when placed between two numbers means that they are to be added. The minus sign (−) when placed in this manner is a signal of subtraction. When the minus sign is placed between two numbers, it must be remembered that the second number is to be subtracted from the first; thus in the subtraction 7 − 6 =

1, the 6 is subtracted from the 7. The multiplication sign ($\times$) signifies that the numbers are to be multiplied; and the division sign ($\div$) that the first number is divided by the second.

The most misused of all these signs is the equals sign ($=$). This sign placed between two sets of numbers simply means that the total calculated value on one side of the equals sign is equal to the total calculated value on the other side, and the whole combination of numbers is called an *equation.* Thus we can write the following equation:

$$6 + 4 + 5 + 3 = 10 + 8$$

This can be further reduced by adding the $10 + 8$,

$$6 + 4 + 5 + 3 = 10 + 8 = 18, \textit{Answer.}$$

This use of the equals sign more than once in an operation is what causes a large number of errors. It must be remembered, in a case like this, that each successive equals sign simply indicates another *equal amount,* and, therefore, the first amount is equal to each succeeding amount. For example:

$1 + 3 + 9 + 4 + 2 = 4 + 9 + 6 = 10 + 9 = 19$, *Answer.*
Therefore, $1 + 3 + 9 + 4 + 2 = 19$, *Answer.*

The equals sign should never be used unless the amounts are equal, nor should it be used in a continuous operation, as above, unless each amount is equal to every other amount. The mistake that is often made is to perform two or more operations and to write down the whole as a continuous equation. For example, if it is desired to multiply 7 times 6, and then to add 4 to the result, the two operations should be performed separately.

Correct method $7 \times 6 = 42$
$42 + 4 = 46$, *Answer.*
Wrong method $7 \times 6 = 42 + 4 = 46$, *Answer.*

In the second method the two operations have been performed continuously, and the equals sign placed without regard to equality.

The second method is wrong because each amount is not equal to the next, although the equals signs indicate that they should be. The equation reads that $7 \times 6 = 42 + 4$, which is not true.

Parentheses ( ) are also much used. When several quantities are placed within parentheses, the operations should be performed upon those quantities *before* operations are performed

upon any quantities outside the parentheses. Thus the expression $5 + 7 + (3 \times 4)$ indicates that the 3 and 4 must be multiplied *before* adding them to the 5 and 7.

Therefore,

$$5 + 7 + (3 \times 4) = 5 + 7 + 12 = 24$$

If we use the same numbers and the same signs, but put the parentheses in different places, we get an entirely different answer. Thus $(5 + 7 + 3) \times 4$ indicates that the 5, 7, and 3 are to be added *before* multiplying by 4.

$$(5 + 7 + 3) \times 4 = 15 \times 4 = 60$$

USEFUL TABLES

Measures of Length

12	inches (in.)	= 1 foot (ft.)
3	feet or 36 inches	= 1 yard (yd.)
$5\frac{1}{2}$	yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320	rods or 5280 feet	= 1 mile (mi.)

Measures of Time

60	seconds (sec.)	= 1 minute (min.)
60	minutes	= 1 hour (hr.)
24	hours	= 1 day (da.)
7	days	= 1 week (wk.)
$365\frac{1}{4}$	days	= 1 average year (yr.)
100	years	= 1 century

Note.—Thirty (30) days are generally considered as 1 month (mo.), though the number of days differs for different months.

Miscellaneous Units

12 things	= 1 dozen (doz.)
12 dozen or 144 things	= 1 gross (gr.)
12 gross	= 1 great gross
20 things	= 1 score

PROBLEMS

1. What is a fraction?
2. Name some fractions of an inch commonly used.
3. Write the following as fractions or mixed numbers:

   Five-sixteenths
   Nine thirty-seconds
   Twenty and one-eighth
   Twenty-one eighths
   Three and three-fourths

4. Write out in words the following:

$$3\frac{1}{6},\ \frac{5}{8},\ \frac{7}{16},\ \frac{8}{8},\ \frac{5}{24},\ 20\frac{1}{4},\ \frac{21}{4}$$

5. Does dividing both the numerator and the denominator of a fraction by the same number change the value of the fraction?

6. Is the following mathematical statement true:

$$\frac{12 \times 2}{4 \times 2} = \frac{6}{2}$$

7. Write three proper fractions, three improper fractions, and three mixed numbers.

8. Reduce to lowest terms:

$$\frac{112}{364},\ \frac{231}{441},\ \frac{1677}{4368}$$

9. Change $\frac{9}{16}$ of an inch to eighths of an inch.

10. How many sixteenths of an inch in $\frac{3}{4}$ in.? How many thirty-seconds of an inch in $\frac{3}{4}$ in.?

11. Which is greater, $\frac{3}{16}$ in. or $\frac{7}{8}$ in.? $\frac{3}{10}$ or $\frac{3}{12}$? $\frac{24}{36}$ or $\frac{25}{37}$?

12. Reduce the following mixed numbers to improper fractions:

$$4\frac{1}{8},\ 7\frac{1}{2},\ 3\frac{3}{4},\ 2\frac{1}{6},\ 1\frac{1}{16}$$

13. Reduce $852\frac{25}{34}$ to an improper fraction.

14. Reduce the following improper fractions to whole or mixed numbers:

$$\frac{21}{16},\ \frac{8}{8},\ \frac{24}{3},\ \frac{7}{2},\ \frac{121}{12}$$

15. How many eggs difference are there between 124 eggs and $\frac{7}{12}$ of a gross?

16. A fireman shovels an average of 6 tons of coal per day into the furnace of a locomotive. What fraction of a carload does the locomotive use per day if an average car contains 30 tons?

17. What fraction of a mile is 550 yd.?

18. How many days will it take for the locomotive in Problem 16 to burn three carloads of coal?

19. If there are 360 degrees in a circle, what fractional part of a circle is 90 degrees?

20. I want to mix up a pound of alloy to be made of 5 parts zinc, 2 parts tin, and 1 part lead. What fraction of a pound of each metal—zinc, tin, and lead—must I have?

21. An apprentice who is drilling and tapping a cylinder for $\frac{7}{8}$ in. studs, tries a $\frac{3}{4}$ in. drill, but the tap binds; so he decides to use a drill $\frac{1}{64}$ in. larger. What size of drill does he ask for?

22. The tubes in a certain boiler are 15 ft. 11 in. long. How many inches long are they?

23. The lowest dimension on the porch post of Fig. 3 reads "two feet five and one-half inches" (2′ 5½″); the next dimension reads "three and one-half inches" (3½″). Write out the remaining dimensions of the post in words.

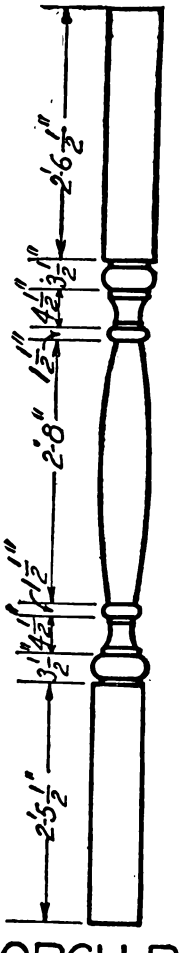


FIG. 3.—Illustration for Problem 23.

24. A man sells his car for $900. If the car cost $1200 when new, how much has the car depreciated, or lost, in value? What fraction is this of the first cost?

25. Wood screws are generally put up in boxes containing 1 gross. If 36 screws are taken from a full box for use on a certain job, what fraction of the gross is used on this job, and what fraction is left in the box? Reduce both fractions to their lowest terms.

26. A steel plate 2 ft. 6 in. wide is to be sheared into four strips of equal width. How wide will each strip be in inches?

27. In one plant all dimensions on the drawings are given in inches, while in another all dimensions above 2 ft. are given in feet and inches. If a dimension is given as 89 in. in the first plant, how would the same dimension be stated in the other plant?

28. A fireman in a power plant shovels into the furnace 3 tons of coal per day. From this coal, there goes into the ashpit refuse and ash to the amount of 500 lb. At this same rate, how many tons of coal must be burned to provide $1\frac{1}{2}$ tons of ash and refuse?

29. A steam engine develops 350 brake horsepower. The steam consumption per hour is 5500 lb. What fraction of a brake horsepower is produced by the use of 1 lb. of steam?

30. Under a system of income taxation, tax payers pay a tax, the amount of which depends upon the yearly income of each individual. On the first \$750 of income, no tax is collected. On all income between \$750 and \$1500, a tax amounting to $\frac{5}{8}$ of a dollar for each \$100 is collected. On all income above \$1500, the tax is $\frac{7}{8}$ of a dollar for each \$100. Calculate the tax to be paid by a man who has an income of \$1800 per year.

CHAPTER II

ADDITION AND SUBTRACTION OF FRACTIONS

15. Common Denominators.—Fractions cannot be added unless they contain the same kind of parts, or, in other words, have the same denominator. When fractions having different denominators are to be added, they must first be reduced to fractions having a common denominator. A number of fractions are said to have a *common* denominator when they all have the same number for their denominators. For example, $\frac{3}{4}$ and $\frac{5}{8}$ cannot be added as they stand, any more than can 3 bolts and 5 washers. Both the fractions must be of the same kind, that is, must have the same denominator. By changing $\frac{3}{4}$ to $\frac{6}{8}$, the fractions are given a common denominator and can be added. Also 6 eighths plus 5 eighths equals 11 eighths, in just the same manner as 6 in. plus 5 in. equals 11 in. The work of this example would be written as follows:

$$\frac{3}{4} + \frac{5}{8} = ?$$

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$\frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}, \textit{ Answer.}$$

Eight (8) is called the *Least Common Denominator* (L. C. D.) of $\frac{3}{4}$ and $\frac{5}{8}$, because it is the smallest number that can be used as a common denominator for these two fractions. In this case, the least common denominator is apparent at a glance. (The student should practice finding the L. C. D. by looking at the problem, as much time can often be saved in this way.) In many other cases it is more difficult to find, especially if there are several fractions to be added. In the case just given, the denominator of one of the fractions can be used for the common denominator. When we have two denominators like 5 and 8, neither of them is an exact multiple of the other number, and so neither can be the common denominator. In such a case, the product of the two numbers can be used as a common denominator.

Example:

$$\frac{3}{5} + \frac{5}{8} = ?$$

$5 \times 8 = 40$, the L. C. D.

$$\frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40}$$

$$\frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40}$$

$$\frac{24}{40} + \frac{25}{40} = \frac{49}{40} = 1\frac{9}{40}, \textit{ Answer.}$$

We can always be sure that the product of the denominators will be a common denominator, to which all the fractions can be reduced, but it will not always be the *least* common denominator. For example, if we wish to add $\frac{5}{12}$ and $\frac{7}{16}$, we can use $12 \times 16 = 192$ for the common denominator, but we readily see that 48 will serve just as well and not make the fractions so cumbersome. In this case 48 is the least common denominator.

16. To Find the L. C. D.—If the L. C. D. cannot be easily determined by examining the denominators, it may be found as follows: Suppose we are to find the L. C. D. of $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{8}{16}$. First place the denominators in a row, separating them by commas, as shown below:

$$\begin{array}{r} 2)\underline{4,\ 3,\ 9,\ 16} \\ 2)\underline{2,\ 3,\ 9,\ 8} \\ 3)\underline{1,\ 3,\ 9,\ 4} \\ 1,\ 1,\ 3,\ 4 \end{array}$$

L. C. D. $= 2 \times 2 \times 3 \times 3 \times 4 = 144$.

Select the smallest number (other than 1) that will exactly divide two or more of the denominators. In this case, 2 will exactly divide 4 and 16. Divide it into all the numbers that are exactly divisible by it, that is, may be divided by it without leaving a remainder. When writing the quotients below, also bring down any numbers which are not divisible by the divisor and write them with the quotients. Now proceed as before, again using the smallest number that will divide two or more of the numbers just obtained. Continue this process until no number (except 1) will exactly divide more than one of the remaining numbers. The product of all the divisors and all the numbers left in the last line of quotients is the least common denominator.

17. To Reduce to the L. C. D.—Having found the least common denominator of two or more fractions, the next step is to reduce the given fractions to fractions having this least common denominator. Let us take $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{3}{8}$. We first find the L. C. D., which is 120. We next proceed to reduce the fractions to fractions having the L. C. D. Divide the common denominator by the denominator of the first fraction. Multiply both numerator and denominator of the fraction by the quotient thus obtained. Do this for each fraction, as shown below.

$$\begin{array}{r} 2)\underline{3,\ 5,\ 4,\ 8} \\ 2)\underline{3,\ 5,\ 2,\ 4} \\ 3,\ 5,\ 1,\ 2 \end{array}$$

$$\text{L. C. D.} = 2 \times 2 \times 3 \times 5 \times 2 = 120$$

$$120 \div 3 = 40 \quad \frac{1}{3} = \frac{1 \times 40}{3 \times 40} = \frac{40}{120}$$

$$120 \div 5 = 24 \quad \frac{1}{5} = \frac{1 \times 24}{5 \times 24} = \frac{24}{120}$$

$$120 \div 4 = 30 \quad \frac{1}{4} = \frac{1 \times 30}{4 \times 30} = \frac{30}{120}$$

$$120 \div 8 = 15 \quad \frac{3}{8} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$$

18. Addition of Fractions.—Addition of fractions is very simple after the fractions have been reduced to fractions with a common denominator. Having done this it is only necessary to add the numerators and place this sum over the common denominator. The sum should always be reduced to lowest terms; and if it happens to be an improper fraction, it should be reduced to a mixed number.

Example:

Find the sum of $\frac{5}{16} + \frac{3}{4} + \frac{9}{32} + \frac{7}{32}$

Common denominator = 32

$$\frac{5}{16} = \frac{5 \times 2}{16 \times 2} = \frac{10}{32}$$

$$\frac{3}{4} = \frac{3 \times 8}{4 \times 8} = \frac{24}{32}$$

$$\text{Adding, } \frac{10}{32} + \frac{24}{32} + \frac{9}{32} + \frac{7}{32} = \frac{50}{32}$$

$$\frac{50}{32} = \frac{25}{16} = 1\frac{9}{16}, \textit{ Answer.}$$

If there are mixed numbers and whole numbers, add the whole numbers and fractions separately. If the sum of the fractions

is an improper fraction, reduce it to a mixed number, and add this to the sum of the whole numbers.

Example:

How long a steel bar is needed from which to shear pieces of the following lengths (one piece of each length):

$7\frac{1}{2}$ in., $5\frac{7}{8}$ in., $4\frac{3}{4}$ in., $6\frac{1}{8}$ in.?

$$\begin{array}{ll} 7\frac{1}{2} & \frac{4}{8} \\ 5\frac{7}{8} & \frac{7}{8} \\ 4\frac{3}{4} & \frac{6}{8} \\ 6\frac{1}{8} & \frac{1}{8} \\ \hline 22 & \frac{18}{8} = 2\frac{1}{4} \\ 2\frac{1}{4} & \\ \hline 24\frac{1}{4}, \textit{ Answer.} & \end{array}$$

Explanation: The sum of the whole numbers is 22. The sum of the fractions is $\frac{18}{8}$, which reduces to $2\frac{1}{4}$. Adding this to the sum of the whole numbers (22), gives $24\frac{1}{4}$ as the sum of the mixed numbers. Hence, we must have a bar $24\frac{1}{4}$ in. long.

19. Subtraction of Fractions.—Just as in addition, the fractions must first be reduced to a common denominator. Then we can subtract the numerators and write the result over the common denominator.

Example:

Subtract $\frac{5}{8}$ from $\frac{15}{16}$.

Common denominator = 16

$$\frac{5}{8} = \frac{10}{16}$$

$$\frac{15}{16} - \frac{10}{16} = \frac{5}{16}, \textit{ Answer.}$$

In subtracting mixed numbers, subtract the fractions first and then the whole numbers.

Example:

How much must be cut from a $15\frac{1}{2}$-in. bolt to make it $12\frac{3}{16}$ in. long?

L. C. D. = 16

$$15\frac{1}{2} = 15\frac{8}{16}$$

$$\begin{array}{r} 12\frac{3}{16} \\ \hline 3\frac{5}{16}, \textit{ Answer.} \end{array}$$

Sometimes, in subtracting mixed numbers, we find that the fraction in the subtrahend (the number to be taken away) is larger than the fraction in the minuend (the number from which

the subtrahend is to be taken). In this case, we borrow 1 from the whole number of the minuend and add it to the fraction of the minuend. This makes an improper fraction of the fraction in the minuend, and we can subtract the other fraction from it.

Example:

Subtract $9\frac{3}{4}$ from $12\frac{1}{8}$.

$12\frac{1}{8} = 11\frac{9}{8}$.

$9\frac{3}{4} = 9\frac{6}{8}$

$2\frac{3}{8}$, *Answer.*

Explanation: $\frac{3}{4}$ cannot be subtracted from $\frac{1}{8}$, so we borrow $1\left(\text{or } \frac{8}{8}\right)$ from 12 and write the minuend $11\frac{9}{8}$.

If the minuend happens to be a whole number, borrow 1 from it and write it as a fractional part of the minuend. Then subtract as before.

Example:

$10 - 8\frac{5}{16} = ?$

$10 = 9\frac{16}{16}$

$8\frac{5}{16}$

$1\frac{11}{16}$, *Answer.*

In Fig. 4, if we were to add to the fraction of an inch *a* (namely, $\frac{1}{2}$ in.) as shown on scale 1, the fraction of an inch *b* (namely, $\frac{3}{8}$

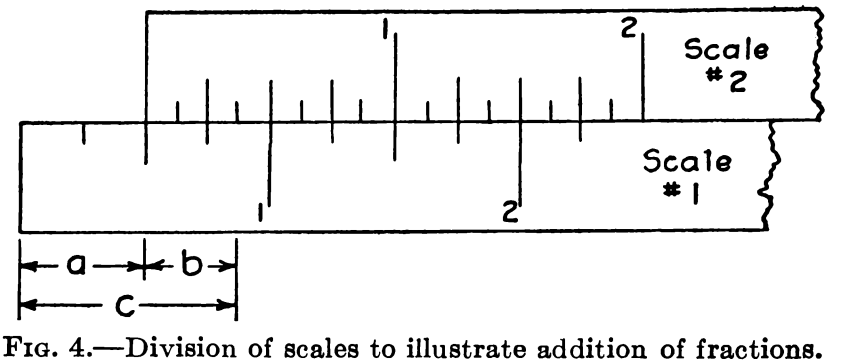


FIG. 4.—Division of scales to illustrate addition of fractions.

in.) as measured on scale 2, we could not read the sum of *a* and *b* (which is *c*) on scale 1 because the fractions of an inch on scale 1 are not of the same units as on scale 2. However, if we reduce the fractional units on scale 1 to the same size as on scale 2 (as has been done in Fig. 5) we can read the sum of *a* and *b* or the distance

c on scale 1. Thus, we have, in a manner, reduced the fractions shown on the two scales to the least common denominator.

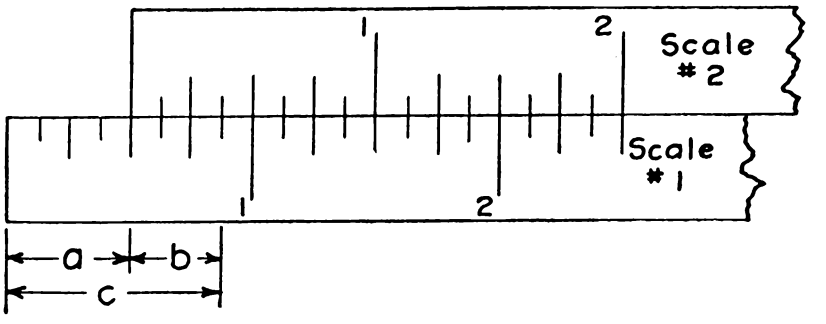


FIG. 5.—Division of scales to illustrate least common denominator.

20. Fractional Dimensions on Drawings.—Quite frequently the machinist, pattern maker, or tinsmith is called upon to add or subtract fractions in order to obtain necessary dimensions from his drawings or blueprints.

Figure 5*a* shows side and end views of a machine bolt. The side view shows the bolt as it appears when looked at from

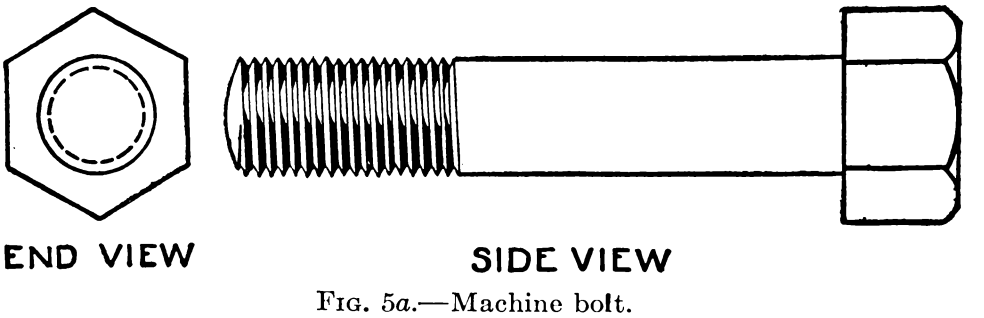


FIG. 5*a*.—Machine bolt.

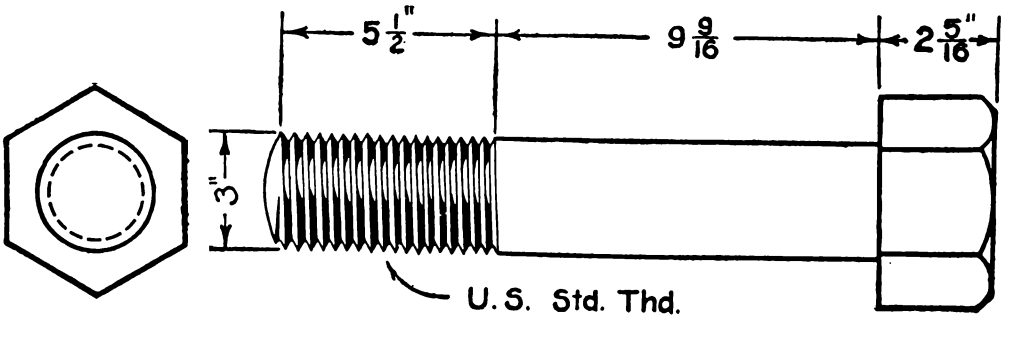


FIG. 5*b*.—Machine bolt with dimensions.

the side; the end view shows the bolt as it appears when looked at from the threaded end. If the machinist were given this drawing and asked to make the bolt, he would be at a loss to know what size of bolt was required, how long it should be, what length of thread to cut, etc.

Consequently, dimensions are placed on the drawing, as in Fig. 5*b*, to furnish all necessary information. The long lines with arrowheads at the ends are called "dimension lines," and the distance from the point of one arrowhead to the point of another is written midway between the arrowheads. Thus the drawing in Fig. 5*b* shows that the bolt has a diameter of 3 in., length of head of $2\frac{5}{16}$ in., length from the edge of the threads to the head of $9\frac{9}{16}$ in., and length of threaded portion of $5\frac{1}{2}$ in.

However, the draftsman failed to place the over-all length of the bolt on the drawing. Consequently, the machinist must calculate this length in order to determine what length of rough stock is required.

Over-all length = $5\frac{1}{2}$ in. + $9\frac{9}{16}$ in. + $2\frac{5}{16}$ in. = 18 in. (finished)

Allowing $\frac{3}{16}$ in. for finish, the length required = $18\frac{3}{16}$ in., *Answer.*

PROBLEMS

31. Reduce to the L. C. D. $\frac{5}{16}$, $\frac{3}{4}$, and $\frac{9}{32}$. *Answer,* $\frac{10}{32}$, $\frac{24}{32}$, $\frac{9}{32}$.

32. Reduce to the L. C. D. $\frac{3}{8}$, $\frac{5}{12}$, $\frac{5}{6}$, and $\frac{7}{16}$.

33. Reduce to the L. C. D. $\frac{3}{4}$, $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{5}{16}$.

34. Add $\frac{10}{24}$, $\frac{11}{12}$, and $\frac{15}{16}$.

35. $\frac{5}{6} + \frac{1}{8} + \frac{2}{3} + \frac{7}{16} = ?$

36. Add $2\frac{3}{8}$, $5\frac{1}{4}$, and $7\frac{1}{16}$.

37. Add $\frac{3}{4}$, $\frac{9}{10}$, and $\frac{5}{13}$, and express the answer both as an improper fraction and as a mixed number.

38. Find the sum of 8, $3\frac{1}{2}$, $4\frac{5}{8}$, and $\frac{3}{16}$.

39. Subtract $\frac{4}{30}$ from $\frac{7}{10}$.

40. $\frac{1}{16} - \frac{1}{24} = ?$

41. Add $11\frac{5}{16}$, $8\frac{3}{4}$, $9\frac{4}{7}$, and from the sum subtract the sum of $4\frac{9}{10}$, $2\frac{4}{5}$, $6\frac{7}{14}$.

42. If a chemist mixes together $3\frac{1}{2}$ g., $4\frac{1}{4}$ g., and $6\frac{7}{8}$ g., respectively, of three chemicals, and then adds an amount of a fourth ingredient equal in weight to $\frac{1}{2}$ the sum of the weight of the first three, what will be the total weight of the mixture?

43. Find the difference between $13\frac{3}{4}$ and $20\frac{1}{2}$.

44. $15 - 11\frac{7}{8} = ?$

45. Subtract $8\frac{3}{16}$ from 14; $9\frac{1}{4}$ from $12\frac{4}{7}$.

46. A boiler maker and his helper renew $\frac{1}{3}$ of the total number of tubes in a boiler the first day on the job; the second day they renew $\frac{1}{4}$ of

the total number of tubes. What fraction of the total number remains to be completed after the second day?

47. The weights of a number of castings are: $412\frac{1}{2}$ lb., $270\frac{1}{2}$ lb., $270\frac{1}{2}$ lb., 1020 lb., $75\frac{1}{2}$ lb., and $68\frac{1}{2}$ lb. What is their total weight?

48. In a certain boiler test about $\frac{1}{8}$ of the total heating value of the coal was unavoidably lost up the chimney; about $\frac{1}{12}$ was lost through unburned coal in the ash; and the remainder was absorbed by the boiler. Compute the fraction of the total heat absorbed by the boiler.

49. A package of nails weighs $5\frac{11}{16}$ lb. After 19 oz. have been used, what is the weight of the package expressed in pounds and in ounces?

50. Four studs are required: $2\frac{3}{4}$ in., $1\frac{7}{8}$ in., $2\frac{5}{16}$ in., and $1\frac{33}{32}$ in. long. How long a piece of steel will be required from which to cut them, allowing $\frac{3}{4}$ in. altogether for cutting off and finishing their ends?

51. Monday morning an engineer bought $48\frac{1}{2}$ gal. of cylinder oil; on Monday, Tuesday, and Wednesday he used $\frac{3}{4}$ gal. per day; on Thursday he used $\frac{7}{8}$ gal.; and on Friday $\frac{1}{2}$ gal. How much oil had he left on Saturday?

52. An electrician in wiring a house finds he lacks 76 ft. of wire to complete the job. He gets a partly used spool of wire from the shop. The spool originally contained 1000 ft. of wire, but $\frac{1}{8}$ of it had been used. If the electrician took the 76 ft. off this partly used spool, what fraction of the wire on the partly used spool did he use?

53. A piece of work on a lathe is 1 ft. in diameter. It is turned down in five cuts: in the first step the tool takes off $\frac{3}{32}$ in. from the diameter; then $\frac{1}{16}$ in.; then $\frac{1}{32}$ in.; then $\frac{1}{32}$ in.; and then $\frac{1}{64}$ in. What is the diameter of the finished piece?

54. A small foundry uses $15\frac{1}{2}$ tons of coke the first week of a month, $14\frac{2}{3}$ tons the second week, 17 tons the third week, and $19\frac{3}{4}$ tons during the remainder of the month. What is the total amount of coke burned during the month?

55. By mistake, the draftsman omitted the thickness of the flange on the drawing of the gas engine cylinder shown in Fig. 6*a*. From the other dimensions given, calculate the thickness of the flange.

56. A millwright has to rig up temporarily a 6-in. belt to be $367\frac{5}{8}$ in long. In looking over the stock of old belting he finds the following pieces of the right width: one piece $126\frac{1}{2}$ in. long, one $142\frac{7}{8}$ in. long, and one $133\frac{3}{8}$ in. long. How many inches must be cut from one of the

pieces so that these pieces can be laced together to give the right length?

57. The time cards for a certain piece of work show 2 hr. and 15 min. lathe work, 3 hr. and 10 min. milling, 1 hr. and 10 min. planing, and 1 hr. and 15 min. bench work; what is the total number of hours to be charged to the job?

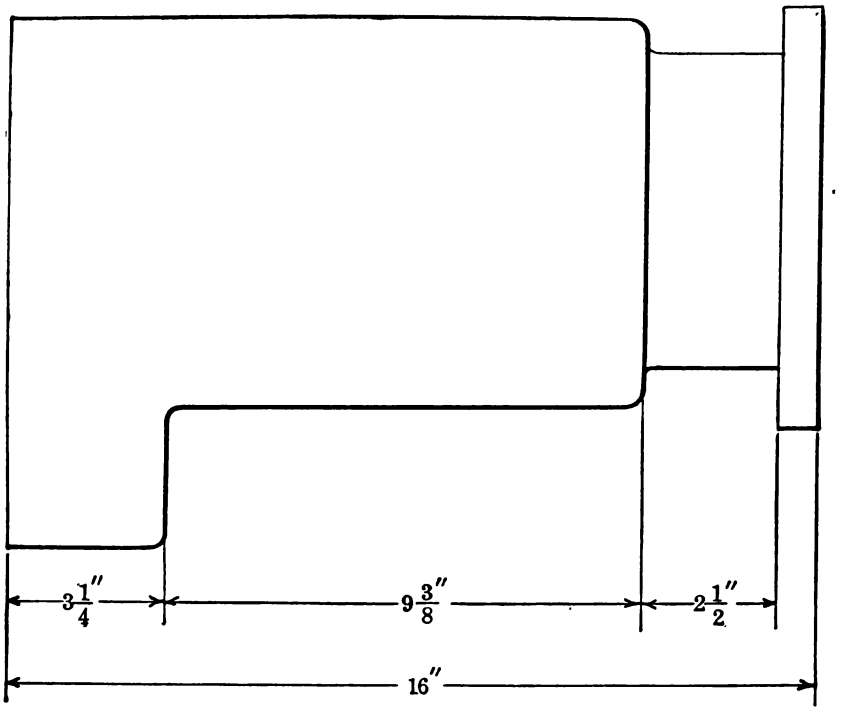


FIG. 6*a*.—Casting for gas engine cylinder.

58. A pole standing in a marsh has $\frac{1}{6}$ of its length in the mud and $\frac{1}{4}$ of its length in the water. The remaining portion of the pole, which is 14 ft., is above the water. Calculate the length of the pole.

59. One side of the roof of a large building is to be equipped with gutters or eave troughing. The entire length of the lower edge of the roof, which is the length of the gutter when erected, is 86 ft. 9 in. If each length of gutter is 10 ft. long, and 2 in. are to be allowed for each joint (not counting caps on ends) how many feet of gutter must be actually used on the job? (In calculating the answer, consider only the gutter actually used. If a length is cut, do not include the amount not used.)

60. A boy has the following pieces of money: one 25-cent piece; one dime; one nickel; and four pennies. What fraction of a dollar does he have?

CHAPTER III

ıULTIPLICATION AND DIVISION OF FRACTIONS

A Whole Number Times a Fraction.—In the study of ;plication, we learn that multiplying is only a short way of ıg. For example, 4×7 is the same as four 7's added her. Either 4×7, or $7 + 7 + 7 + 7$ will give 28. If we ' this same principle to the multiplying of fractions, we see $4 \times \frac{7}{8}$ is the same as four of these fractions added together.

$$4 \times \frac{7}{8} = \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} = \frac{28}{8}$$

is shows that multiplying a fraction by a whole number is rmed by multiplying the numerator by the whole number ılacing the product over the denominator of the fraction. other words, the *size* of the parts is not changed, but the *er* of parts is increased by the multiplication. After plying, the product should be reduced to lowest terms; and, improper fraction, should be reduced to a whole or mixed ıer.

ıle:

What would be the total weight of 12 brass castings each ıg $\frac{3}{4}$ of a pound?

$$12 \times \frac{3}{4} = \frac{36}{4} = 9 \text{ lb., } \textit{Answer.}$$

"Of" Means "Times."—The word "of" is often seen in ;ms in fractions, as, for instance, "What is $\frac{3}{4}$ of 5 in.?" ;h a case, we work the problem by multiplying, so we say 'of" means "times." It may be seen that this is so by ; a piece of wood 5 in. long and cutting it into 4 equal parts ıen taking 3 *of* these parts. These 3 parts will be $\frac{3}{4}$ of 5 in., y actual measurement will be $3\frac{3}{4}$ in. long, so we know that $\frac{3}{4}$: $3\frac{3}{4}$. Now see what $\frac{3}{4}$ *times* 5 is:

$$\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$$

is the same value. Therefore, we see that the word "of"

23. A Fraction Times a Fraction.—To multiply two or more fractions together, multiply the numerators together for the numerator of the product, and multiply the denominators together for the denominator of the product.

Example:

Multiply $\frac{7}{8} \times \frac{2}{3}$.

$\frac{7}{8} \times \frac{2}{3} = \frac{14}{24} = \frac{7}{12}$

Explanation: The numerator of the product is obtained from multiplying the numerators together: $7 \times 2 = 14$. The denominator of the product, in the same manner, is $8 \times 3 = 24$. This gives the product $\frac{14}{24}$, which can be reduced to $\frac{7}{12}$.

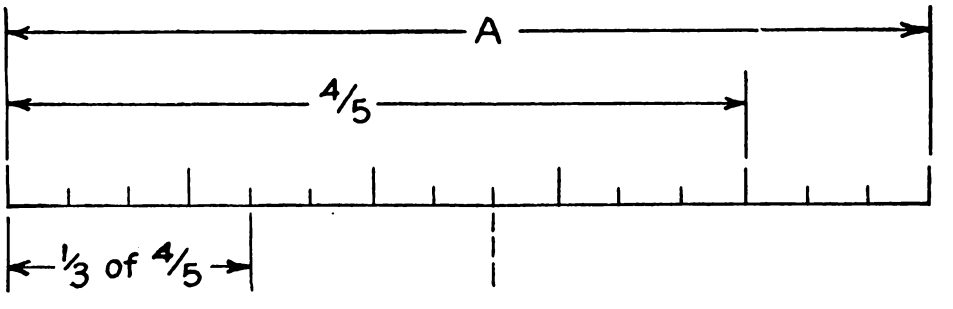


FIG. 6*b*.

Figure 6*b* illustrates the operation of multiplying $\frac{1}{3}$ by $\frac{4}{5}$, or taking $\frac{1}{3}$ of $\frac{4}{5}$. *A* is a unit of length divided into 5 major parts. Four of these 5 major parts, or $\frac{4}{5}$ of *A*, is shown. This $\frac{4}{5}$ of *A*

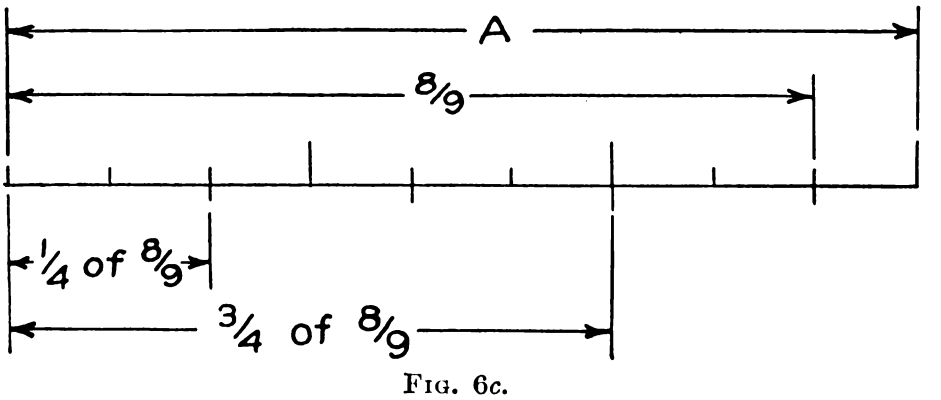


FIG. 6*c*.

is then divided into 3 equal parts, with the first $\frac{1}{3}$ part indicated. It can be seen that the one-third parts of $\frac{4}{5}$ are not contained in *A* a whole number of times. It remains then to divide the one-third part into equal divisions so that these divisions are also

contained in A a whole number of times. This is seen to be 15th's of A and 4th's of the one-third parts. It will be seen also that the number of parts into which A is divided is equal to the L. C. D. of the two fractions, namely, $\frac{1}{3}$ and $\frac{4}{5}$, or 15. $\frac{1}{3}$ of $\frac{4}{5}$, or $\frac{1}{3} \times \frac{4}{5}$, is, thus, $\frac{4}{15}$.

Figure 6*c* illustrates the product of $\frac{3}{4}$ and $\frac{8}{9}$; $\frac{1}{4}$ of $\frac{8}{9}$ was found in the same manner as $\frac{1}{3}$ of $\frac{4}{5}$ was found. Three of these divisions (representing $\frac{1}{4}$ of $\frac{8}{9}$) were then taken to make $\frac{3}{4}$ of $\frac{8}{9}$. By reducing to lowest terms, $\frac{3}{4} \times \frac{8}{9}$ or $\frac{24}{36} = \frac{2}{3}$; $\frac{3}{4}$ of $\frac{8}{9}$ is thus seen to be $\frac{2}{3}$ of A.

24. Multiplying Mixed Numbers.—This is one of the most difficult operations in the study of fractions, unless one adopts a fixed rule and follows it in all cases. The student will have no trouble if he will first reduce the mixed numbers to improper fractions, and then multiply these like any other fractions.

Example:

Find the product of $3\frac{1}{4}$ and $2\frac{1}{2}$.

$$3\frac{1}{4} = \frac{13}{4}; \text{ and } 2\frac{1}{2} = \frac{5}{2}.$$

$$3\frac{1}{4} \times 2\frac{1}{2} = \frac{13}{4} \times \frac{5}{2} = \frac{13 \times 5}{4 \times 2} = \frac{65}{8} = 8\frac{1}{8}, \textit{ Answer.}$$

To multiply a mixed number by a whole number, we can either reduce the mixed number to an improper fraction and then perform the multiplication, or we can multiply the fractional part and the whole number part separately by the number and then add the products.

Example:

What would be the cost of ten $\frac{1}{2}$-in. by 6-in. machine bolts at $1\frac{3}{8}$ cts. apiece?

First method:

$$10 \times 1\frac{3}{8} = 10 \times \frac{11}{8} = \frac{110}{8} = 13\frac{6}{8} = 13\frac{3}{4} \text{ cts.}$$

Second method:

$$\begin{array}{r} 10 \\ 1\frac{3}{8} \\ \hline 3\frac{3}{4} \\ 10 \\ \hline 13\frac{3}{4} \text{ cts., } \textit{Answer.} \end{array}$$

Explanation: First multiply 10 by $\frac{3}{8}$. This gives $\frac{30}{8}$, or $3\frac{6}{8}$, or $3\frac{3}{4}$. Set this down. Then multiply 10 by 1. This gives 10, and we add this to the $3\frac{3}{4}$, giving a total of $13\frac{3}{4}$ cts.

25. Cancellation.—Very often the work of multiplying fractions may be lessened by cancellation, as it avoids the necessity of reducing the product to lowest terms. To get an idea of cancellation we must first understand what a "factor" is. A factor of a number is a number which will divide it evenly. Thus, 2 is a factor of 8, 3 is a factor of 27, 5 is a factor of 35, etc. When the same number will exactly divide two or more numbers, it is called a *common factor* of those numbers. Thus, 2 is a common factor of 8 and 12, because it will divide into both 8 and 12 without leaving a remainder. Four (4) is also a common factor of 8 and 12. Similarly, 7 is a common factor of 14 and 21.

This idea of common factors we have already used in reducing fractions to lowest terms. Thus, when we have $\frac{8}{12}$ we divide both 8 and 12 by 4 and get

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

Cancellation is a process of shortening the work of reduction by removing or cancelling the equal factors from the numerator and denominator.

Suppose we wish to multiply $\frac{3}{4}$ by $\frac{2}{3}$.

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} \qquad \frac{6 \div 6}{12 \div 6} = \frac{1}{2}, \textit{ Answer.}$$

Now, by cancelling out the common factors before multiplying, we can obtain the same product more quickly and with less work, as follows:

$$\begin{matrix} 1 & 1 \\ \cancel{3} & \cancel{2} \\ \cancel{4} & \cancel{3} \\ 2 & 1 \end{matrix} \times \frac{}{} = \frac{1 \times 1}{2 \times 1} = \frac{1}{2}, \textit{ Answer.}$$

Explanation: First the 3 in the numerator and the 3 in the denominator are cancelled. In other words, a common factor, 3, is divided into the 3 in the numerator and the 3 in the denominator; $3 \div 3 = 1$, so we cross out the 3's and place 1's in their places. Then a common factor, 2, is cancelled from the 2 in the numerator and the 4 in the denominator, leaving a 1 in place of the 2 and a 2 in place of the 4. No more factors can be cancelled, so we multiply together the remaining figures above the line for the numerator of the answer, and below the line for the

denominator, giving us the answer, $\frac{1}{2}$, as before. Note that, *whenever a factor is cancelled from the numerator, the same factor must be cancelled from the denominator.*

Examples:

1. Suppose we have several fractions to multiply together, as $\frac{3}{4} \times \frac{2}{3} \times \frac{3}{14} \times \frac{21}{32}$.

Their product is $\frac{3 \times 2 \times 3 \times 21}{4 \times 3 \times 14 \times 32} = \frac{378}{5376}$

Reduced to its lowest terms $\frac{378}{5376} = \frac{9}{128}$, *Answer.*

By cancellation, we could obtain the product in its lowest terms immediately, without reduction, as follows:

$$\begin{matrix} 1 & 1 & & 3 \\ \cancel{3} & \cancel{2} & & \cancel{21} \\ \cancel{4} & \cancel{3} & \cancel{3} & \cancel{14} \\ 2 & 1 & 2 & \end{matrix} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{14}} \times \frac{\cancel{21}}{32} = \frac{1 \times 1 \times 3 \times 3}{2 \times 1 \times 2 \times 32} = \frac{9}{128}, \textit{ Answer.}$$

Explanation: First a 3 in the numerator is cancelled with a 3 in the denominator, leaving a 1 in place of each 3 thus cancelled. Next a common factor 2 is cancelled from the 2 and the 4; in other words, they are both divided by 2. We see that 7 is a common factor of 14 and 21, so we cancel out a 7 by dividing each of these terms by 7. This leaves 2 in place of the 14 and 3 in place of the 21. There are no more common factors; therefore, we multiply together the remaining numbers and find the answer to be $\frac{9}{128}$.

2. Multiply $\frac{500}{250} \times \frac{36}{63} \times \frac{42}{20} \times 50$.

$$\begin{matrix} 1 & & & & \\ \cancel{2} & 4 & 6 & 5 & \\ \cancel{500} & \cancel{36} & \cancel{42} & \cancel{50} & \\ \cancel{250} & \cancel{63} & \cancel{20} & 1 & \\ 1 & \cancel{7} & \cancel{2} & & \\ & & 1 & & \end{matrix} = \frac{120}{1} = 120, \textit{ Answer.}$$

Explanation: First cancel 250 out of 500 and also out of 250, then 9 out of 36 and 63; then 10 out of 20 and 50, and finally 2 out of 2 and 2, and 7 out of 7 and 42.

26. Division—The Reverse of Multiplication.—Division is just the opposite of multiplication, and this fact gives us the cue to a very simple method of dividing fractions.

To divide one fraction by another, invert the divisor and then multiply. To invert means to turn upside down. Invert $\frac{3}{4}$ and we get $\frac{4}{3}$; invert $\frac{1}{2}$ and we get $\frac{2}{1}$.

Example:

Divide $\frac{27}{32}$ by $\frac{3}{4}$.

$$\frac{27}{32} \div \frac{3}{4} = \frac{\overset{9}{\cancel{27}}}{\underset{8}{\cancel{32}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{9}{8} = 1\frac{1}{8}, \textit{ Answer.}$$

Explanation: The divisor is $\frac{3}{4}$. Inverting this gives $\frac{4}{3}$. In multiplying, we make use of cancellation to simplify the work, and we get $\frac{9}{8}$ or $1\frac{1}{8}$ for the result.

Suppose we have a fraction to divide by a whole number; as $\frac{14}{16} \div 2 = ?$ Two (2) is the same as $\frac{2}{1}$. If we invert this we get $\frac{1}{2}$. Therefore,

$$\frac{14}{16} \div \frac{2}{1} = \frac{\overset{7}{\cancel{14}}}{16} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{7}{16}, \textit{ Answer}$$

27. Compound Fractions.—Sometimes we see a fraction which has a fraction for the numerator and another fraction for the denominator. This is called a *Compound Fraction.* If we remember that a fraction indicates the division of the numerator by the denominator, we will see that a compound fraction can be simplified by performing this division.

Example:

What is $\dfrac{\frac{27}{32}}{\frac{3}{4}}$?

This means the same as $\frac{27}{32} \div \frac{3}{4}$ and, therefore, would be solved as follows:

$$\frac{\frac{27}{32}}{\frac{3}{4}} = \frac{27}{32} \div \frac{3}{4} = \frac{\overset{9}{\cancel{27}}}{\underset{8}{\cancel{32}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \frac{9}{8} = 1\frac{1}{8}, \textit{ Answer.}$$

28. How to Analyze Practical Problems.—The chief trouble that students have in working practical problems is in analyzing the problems to find out just what operations they should use to solve them. Problems in multiplication or division of fractions will fall in one of the three following cases:

1. Given a whole; to find a part (multiply).
2. Given a part; to find the whole (divide).
3. To find what part one number is of another (divide).

Example of Case 1:

The total weight of a shipment of steel bars is 3425 lb. $\frac{7}{25}$ of this consists of $\frac{3}{4}$-in. round bars and the balance is $\frac{1}{2}$-in. round. What weight is there of each size?

$$\frac{7}{\cancel{25}} \times \frac{\overset{137}{\cancel{3425}}}{1} = 959 \text{ lb. of } \tfrac{3}{4}\text{-in. bars.}$$

$3425 - 959 = 2466$ lb. of $\frac{1}{2}$-in. bars.

Or

$$\frac{18}{\cancel{25}} \times \frac{\overset{137}{\cancel{3425}}}{1} = 2466 \text{ lb. of } \tfrac{1}{2}\text{-in. bars.}$$

Explanation: In this example we have the whole (3425 lb.); to find a part $\left(\frac{7}{25}\right)$. If the whole is 3425, then $\frac{7}{25}$ of 3425 = 959 lb. The balance, which consists of $\frac{1}{2}$-in. bars, will be $3425 - 959$. Or it will be $1 - \frac{7}{25} = \frac{25}{25} - \frac{7}{25} = \frac{18}{25}$ of the whole. $\frac{18}{25}$ of 3425 = 2466 lb.

Example of Case 2:

The base of a dynamo weighs 270 lb.; the base is $\frac{3}{11}$ of the total weight; find the total weight.

$\frac{3}{11}$ of the whole = 270 lb.

$$\text{whole} = 270 \div \frac{3}{11}$$

$$270 \div \frac{3}{11} = \frac{\overset{90}{\cancel{270}}}{1} \times \frac{11}{\cancel{3}} = 990 \text{ lb., } \textit{Answer.}$$

Explanation: Here we have a part given to find the whole. $\frac{3}{11}$ of the whole is 270 lb. This means that if the whole machine were divided into 11 equal parts, 3 of these parts together would weigh 270 lb. Then 1 part would weigh $270 \div 3 = 90$ lb. Since there are 11 of these equal parts, the whole machine weighs

$$11 \times 90 = 990 \text{ lb.}$$

This is the same as dividing 270 by the fraction $\frac{3}{11}$.

Example of Case 3:

A molder who is on piece work sets up 91 flasks, but the castings from 7 of them are defective. What fractional part of his work does he get paid for?

$91 - 7 = 84$ sound castings

$\frac{84}{91} = \frac{84 \div 7}{91 \div 7} = \frac{12}{13}$, *Answer.*

Explanation: The problem is: What part of 91 is 84? There are 91 parts in his whole work and he gets paid for 84 parts, or $\frac{84}{91}$ of the whole. This can be reduced to $\frac{12}{13}$, which is the answer.

PROBLEMS

61. $7 \times \frac{3}{16} = ?$

62. $8 \times 1\frac{7}{32} = ?$

63. (*a*) $2\frac{5}{56} \times \frac{28}{65} = ?$ (*b*) $33\frac{11}{16} \times 1\frac{3}{19} = ?$

64. (*a*) $\frac{2}{3} \times \frac{3}{8} \times \frac{5}{7} = ?$ (*b*) $13\frac{1}{3} \times \frac{4}{7} \times \frac{1}{2} = ?$

65. Multiply $\frac{7}{8} \times \frac{9}{32} \times \frac{16}{27} \times 41\frac{4}{14}$.

66. Multiply $2\frac{3}{32}$ by $\frac{5}{16}$.

67. Find the product of $\frac{3}{4}$ and $\frac{5}{8}$.

68. Does $\frac{1}{2} \times 6 = \frac{6}{3} + \frac{3}{2}$?

69. What is $\frac{3}{8}$ of $2\frac{2}{3}$? Work by cancellation.

70. What is $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{5}{16}$ of $\frac{4}{5}$? Work by cancellation.

71. Multiply $81 \times \frac{3}{9} \times 1\frac{3}{27} \times \frac{150}{65}$. Work by cancellation.

72. Perform the indicated operation:

$$\frac{\frac{1}{4} \times \frac{6}{9} \times \frac{8}{12}}{\frac{7}{8} \times \frac{1}{8} \times \frac{31}{21}}$$

73. Perform the indicated operation:

$$\frac{\frac{1}{4} + \frac{1}{8} + \frac{7}{16}}{\frac{7}{8} \div \frac{1}{16}} \times \frac{\frac{4}{5} \times \frac{2}{7} \times \frac{1}{2}}{\frac{7}{16} - \frac{1}{24}}$$

74. Which is greater $\frac{7}{16} \times 4$ or $\frac{7}{16} \div \frac{1}{3}$?

75. Divide 175 by $\frac{3}{4}$.

76. Divide $1\frac{5}{16}$ by $1\frac{7}{8}$.

77. Divide $3\frac{3}{4}$ by $5\frac{4}{5}$.

78. Find the quotient of $21 \div \frac{7}{8}$.

79. $\frac{\frac{7}{8}}{\frac{3}{4}} = ?$

80. A gallon is about $\frac{2}{5}$ of a cubic foot. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how much does a gallon weigh?

81. A machine part requires two "finishing" operations on a milling machine, and the cost of these two operations on every piece is $\frac{1}{3}$ of the total cost of manufacturing. If the total cost of each is 60 cts., what will be the cost of finishing operations on 10,000 of these parts?

82. If a certain sized steel bar weighs $2\frac{1}{5}$ lb. to the foot, how long must a piece be to weigh $8\frac{1}{4}$ lb.?

83. How many steel pins to be finished $1\frac{1}{8}$ in. long can be cut from an 8-ft. rod if we allow $\frac{3}{16}$ in. to be wasted in cutting off and finishing each pin?

84. A steam engine during a test developed $148\frac{1}{2}$ horsepower. If the steam consumed by the engine in a hour was 2277 lb., how much steam did it consume per horsepower hour?

85. A firm manufacturing pumps finds that it costs $7\frac{1}{3}$ cts. a pound to make its castings. A near-by foundry and machine company offers to make the castings at a price of $5\frac{3}{4}$ cts. per pound. The actual cost per pound, however, when the castings are bought from this firm is found to be $1\frac{1}{3}$ times the net cost of $5\frac{3}{4}$ cts. Is it cheaper for the pump company to buy its castings or to make them in its own foundry?

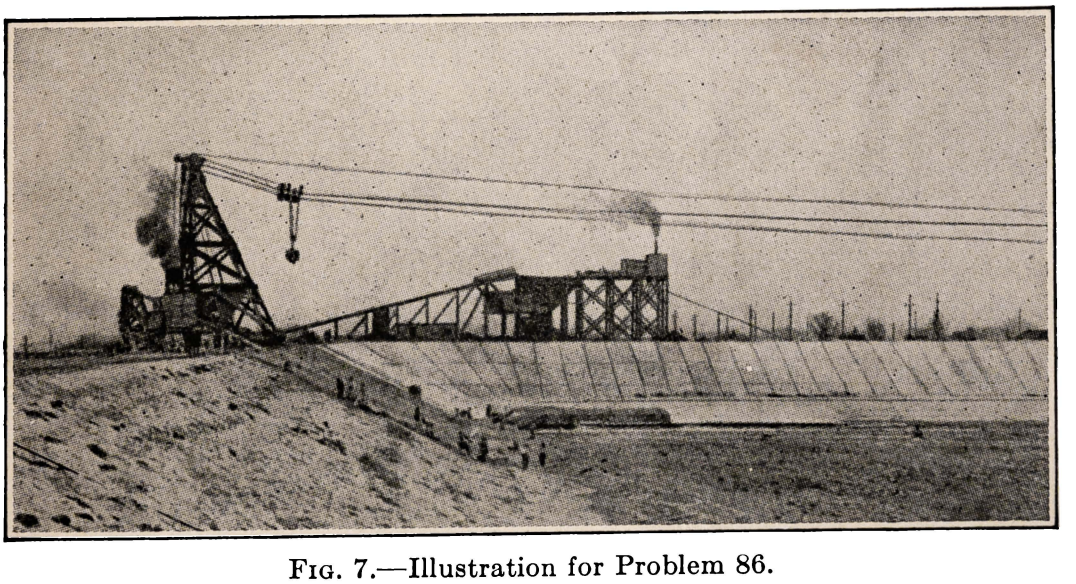


FIG. 7.—Illustration for Problem 86.

86. It was estimated that the cableway of Fig. 7 could convey 24 loads of excavated earth per hour, each load containing about $2\frac{1}{4}$ cu. yd. If the crew works $9\frac{1}{3}$ hr. daily, how many days will it take to remove an excavation of about 12,000 cu. yd.?

87. I want to cut 300 pieces of steel each 112 in. long for wagon tires. I have in stock a sufficient number of bars of the same size but they are 120 in. long; and I also have a sufficient number 235 in. long. Which length should I use in order to waste the less material? Calculate the total number of inches of stock that would be wasted in each case.

88. The outside circumference of the tires on the wheels of an automobile is $83\frac{1}{2}$ in. How many revolutions per minute do the wheels make when the car is traveling at a speed of 30 miles per hour?

89. A man withdrew a sum of money from the bank. He lost $\frac{3}{4}$ of the sum which he withdrew. Later he found $\frac{2}{5}$ of the sum which he lost. Adding the amount he found to the sum which he did not lose, he had a total of \$44. How much did he draw from the bank?

90. The cost of two loads of coal, one load containing $1\frac{5}{12}$ tons and the other $1\frac{2}{3}$ tons, is \$31. At the same rate per ton, how much will $5\frac{1}{2}$ tons cost?

CHAPTER IV

MONEY AND WAGES

29. United States Money.—Nearly every country has a money system of its own. The unit of money in the United States is the *dollar.* To represent parts of a dollar, we use the *cent,* which is $\frac{1}{100}$ of a dollar. Fifty cents is $\frac{50}{100}$ of a dollar; it is also one-half dollar ($\frac{50}{100} = \frac{1}{2}$). Likewise, twenty-five cents is $\frac{25}{100}$ dollar, or one-quarter dollar.

In writing the symbol for United States money, the dollar sign ($) is written before the number; a period, called the *decimal point,* is placed after the number of dollars; following this decimal point is placed the number of cents.

Two dollars and seventy cents is written $2.70
Fifteen dollars and seven cents is written $15.07
One hundred twenty-five dollars is written $125.00
One dollar and twenty-five cents is written $1.25
Thirty-five cents is written $.35
Eight cents is written $.08

Since one cent is $\frac{1}{100}$ dollar, it follows that the figures to the right of the decimal point represent a fraction of a dollar. These figures are the numerator, and the denominator is 100.

$ 2.70 is the same as $ $2\frac{70}{100}$
$15.07 is the same as 15\frac{7}{100}$
$ .08 is the same as $ $\frac{8}{100}$

The first figure following the decimal point can be said to indicate the number of dimes, because 1 dime = 10 cents, and this figure indicates the number of tens of cents. Also this number represents tenths of a dollar, because 1 dime = 10 cents = $\frac{10}{100}$ dollar = $\frac{1}{10}$ dollar. The second figure after the decimal point indicates cents, or hundredths of a dollar.

This decimal system of writing amounts of money has great advantages in performing the operations of addition, subtraction, multiplication, and division, because we can perform these opera-

tions just as if we were dealing with whole numbers, which makes the work much simpler than if we had fractions to deal with.

30. Addition.—We can add numbers made up of dollars and cents and carry forward just as in simple addition. The number of *tens* of cents will represent dimes (10 cents = 1 dime; 30 cents = 3 dimes, etc.) and thus can be carried forward and added into the dime column. Likewise, the number of *tens* of dimes will represent dollars (10 dimes = 1 dollar) and, therefore, this number can be added into the dollar column.

Example:

Add \$5.20, \$2.65, \$3.25, and \$.35.

\$ 5.20	*Explanation:* Adding the cents column, we get 5 + 5 + 5 + 0 = 15 cts. Put down the 5 and carry the 1 into the next column (since 15 cts. = 1 dime + 5 cts.) Adding the dimes, we get 1 + 3 + 2 + 6 + 2 = 14 dimes. Put down the 4 and carry the 1 into the dollar column (since 14 dimes = \$1 + 4 dimes). 1 + 3 + 2 + 5 = 11 dollars, which we put down complete. The decimal point we now place in the sum directly under the decimal points in the numbers added, so that it properly separates dollars from cents.
2.65	
3.25	
.35	
\$11.45, *Answer.*	

The only precaution to be observed is to see that the dollars, dimes, and cents are properly lined up vertically before adding. To do this it is only necessary to see that the decimal points are kept in a straight vertical line.

Example:

What is the total cost of three articles priced as follows: \$2.25, \$1, and \$1.75?

\$2.25	*Explanation:* Here the \$1 does not have any decimal point or cents after it, and care should be taken to see that it is put down in the dollar column and not in the cents column. \$1 may be written \$1.00, if desired, to avoid any danger of a mistake.
1	
1.75	
\$5.00, *Answer.*	

31. Subtraction.—The same rules should be followed in subtracting. If any figure in the subtrahend is larger than the corresponding figure in the minuend, we can borrow 1 from the figure to the left, just as in ordinary subtraction.

Example:

A man draws $49.50 on pay day and immediately pays bills amounting to $17.72. How much does he have left to put in the bank?

$49.50
17.72
$31.78, *Answer.*

Explanation: Having set down the numbers properly, we subtract just as if we were subtracting whole numbers. The only difference is that, after subtracting, we put the decimal point in the remainder directly below the other decimal points, to separate the dollars and cents in the remainder.

32. Multiplication.—In multiplying an amount of money by any number, the process is the same as in simple multiplication, remembering, however, that the decimal point separates the dollars from the cents. The reasoning for this is just the same as in addition. Multiplying cents, gives cents; multiplying dimes, gives dimes; and multiplying dollars, gives dollars. The figures left over are carried forward just as in plain multiplication, because 10 cents = 1 dime, and 10 dimes = 1 dollar.

Examples:

1. What should a machinist receive for finishing 48 gas engine pistons at 25 cts. each?

$.25
48
200
100
$12.00, *Answer*

Explanation: The 25 cts. is put down as $.25. The multiplication of 25 by 48 is performed as usual. Then the decimal point is placed in the product to separate the dollars and cents by leaving the two places, counting from the right, to represent cents.

2. If you owe $2\frac{1}{2}$ weeks' board at $6.25 a week, how much money will it take to settle the bill?

$ 6.25
$2\frac{1}{2}$
$3.12\frac{1}{2}$
12.50
$15.62\frac{1}{2}$

Explanation: First, we put down the numbers and then multiply as if we were simply multiplying $625 \times 2\frac{1}{2}$. The product is $1562\frac{1}{2}$. But since we are dealing with dollars and cents, we must put the decimal point in the product to show what part of it represents dollars and what part cents.

33. Division.—In dividing an amount of money by any number, the division is carried out as in ordinary division. The decimal point is then placed in the quotient in the same position (from the right) that it had in the dividend.

Example:

The weekly payroll of a company employing 405 men is $10,319.40. What is the average amount paid to each man?

```
405)$10319.40($25.48, Answer.
    810
    ----
    2219
    2025
    ----
     1944
     1620
     ----
     3240
     3240
     ----
```

Explanation: The division is carried out just as if we were dividing 1,031,940 by 405. The quotient obtained is 2548. Then, since two of the figures in the dividend were cents, we place the decimal point in the quotient so as to point off the cents, making the quotient read $25.48.

Another way of locating the decimal point is to place it in the quotient as soon as the number of dollars in the dividend has been divided. Taking the same example; we first divide 405 into 1031 and get 2, with a remainder of 221. Annexing the next figure, 9, and dividing again, we get 5 for the quotient. We have now divided the number of whole dollars (10,319) and have $25 for the quotient, with a remainder of 194. The 25 is, therefore, the number of whole dollars in the quotient. We now bring down the next figure (4) from the dividend and find that 405 goes into 1944, 4 times, so we have 4 dimes. Then bringing down the 0 cents, we get 3240 cents, which, divided by 405, gives just 8 cents. The men, therefore, get an average of $25.48 each per week.

34. Reducing Dollars to Cents.—Sometimes we find it desirable to change a number of dollars and cents all into cents. To do this, merely remove the decimal point from between the dollars and cents and you will have the number of cents. Everyone knows that:

$1.00 is 100 cents
$1.25 is 125 cents
$ .25 is 25 cents

Likewise:

$ 12.75 is 1275 cents
$ 247.86 is 24786 cents
$1000.00 is 100000 cents

What we have really done in making these changes is to multiply the dollars by 100 to get the equivalent cents. We have taken a mixed number and multiplied it by 100 because there are 100 cents in a dollar. This operation is performed by moving the decimal point two figures to the right, or placing

it after the cents, where it is, of course, useless and is seldom written.

In many problems it is very desirable to change the dollars to cents and carry the work through as cents. The following example shows clearly such a case.

Example:

During one month a foundry turned out 312,000 lb. of iron castings. The total cost of the iron used, including the cost of melting and pouring, was \$3900. What was the cost, in cents, of 1 lb. of iron, melted and poured?

\$3900 = \$3900.00 = 390,000 cts.

$390{,}000 \div 312{,}000 = 1\frac{78}{312} = 1\frac{1}{4}$ cts., *Answer.*

Explanation: Since the cost of iron per pound melted and poured is but 1 or 2 cts., we might as well change the total cost to cents before we divide by the number of pounds. Then we will get the cost directly in cents per pound, as we want it.

35. Reducing Cents to Dollars.—The reduction of cents to dollars is really performed by dividing the number of cents by 100, since there are 100 cents in 1 dollar.

$$217 \text{ cents} = \frac{217}{100} \text{ dollars} = \$2\frac{17}{100} = \$2.17$$

Hence, 217 cents = \$2.17

This shows us that the following simple rule can be adopted for this reduction:

To reduce cents to dollars, place a decimal point in the number so as to have two figures to the right of the decimal point.

36. The Mill.—There is another division of United States money called the *mill.* A mill is one-tenth of a cent or one one-thousandth of a dollar.

10 mills = 1 cent
100 mills = 10 cents = 1 dime
1000 mills = 100 cents = 10 dimes = 1 dollar

There is no coin smaller than the cent and, therefore, the mill is merely a name applied in calculations where it is desirable to have some unit smaller than the cent. For example, tax rates are usually given in mills per dollar. A tax rate of 15 mills on the dollar would mean that a person would have to pay 15 mills (or $1\frac{1}{2}$ cents) on each dollar of assessed valuation.

Cost accountants generally figure costs down as fine as mills and even, in some cases, to tenths of mills or finer.

In sums of money containing mills, there are three figures following the decimal point. The first and second figures after the decimal point indicate dimes and cents, as before. The third figure indicates mills.

$.014 is 1 cent and 4 mills
It is also 14 mills (since 1 cent = 10 mills).

In multiplying or dividing numbers containing mills, we must place the decimal point in the answer in the same position, that is, three places from the right of the number.

Example:

If your house and lot were assessed at $2000, and the tax rate was 15 mills on the dollar, what would be the amount of your taxes?

15 mills = $.015.

$.015	*Explanation:* 15 mills is 1 ct. and 5 mills, or $.015. This is the amount you must pay on each dollar of assessed value. For an assessment of $2000, you would pay 2000 times $.015, which is $30.
2000	
$ 30.000, *Answer.*	

37. Wage Calculations.—The chief use that shop men have, within the shop, for calculations concerning money is in connection with their time and wages. With some wage systems the calculation of one's earnings is comparatively simple; with others it seems rather complicated until the systems are thoroughly understood. The simplest systems are, of course, the well-known day-rate and piece-rate systems. By the old day-rate system, the men are paid according to the time put in, without any reference to the work accomplished. The rate may be so much per hour, per day, or per week, but the method of calculating is the same, and the timekeeper will use exactly the same process in each case. The payroll calculations consist merely in multiplying the number of units of time which each man has to his credit by his rate per unit of time; hours by rate per hour, or days by rate per day.

Examples:

1. A machinist puts in 106 hours at 90 cts. per hour. How much money is due him from the company?

106	*Explanation:* The amount due is the product of 106 × .90. Since the amount will run into dollars, we write the 90 cts. as dollars, $.90. The product is $95.40, which is the amount due.
$ .90	
000	
954	
$95.40, *Answer.*	

2. The toolroom foreman gets \$8 a day and has worked 12 days. What is the amount due him?

12
\$8
\$96, *Answer.*

Explanation: The same process is carried out here except that we have the rate per day times the number of days.

The piece-work system, by setting prices for certain pieces of work and paying according to the work done, rewards the man in exact proportion to the work that he does. In this system, an account is kept of the amount of a man's work, and his pay is calculated by multiplying the numbers of pieces by the piece rates.

Example:

The price for assembling a certain sized commutator is 73 cts. If a man assembles an order of 22, how much does he receive? What will he average per day if he does the job in 3 days?

73 cts. = \$.73
\$.73 × 22 = \$16.06, amount he receives for job.
\$16.06 ÷ 3 = \$5.35$\frac{1}{3}$, amount he averages per day.

There are many other wage systems, too numerous, however, to be explained here. They all are planned to take into account both the amount of work a man does and the time that he puts in to do it. One of these systems, the Premium System, is so well known and so successful as to warrant brief mention here. In its most common form this system is as follows:

The men are all placed on a time rate, usually a rate per hour.

A record is kept of every man's time and also of the work done by him.

Every job has a standard time (sometimes called "the limit") allowed for its completion.

On each job, the man's actual time is recorded and also the limit for the job.

The man is paid straight wages for the time put in on the job and, in addition, is paid a premium of usually one-half of the time that he saves below the limit.

If it takes a man a longer time than the limit, he is paid full wages for the time he puts in.

This system is planned to satisfy both parties by giving the efficient man an increased earning, and by giving the firm a share of the time saved. This gives a reduced cost whenever higher wages are paid.

Let us take an example to see how one's pay would be figured on this plan.

Example:

In 1 day a man whose rate is 75 cts. an hour does the following premium jobs: the first has a limit of 7 hr. and is done in 5 hr.; the second has a limit of 3 hr. and is done in 2 hr.; the third has a limit of 2 hr. and is finished in 1 hr. What is his pay for the day?

$$5 + 2 + 1 = 8 \text{ hr. actually put in.}$$
$$7 - 5 = 2 \text{ hr. saved on first job.}$$
$$3 - 2 = 1 \text{ hr. saved on second job.}$$
$$2 - 1 = 1 \text{ hr. saved on third job.}$$
$$2 + 1 + 1 = 4 \text{ hr. saved on the day's work.}$$

He will get paid for the 8 hr. and, in addition, for half of the 4 hr. that he saved. Altogether he will be paid for $8 + \frac{1}{2}$ of $4 = 10$ hr.

$$10 \times \$.75 = \$7.50, \textit{Answer.}$$

He gets \$7.50 for his 8 hr. work and, therefore, makes a premium of \$1.50. Meanwhile, the company gets the work done for \$7.50 instead of paying \$9.00 which it would have cost if the workman had taken the full time limit for the work.

PROBLEMS

91. Write in figures the following amounts of money:

One dollar and twelve cents
Two dollars and twenty-five cents
Eight cents
Fifteen dollars and thirty-seven and one-half cents
Twenty-five mills

92. Add the amounts of Problem 91.

93. Read the following, and write them out in words:

\$ 2.75 \$ .008

\$ $.03\frac{1}{2}$ \$1.08

\$16.25

94. Find the sum of the amounts in Problem 93, and subtract from it the answer to Problem 92.

95. What is the difference between \$24.00 and \$13.07½?

96. In the manufacture of a certain piece of machinery, the cost of producing the rough castings in the foundry is \$12.35, the cost of finishing the parts in the machine shop is \$46.12, the cost of assembling the completed parts is \$3.08, and all other expenses involved in its manufacture, included in the term "overhead,"

amount to $6.92. If the manufacturer desires to make a profit of $11.53 on each machine sold, for what price does he sell the machine?

97. A man bought a small supply of lumber at $40 per thousand board feet. The itemized bill included 60 bd. ft. of 2 in. by 8 in. planks, 40 bd. ft. of 2 in. by 4 in., and 100 bd. ft. of 1 in. by 8 in. ship-lap. What was the total cost of the lumber?

ERECTING SHOP—TWO WEEKS ENDING JAN. 24, 1931

Name	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total hours	Rate per hour	Amount for the 2 weeks
Randall, J. F.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	1.00	
Nelson, O. C.	..	8	8	8	8	8	4	..	4	..	8	8	8	4	..	.85	
Baker, J. C.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.75	
Carter, L. B.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.75	
Mullen, T. F.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.75	
Holmes, Wm.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.75	
Murphy, P. J.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.65	
Kelly, T. P.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.65	
Bodenstab, J.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.50	
Schaub, F. T.	6	8	8	8	8	8	10	6	8	8	8	8	8	4	..	.50	
Strozinski, P.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.40	
Rynkiewicz, J. D.	..	8	8	8	8	8	4	..	8	8	8	4	8	4	..	.40	
Skawaske, G. J.	..	8	8	8	8	8	4	..	8	8	8	8	8	4	..	.40	
Total																	

FIG. 8.—Payroll for erecting shop.

98. The average yield of corn in the United States for a recent year was $25\frac{9}{10}$ bu. per acre; the average value per bushel was $.618. The value of the corn raised averaged how much per acre?

99. A certain job calls for four $\frac{3}{4}$ in. by 3 in. machine bolts, two $\frac{5}{8}$ in. by $1\frac{3}{4}$ in. set screws, and two $\frac{1}{2}$ in. by 2 in. cap screws. What will be the total cost of the bolts and screws if the machine bolts are worth $2\frac{1}{4}$ cts. each, the set screws $1\frac{1}{4}$ cts. each, and the cap screws $1\frac{3}{4}$ cts. each?

100. In putting up a radio aerial a man used the following: 125 ft. of wire costing 3 mills a ft.; 10 porcelain insulators at $.25 apiece; and two 25 ft. poles at $.75 apiece. How much did this material cost?

101. If you bought a house for $6000 and it was assessed at two-thirds of what it cost you, what taxes would you have to pay if the tax rate was 34 mills on the dollar?

102. A painter bids on a contract to paint six bungalows. By a rough estimate he concludes that there are about 2500 sq. ft. of surface

to be painted in each bungalow, and he figures that a gallon of paint will cover about 200 sq. ft. allowing for three coats of paint. If paint costs $3.50 per gallon, what is the total estimated cost of the paint alone?

103. An apprentice and a machinist are working together on a piece-work job and they earn $30. They are prorated on the job, which means that the money is divided according to their day work rates. If the apprentice's rate is 30 cts. an hour and the machinist's is 90 cts. an hour, what fraction of the money will each get, and how much money will each get?

104. A man rated at 75 cts. an hour is working under the premium system. In one day of $9\frac{1}{2}$ hr. he performs three operations, the limits of which are $4\frac{1}{2}$ hr. each. If he gets paid a premium of half the time saved, how much will he make for the day?

105. Figure 8 represents a page from a time book of a shop working entirely on day work. Calculate the total time of each man, the amount of money due him, and the total payroll for the shop.

CHAPTER V

DECIMAL FRACTIONS

38. What Are Decimals?—In the old days, when no machinist pretended to work much closer than $\frac{1}{64}$ in. and the micrometer was unknown, the mechanic had little use for decimals except in figuring his pay. Now, however, we find that micrometer measurements are used so generally that a knowledge of decimal fractions is essential.

A *Decimal Fraction* is merely a fraction having a denominator of 10, 100, 1000, or some similar multiple of 10. The denominator is never written, however, but a system similar to that used in writing United States money is used. A decimal fraction is written by first putting down a period or "decimal point" and then writing the numerator of the fraction after the decimal point in such a manner that the denominator can be understood. Everything that comes after the decimal point (to the right of it) is a fraction, or part of a unit.

In writing sums of money, the first figure after the decimal point indicates dimes or tenths of a dollar; the second figure indicates cents, or hundredths of a dollar; the third figure, if any, indicates mills or thousandths of a dollar. This system has proved so convenient that it has been extended to represent fractions of any sort of a unit (not necessarily dollars).

Thus

$$.08 \text{ in. means } \frac{8}{100} \text{ in.}$$

$$.25 \text{ in. means } \frac{25}{100} \text{ in.}$$

$$1.256 \text{ in. means } 1\frac{256}{1000} \text{ in.}$$

Let us take a decimal, say .253, and find out its meaning. We said that the first figure was tenths; the second, hundredths, the third, thousandths; and so on. Then .253 would be $\frac{2}{10} + \frac{5}{100} + \frac{3}{1000}$. This is not a very handy system unless there is

some easier way to read it. If we reduce these to a common denominator and add them, we get:

$$.253 = \frac{2}{10} + \frac{5}{100} + \frac{3}{1000} = \frac{200}{1000} + \frac{50}{1000} + \frac{3}{1000} = \frac{253}{1000}$$

Then, .253 is $\frac{253}{1000}$

This shows that one place to the right of the decimal point indicates a denominator of 10, two places a denominator of 100, three places a denominator of 1000, and so on. In other words, the denominator of a decimal fraction contains as many zeros as there are figures to the right of the decimal point in the decimal number. For example,

.2 is $\frac{2}{10}$, or *two-tenths.*

.37 is $\frac{37}{100}$, or *thirty-seven one-hundredths.*

.526 is $\frac{526}{1000}$, or *five hundred twenty-six one-thousandths.*

.2749 is $\frac{2749}{10{,}000}$, or *two thousand seven hundred forty-nine ten-thousandths.*

.042 is $\frac{42}{1000}$, or *forty-two one-thousandths.*

The last case (.042) presents an interesting problem. Here we have a numerator so small in respect to the denominator that it is necessary to have a cipher, or zero (0) between it and the decimal point, in order that the denominator can be indicated correctly. Let us see how we would go about writing such a common fraction as a decimal. Take $\frac{5}{1000}$. If we merely wrote .5 that would be $\frac{5}{10}$ and would, therefore, not be right. From the rule for finding denominators of decimals we see that there must be as many figures after the decimal point as there are ciphers in the denominator. In this case the denominator (1000) has 3 ciphers, so we must have 3 figures in our decimal. We, therefore, put 2 ciphers to the left of the 5 and then put down the decimal point. We now have .005, which can be easily seen to be $\frac{5}{1000}$.

One thing that must be carefully borne in mind is that adding ciphers *after* a decimal does not change the value of the fraction .5 is the same in value as .50 or .500, because $\frac{5}{10}$ is the same in value as $\frac{50}{100}$ or $\frac{500}{1000}$. On the other hand, ciphers immediately

following the decimal point do affect the value of the fraction, as has just been shown.

Mixed numbers are especially easy to handle by decimals, because the whole number and the fraction can be written out in a horizontal line with the decimal point between them. We read mixed decimals just as we would any mixed number—first the whole number, then the numerator, and lastly the denominator.

Example:

42,137.24697.

In this example, 42,137 is the whole number, and .24697 is the fraction. The number reads "forty-two thousand one hundred thirty-seven *and* twenty-four thousand six hundred ninety-seven *hundred-thousandths.*

The names and places to the right and left of the decimal point are as follows:

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
7	6	5	4	3	2	1	.	1	2	3	4	5	6

When decimal fractions are written in words, the word "and" is used to imply the decimal point as indicated in the following:

2.5 is read "two *and* five-tenths."

652.47 is read "six hundred fifty-two *and* forty-seven hundredths."

42,123.9674 is read "forty-two thousand one hundred twenty-three, *and* nine thousand six hundred seventy-four ten-thousandths."

39. Addition and Subtraction.—Knowing that all figures to the right of the decimal point are decimal parts of one thing and that all figures to the left are whole numbers and represent whole things, it will be seen readily that in addition and subtraction the figures must be so placed that the decimal points come under each other. As was shown under United States Money, the

operations can then be carried out just as if we were dealing with whole numbers.

Examples:

Addition	Subtraction
783.5	22.7180
21.473	1.7042
804.973	21.0138

Pay no attention to the number of figures in the decimal. Place the decimal points in line vertically. You can, if you desire, add ciphers to make the number of decimal places equal in the two numbers. Remember, however, that the ciphers must be added to the *right* of the figures in the decimal. Proceed as in ordinary addition and subtraction, carrying the tens forward in addition, and borrowing, where necessary, in subtraction just as with whole numbers.

40. Multiplication.—In multiplication forget all about the decimal point until the work is finished; multiply as usual with whole numbers. Then point off in the product as many decimal places, counting from the right, as there are decimal places in the multiplier and multiplicand together.

Example:

6.685	Multiplicand	(3 places)
5.2	Multiplier	(1 place)
13370		
33425		
34.7620	Product	(4 places)

Explanation: Since there are three decimal places in one number, and one in the other, we count off in the product four (3 + 1) places from the right and place the point between the 7 and the 4. The last 0 can be dropped after pointing off the product, giving the result 34.762 (or $34\frac{762}{1000}$). The reason for this can be seen from the following: The whole numbers are 6 and 5. The result must be a little more than $6 \times 5 = 30$, and less than $7 \times 6 = 42$, since the numbers are more than 6 and 5, and less than 7 and 6. The actual result is 34.762.

The position of the decimal point can be reasoned out in this way for any example, but the quickest way is to point off from the right a number of decimal places equal to the sum of the numbers of decimal places in the multiplier and multiplicand.

Examples:

(*a*) .0045 × 2.7

.0045 (4 places)
2.7 (1 place)
315
90
.01215 (5 places)

(*b*) .000402 × 4.26

.000402 (6 places)
4.26 (2 places)
2412
804
1608
.00171252 (8 places)

In the above examples it was necessary to put ciphers before the product in order to get the required number of decimal places. To see the reason for this take a simple example such as .2 × .3. The product is .06 or $\frac{6}{100}$, as can be readily seen if they are multiplied as common fractions ($\frac{2}{10} \times \frac{3}{10} = \frac{6}{100}$). This checks with the rule of adding the number of decimal places in the two numbers to get the number in their product. The product of two proper fractions is always less than either of the fractions, because it is part of a part.

41. Short Cuts.—If we want to multiply or divide a decimal by 10, 100, 1000, or any similar number, the process is very simple. Suppose we had a decimal .145 and then moved the decimal point one place to the right and made it 1.45. The number would then be $1\frac{45}{100}$ or $\frac{145}{100}$ instead of $\frac{145}{1000}$, so we see that moving the decimal point one place to the right has multiplied the original number by 10. Therefore, we see that:

To multiply by 10 move the decimal point *one* place to the *right.*

To multiply by 100 move the decimal point *two* places to the *right.*

For other similar multipliers move the decimal point one place to the right for each cipher in the multiplier. This process is reversed in division, the rules being:

To divide by 10 move the decimal point *one* place to the *left.*

To divide by 100 move the decimal point *two* places to the *left,* etc.

Example:

Reduce 10,275 cts. to dollars.

10,275 ÷ 100 = $102.75 (decimal point moved two places to left).

42. Division.—The division of decimals is just as easy as the multiplication of them after one learns to forget the decimal point entirely until the operation of dividing is finished. Divide as in simple numbers, disregarding the decimal point. Then to locate the decimal point, subtract the number of decimal places

in the divisor from the number of decimal places in the dividend. The difference is the number of decimal places in the quotient.

Example:

Divide 105.587 by .93

.93)105.587(113.5 +, *Answer.*

93

125

93 Dividend, 3 places

328 Divisor, 2 places

279 Quotient, 1 place

497

465

32, *Remainder.*

Explanation: The number of places in the dividend is 3, and in the divisor 2. Hence, when the division is completed, we point off 3 − 2 = 1 place in the quotient. The + sign after the quotient means that the division did not come out even, but that there was a remainder and that the quotient given is not complete. If desired, ciphers could be placed after the dividend and the division carried further, giving more decimal places in the quotient.

If there are more decimal places in the divisor than in the dividend, as, for example, .0064 ÷ .00008, then a few ciphers can be added to the right of the dividend, making it .006400 ÷ .00008. This does not change the value of the dividend, and makes it possible to perform the division and locate the decimal point as before.

Example:

0064 ÷ .00008 = ?

.00008).006400(80.0, *Answer.*

64

0

0

0

0

Dividend = 6 places

Divisor = 5 places

Quotient = 1 place

When counting the decimal places, be sure to count only the ciphers actually used in the division.

It makes no difference if the divisor is larger than the dividend, for in such a case the quotient will be entirely a decimal.

Example:

22.762 ÷ 84.25 = ?

84.25)22.762000(.2701 + or .2702, *Answer.*

16 850

5 9120

5 8975

14500

8425

6075

Explanation: The divisor being larger than the dividend, the quotient turns out to be an entire decimal. In this case we will presume that we wanted the answer to 4 decimal places. We have, therefore, added ciphers to the dividend until we have 6 decimal places. When these have all been used in the division, we have 6 − 2 = 4 places in the quotient. The remainder is more than half of the divisor, showing that if we had carried the division to another place, the next figure would have been more than 5. We, therefore, raise the last figure (1) of the quotient to 2, because this is nearer the exact quantity.

In ending any division this way, if the next figure of the quotient would be less than 5, let the quotient stand as it is, but if the next figure would be 5 or more, as in the example just worked, increase the last figure of the quotient to the next higher figure.

Sometimes the decimal places are equal in dividend and divisor, as, for instance, if we divide .28 by .07.

$$.07\overline{).28}$$
$$4$$

As the numbers of decimal places in the dividend and divisor are the same, the difference between them is zero, and there are no decimal places in the quotient. The answer is simply 4. The decimal point would come after the 4, where it would, of course, be useless.

43. Reducing Common Fractions to Decimals.—Common fractions are easily reduced to decimals by dividing the numerator by the denominator. In the case of $\frac{1}{2}$, we divide 1.0 by 2 and get .5. All that is necessary is to take the numerator and place a decimal point after it, adding as many ciphers to the right of the decimal point as are likely to be needed, 4 being a common number to add, as 4 decimal places (ten thousandths) are accurate enough for almost any calculations.

If $\frac{1}{32}$ is to be reduced to a decimal, the work is simply an example in long division, the placing of the point being the main thing to consider. Simply divide 1.00000 by 32. This gives .03125, or 3125 one-hundred-thousandths.

$$32\overline{)1.00000(.03125}$$
$$\begin{array}{r} 96 \\ \hline 40 \\ 32 \\ \hline 80 \\ 64 \\ \hline 160 \\ 160 \\ \hline \end{array}$$

44. Complex Decimals.—A complex decimal is a decimal with a common fraction after it, such as .12½, .0312½, etc. The fraction is not counted in determining the number of places in the decimal; .12½ is read "twelve and one-half hundredths;" .0312½ is read "three hundred twelve and one-half ten-thousandths." To change a complex decimal to a straight decimal, reduce the

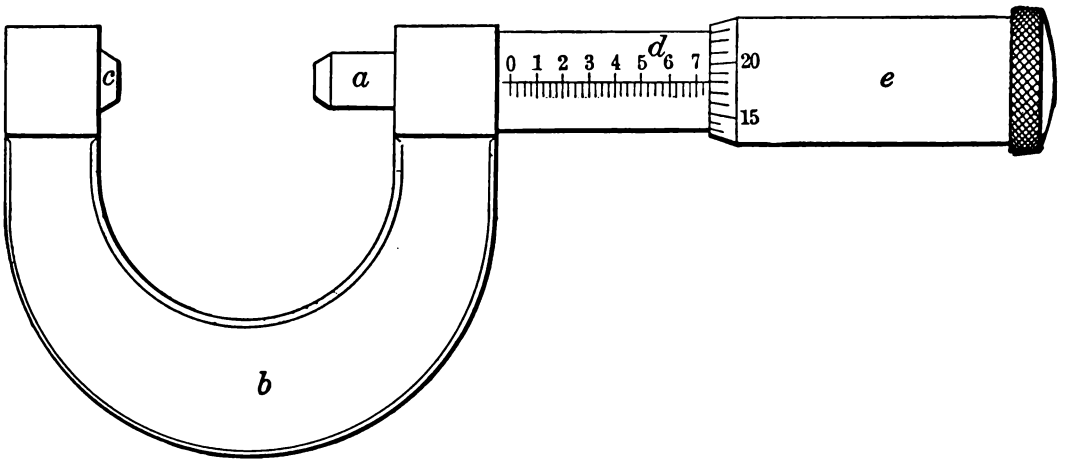


FIG. 9.—Micrometer caliper.

common fraction to a decimal and write it directly after the other decimal, without any decimal point between them.

Examples:

$$.06\tfrac{1}{2} = .065 \quad .8\tfrac{3}{4} = .875 \quad .03\tfrac{1}{8} = .03125$$

To read complex decimals, it is necessary to use the word "and" just preceding the fractional part of the decimal. This, however, does not indicate the decimal point.

Example:

.06½ is read "six and one-half hundredths."
3.06½ is read "three *and* six and one-half hundredths."

45. The Micrometer.—The micrometer is a device to measure to the thousandth of an inch and is best known to shop men in the form of the micrometer caliper shown in Fig. 9. The whole principle of the micrometer, as generally made, can be said to depend on the fact that $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}$. The micrometer, as shown in Fig. 9, is made up of the frame or yoke *b*, the anvil *c*, the screw or spindle *a*, the barrel *d*, and the thimble *e*. The spindle *a* is threaded inside of *d*. The thimble *e* is attached to the end of the spindle *a*. The piece to be measured is inserted between *c* and *a*, and the caliper closed on it by screwing *a*

against it. The screw on *a* has 40 threads to the inch, so if it is open one turn, it is open $\frac{1}{40}$ in., or $\frac{25}{1000}$, or .025. Along the barrel *d* are marks to indicate the number of turns or the number of fortieths inch that the caliper is open. Four of these divisions ($\frac{4}{40}$) will represent one-tenth of an inch, so the tenths of an inch are marked by marking every fourth division on the barrel. Around the thimble *e* are 25 equal divisions to indicate parts of a turn. One of these divisions on *e* will, therefore, indicate $\frac{1}{25}$ of a turn, and the distance represented will be $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}$ in.

The micrometer measurement, or the distance between *c* and *a*, is indicated by the length of scale which is exposed on *d*; that is, the distance between the zero point and the edge of the thimble. To read the micrometer is simply a matter of reading this length. The large numbered divisions, as explained, are equal to tenths of an inch, so we first set down the numbers of tenths of an inch, as shown by the last number exposed on the barrel *d*. The small divisions are equal to .025 in., so for each additional small division entirely exposed to the right of the last tenths mark, we add .025 to the number of tenths. The number of thousandths corresponding to the last small division which is only partly exposed is read from the scale on the thimble at the point where the two scales intersect. This added to the rest gives the complete measurement.

Example:

Let us read the micrometer shown in Fig. 9.

.7
.025
.018
.743 in., *Answer.*

Explanation: First we find the figure 7 exposed on the barrel, indicating that we have over $\frac{7}{10}$ in. This we put down as a decimal. In addition, there is one of the smaller divisions uncovered. This is .025 in. more. And on the thimble we find that it is 3 divisions beyond the 15 mark toward the 20 mark. This would be 18, and indicates .018 in. more. Adding the three, .7 + .025 + .018 = .743 in., *Answer.* This can perhaps be better understood as being 7 thousandths less than $\frac{3}{4}$ in. Many men locate a decimal in their minds by its being just so far from some common fraction.

Most micrometers have stamped in the frame the decimal equivalents of the common fractions of an inch by sixty-fourths from $\frac{1}{64}$ in. to 1 in. A table of these decimal equivalents as given in this text will be found very useful. Everyone should remember the decimal equivalents of one eighth, one quarter, and one-half, or, at least, that one-eighth is .125. Then $\frac{5}{8} = 5 \times .125 = .625$; and $\frac{7}{8} = 7 \times .125 = .875$, etc. Also, if possible,

TABLE OF DECIMAL EQUIVALENTS FROM $\frac{1}{64}$ TO 1

Fraction	Decimal equivalent	Fraction	Decimal equivalent
$\frac{1}{64}$	.015625	$\frac{33}{64}$	.515625
$\frac{1}{32}$	.03125	$\frac{17}{32}$	.53125
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.0625	$\frac{9}{16}$	.5625
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.09375	$\frac{19}{32}$	.59375
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.125	$\frac{5}{8}$	.625
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.15625	$\frac{21}{32}$	.65625
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.1875	$\frac{11}{16}$	.6875
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125
$\frac{7}{32}$	.21875	$\frac{23}{32}$	.71875
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.25	$\frac{3}{4}$	.75
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.28125	$\frac{25}{32}$	.78125
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.3125	$\frac{13}{16}$	.8125
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.34375	$\frac{27}{32}$	.84375
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.375	$\frac{7}{8}$	.875
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.40625	$\frac{29}{32}$	.90625
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.4375	$\frac{15}{16}$	.9375
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.46875	$\frac{31}{32}$	.96875
$\frac{31}{64}$	.484375	$\frac{63}{64}$	.984375
$\frac{1}{2}$	.5	1	1.00000

learn that $\frac{1}{16} = .062\frac{1}{2}$, or .0625. To get the decimal equivalent of a number of sixteenths, add $.062\frac{1}{2}$ to the decimal equivalent of the eighths next below the desired sixteenths.

Example:

What is the decimal equivalent of $\frac{13}{16}$ in.?

$$\frac{13}{16} = \frac{3}{4} + \frac{1}{16} = .750 + .062\frac{1}{2} = .812\frac{1}{2} \text{ in., or } .8125 \text{ in.}$$

To set a micrometer to a certain decimal, first unscrew the thimble until the number is uncovered on the barrel corresponding to the number of tenths in the decimal. (Thus in the decimal .678, the number of tenths would be 6.) Then divide the remaining part of the decimal by .025. The quotient will be the additional number of small divisions to be uncovered on the barrel. The number of thousandths in the remainder will give the number of divisions that the thimble should be turned from the zero point on its edge. In the decimal .678, as above stated, the number of tenths are 6. The remaining part of the decimal is .078, which divided by .025 equals 3, plus a remainder of .003. Therefore, to set the micrometer to .678, uncover 6 tenths on the barrel, plus 3 additional small divisions, and turn the thimble 3 divisions past the zero on its edge.

Example:

Calculate the setting for $\frac{7}{16}$ in. ($= .4375$ or $.437\frac{1}{2}$).

First unscrew the micrometer until the 4 is uncovered on the barrel. Then divide the remainder .0375 by .025. This gives 1 and leaves a remainder of .0125. The thimble should, therefore, be unscrewed one full turn or 1 division beyond 4 on the barrel, plus 12.5 divisions on the thimble.

46. The Decimal Fraction in Machine Work.—In modern machine shop practice, all close machine work is done to the thousandth of an inch, and very often to the ten-thousandth. Dimension limits, or "tolerances," are specified on drawings in decimal fractions. For example, suppose that a shaft is to be turned to a diameter of 3 in., and the machinist is allowed a variation of one-half thousandth, or 5 ten-thousandths of an inch (.0005) greater or less than 3 in. This means that he must make the diameter within .0005 in. of 3 in. The dimension on the drawing or blueprint might be written $\frac{+3.0005}{-2.9995}$, or $3.0000 \frac{+.0005}{-.0005}$. These are two very common methods of indicating limits,

although there are many others used. The first type of dimension $\left(\frac{+3.0005}{-2.9995}\right)$ gives the high and low limits allowed, and to the machinist it means "between 2.9995 and 3.0005 in." The second type $\left(3.000 \frac{+.0005}{-.0005}\right)$ gives the exact dimension with the allowable variations from that dimension. The dimension is read as "3.000 in. plus or minus .0005 in.," and it means that the diameter desired is 3.000 in., but it may be anywhere between a maximum of .0005 in. *more* than 3.000 in. and a minimum of .0005 in. *less* than 3.000 in.

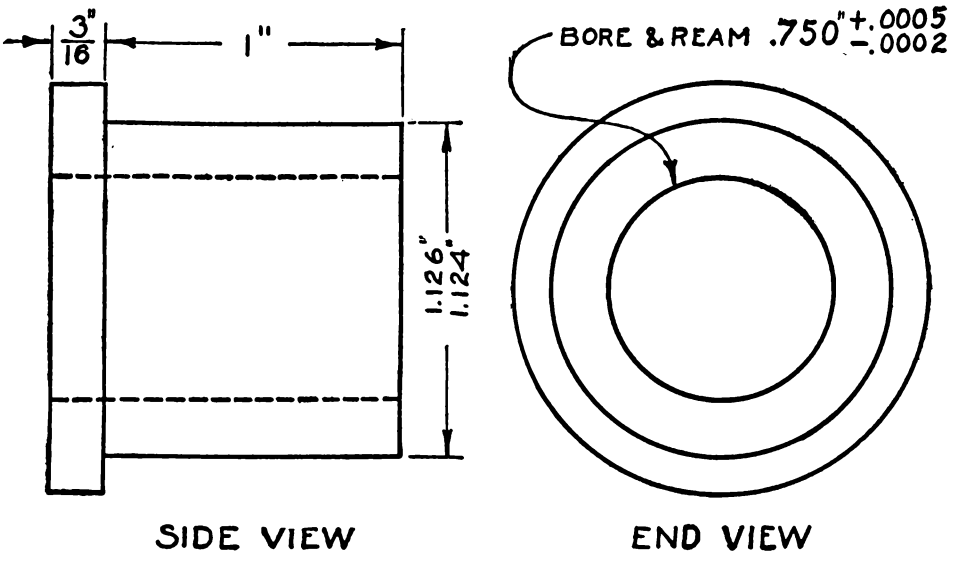


FIG. 10.—Illustration for Problem 124.

Figure 10 illustrates a shop drawing of a small machine part with the limits specified on the drawing. The hole through the center is to be "bored and reamed" to a diameter of .750 in. with an allowable maximum diameter of .7505 in. and a minimum diameter of .7498 in. The machinist will have either a micrometer or a limit gage to check the dimensions on the piece when completed.

46*a*. *The Metric System.*—In working some of the problems given in previous chapters, it was, no doubt, apparent to you that many of the units of measurement used are related to one another but that this relationship is hard to remember and somewhat difficult to apply. A glance at the following list, will illustrate what is meant.

36 inches = 1 yard
$5\frac{1}{2}$ yards = 1 rod
5280 feet = 1 mile
144 things = 1 gross
16 ounces = 1 pound
4 quarts = 1 gallon

It would seem that if we had some systems of units of measurement that had simple relations to each other, they would be much easier to remember and possibly less difficult to use. It seems out of the way when we stop to think of it, that a yard should be divided into 36 in. and a foot into 12 in.; a pound into 16 oz. and a ton into 2000 lb.; a quart into 2 pt. and a gallon into 4 qt. Why not choose a system of measurements such that the units are related by numbers easier to remember and easier to work with mathematically? Why not choose units that are related by the number 10 or multiples and decimal parts of ten? Surely these would not be so hard to remember. We have seen also that it is very simple to divide or multiply any decimal by ten.

This is precisely what has been done in the system of units known as the metric system. The metric system is used in many European countries and to some extent in this country, especially in scientific work. Due to international trade and the World War the metric system is being more widely used. Some industrial plants are now using this system. The objection to adapting it for the sole system of units in this country is not only that everyone would have to learn the new system, but that millions of dollars worth of equipment are graduated in the English system of units.

The base of the linear measurements is the *meter*, which was intended to be $\frac{1}{10{,}000{,}000}$ of the distance from the equator to the poles. It has since been found to be slightly less than this. This relationship was adopted since it was felt that some standard was necessary which would not change and which could not be "lost."

Using the *meter* as the *base*, the linear measurements in the metric system have the following relationships:

10 millimeters (mm.)	= 1 centimeter	(cm.)	= .01 meter
10 centimeters	= 1 decimeter	(dm.)	= .1 meter
10 decimeters	= 1 *meter*	(m.)	=
10 meters	= 1 dekameter	(dm.)	= 10 meters
10 dekameters	= 1 hectometer	(Hm.)	= 100 meters
10 hectometers	= 1 kilometer	(Km.)	= 1000 meters
10 kilometers	= 1 myriameter	(mm.)	= 10,000 meters

A careful study of the above table will reveal that not only are the various units obtained from the *meter*, but that the names of the units themselves are also formed from the word (meter). This is done by adding to the word (meter) a prefix, which means to put something before. Thus when we put (milli) before (meter) we get (millimeter), just as we get a millimeter by dividing a meter by 1000. Continuing this reasoning, we may write the following table:

milli-	denotes .001
centi-	denotes .01
deci-	denotes .1
deka-	denotes 10
hecto-	denotes 100
kilo-	denotes 1,000
myria-	denotes 10,000

If we define a unit of volume and a unit of weight in some such manner as we defined the *meter*, we can use the above table immediately to set down the tables of units of measure, of volume, of capacity, or of weight. The unit of capacity is called the *liter* and is defined as the volume of 1 cubic decimeter, that is, the same volume as a box 1 decimeter wide, 1 decimeter long, and 1 decimeter deep, inside dimensions. The unit of weight is called the *gram* and is defined as $\frac{1}{1000}$ of the weight of 1 liter of pure water at the temperature of its greatest density. Using these two standards or *bases*, and the table of prefixes, we can write the two following tables:

UNITS OF CAPACITY

10 milliliters (ml.)	= 1 centiliter (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 *liter* (l.) = 1 $dm.^3$
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.) = 1 $m.^3$

UNITS OF WEIGHT

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 *gram* (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 kilogram (Kg.)

In Chapter XI it is shown that area and volume can be obtained by multiplying units of linear measurement. The same thing, of course, holds true for the metric system as well as for the English system. Thus we have the following two tables:

UNITS OF SURFACE (AREA)

100 square millimeters (mm.^{2})	= 1 square centimeter (cm.^{2})
100 square centimeters	= 1 square decimeter (dm.^{2})
100 square decimeters	= 1 square meter (m.^{2})
100 square meters	= 1 square dekameter (Dm.^{2})
100 square dekameters	= 1 square hektometer (Hm.^{2})
100 square hektometers	= 1 square kilometer (Km.^{2})

UNITS OF VOLUME

1000 cubic millimeters (mm.^{3})	= 1 cubic centimeter (cm.^{3})
1000 cubic centimeters	= 1 cubic decimeter (dm.^{3})
1 cubic decimeter	= 1 liter
1000 cubic decimeters	= 1 cubic meter (m.^{3})

46*b*. *Changing from One Metric Unit to Another.*—Because the metric system is based on the decimal system of numbers, it is very simple to change from one unit of the metric system to another unit of like dimension in the same system. To change some number expressed in one unit to a number in a larger unit, we divide by 10, 100, 1000, etc. To change to a smaller unit we multiply by 10, 100, 1000, etc.

Examples:

1. To change 4729 cm. to meters we divide by 100 by moving the decimal point two places to the left as in any decimal number. Thus we have 47.29 meters. In like manner, 42197 g. = 42.197 kg.

2. To change 26.5 cm. to millimeters we multiply by 10 by moving the decimal point one place to the right, or, 26.5 cm. = 265 mm. Likewise, 6.3 kg. = 630 dg.

Because both systems of measurement, the English and the metric, are now used in this country and in our scientific, industrial, and commercial dealings with other nations, it very often

becomes necessary to convert one system to the other. To make this conversion, we multiply one unit by some fixed number or *conversion factor* as it is called. Thus we can put an equals sign between two entirely different number of units provided we multiply one of the numbers by the conversion factor. These conversion factors are called equivalents. A table of equivalents or conversion factors is shown below. An example of the use of equivalents is also given.

Given: 1 in. = 2.54 cm.

Desired to convert 14.7 in. to cm.

Solution: For every inch there will be 2.54 cm.; therefore, 14.7 in. = $14.7 \times 2.54 = 37.338$ cm. (Here 2.54 is the conversion factor.)

CONVERSATION TABLES

Metric and English Units

Units of Length

Metric to English Units	English to Metric Units
1 meter = 39.37 inches	1 inch = 2.54 centimeters
1 meter = 3.281 feet	= 25.4 millimeters
1 meter = 1.0936 yards	1 foot = .3048 meter
1 centimeter = .3937 inch	
1 millimeter = .03937 inch	
1 kilometer = .6214 mile	1 mile = 1.609 kilometer

Units of Capacity

1 liter = 1 cubic decimeter	1 gallon (U. S.) = 3.785 liters
= 61.023 cubic inches	1 quart (U. S.) = .94625 liter
= .035 cubic foot	1 cubic foot = 28.317 liters
= 1.0567 quarts (U. S.)	
= .2642 gallon (U. S.)	
= 2.202 pounds of water	
1 cubic meter = 264.2 gallons	

Units of Weight

1 gram = .0022 pound	1 pound = 453.6 grams
1 kilogram = 2.2046 pounds	1 ounce (avoirdupois) = 28.35 grams
	1 pound = .4536 kilogram

Units of Surface (Area)

1 sq. meter = 10.764 square feet	1 sq. yard = .836 sq. meter
= 1.196 square yards	1 sq. foot = .0929 sq. meter
1 sq. centimeter = .155 sq. inch	1 sq. inch = 6.452 sq. centimeters
1 sq. millimeter = .00155 sq. inch	= 645.2 sq. millimeters

Units of Volume

1 cubic meter = 35.314 cubic feet	1 cubic yard = .7645 cubic meter
= 1.308 cubic yards	1 cubic foot = .02832 cubic meter
= 61023 cubic inches	= 28.317 cubic decimeters
1 cubic decimeter = 61.023 cubic inch	1 cubic inch = 16.387 cubic centimeters
= .0353 cubic feet	
1 cubic centimeter = .061 cubic inch	

PROBLEMS

106. Write the following fractions as decimals:

$$\frac{9}{10},\ \frac{76}{100},\ \frac{3}{4},\ \frac{770}{1000},\ \frac{7}{8},\ \frac{1}{100}$$

107. Write as decimals:

(*a*) 75 thousandths
(*b*) 129 and 78 ten-thousandths
(*c*) $27\frac{1}{8}$; $\frac{44}{66}$
(*d*) $6 \times \frac{7}{8}$; $10 \div 8$

108. Write as decimals:

One and twenty-five one-hundredths
Three hundred seventy-five one-thousandths
Three hundred and seventy-five one-thousandths

109. Read the following decimals, and write them out in words:

.075
.137
100.037
$.12\frac{1}{2}$
1.09375

110. Write as a decimal the difference between $24\frac{7}{8}$ and $12\frac{6}{11}$.

111. A bill for repairs included the following items: labor, \$35; lumber, \$3.28; pipe and pipe fittings, \$15.42; bathroom fixtures, \$19. What was the gross amount of the bill? If a discount of \$3.23 was allowed for immediate cash payment, what was the actual amount paid?

112. A fast train travels at the rate of 1.25 mi. a minute. How many miles will it travel in 3 min.; in .75 of an hour?

113. Reduce the following fractions to decimals:

$$\frac{1}{16},\ \frac{15}{32},\ \frac{3}{64}$$

114. A steam pump delivers .965 cu. ft. of water at every stroke. If it runs 48 strokes per minute, how many cubic feet of water does it deliver in 1 min.? If there are 7.47 gal. in a cubic foot, how many gallons does it deliver in 1 min.?

115. Calculate the decimal equivalent of $\frac{1}{24}$ in.

116. Write .8125 as a common fraction, and reduce it to the lowest terms.

117. What is a convenient method of multiplying a decimal by 10, 100, 10,000?

118. Express $\frac{1}{4}$ of 43.9 as a decimal.

119. What is the rate of speed of a train traveling 407 mi. in 12.4 hr.?

120. A kilometer is about 3281 ft. What decimal part of 3.2 mi. is this?

121. What is a convenient method of dividing a decimal by 10, 100 1000, 10,000?

122. If an alloy is .67 copper and .33 zinc, how many pounds of each metal will there be in a casting weighing 75 lb.?

123. A tank car loaded with fuel oil is found to contain 80,000 lb. of oil. If a gallon of fuel oil weighs 7.396 lb., how many gallons does the car contain?

124. The reamed hole in one of the finished glands of Fig. 10 was found upon inspection to be only .749 in. in diameter. How much larger must the hole be made in order to pass inspection?

125. The diameter of No. 8 B. W. G. wire is .165 in. and of No. 12 wire is .109 in. What is the difference in diameter of the two wires?

126. A machinist whose rate is 87.5 cts. per hour puts in a full day of 10 hr. and also 3 hr. overtime. If he is paid "time and a half" for overtime, how much should he be paid altogether?

127. An electrician purchased a roll of No. 2 copper wire weighing 22.1 lb. at a cost of 25 cts. a pound. What did the roll cost him?

128. If No. 2 copper wire weighs .2009 lb. per foot, how many feet of wire did the electrician buy in the previous problem?

129. In the analysis of a shipment of Illinois coal, the chemist used a sample weighing 1 g. He found that the sample contained, by actual weight, .0221 g. of moisture, .2712 g. of volatile combustible, and .6307 g. of fixed carbon. If the remainder of the sample consisted of ash, what was the weight of the ash in grams?

130. Explain how you would set a micrometer for $\frac{2}{100}$ in. over $\frac{7}{8}$ in.

131. The depth of thread on a $\frac{3}{4}$-in. bolt with U. S. Standard threads is .065 in. What is the diameter of the bolt at the bottom of the threads?

132. A 2-in. pipe has an actual inside diameter of 2.067 in. The metal of the pipe is .154 in. thick. What is the outside diameter of the pipe?

133. (*a*) Read the micrometer shown in Fig. 11.

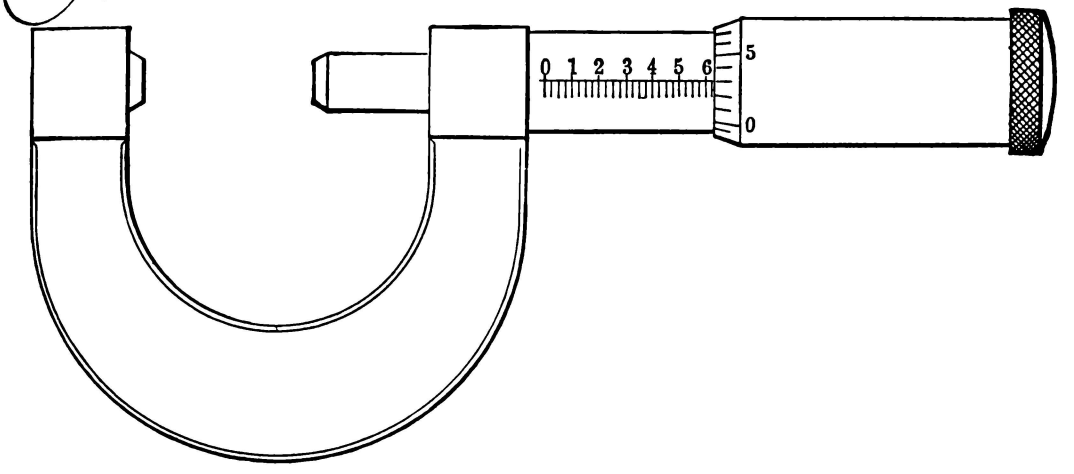


FIG. 11.—Illustration for Problem 133 (*a*).

(*b*) Figure 12 shows a special form of micrometer for measuring pitch diameters of screw threads. What pitch diameter does the micrometer indicate?

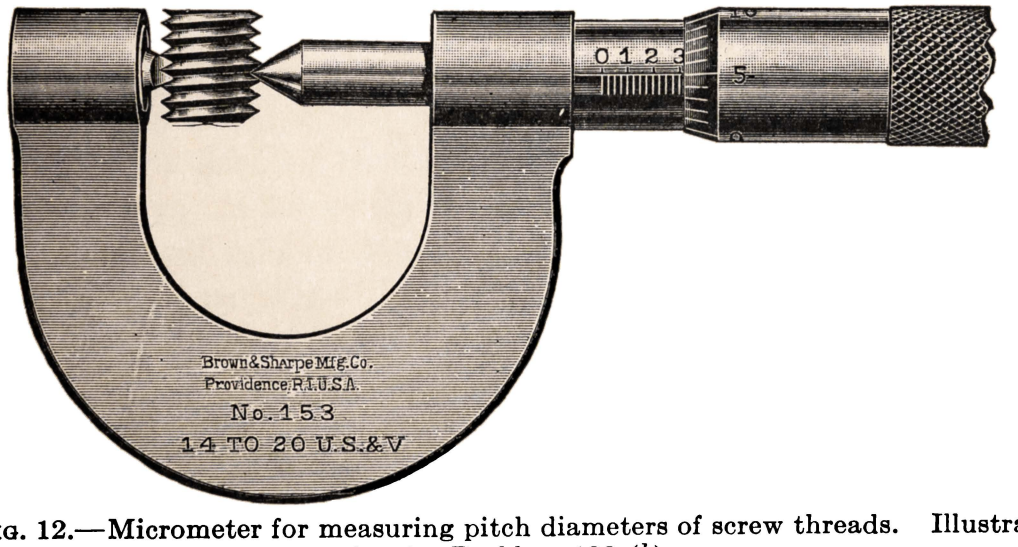


FIG. 12.—Micrometer for measuring pitch diameters of screw threads. Illustration for Problem 133 (*b*).

134. Express the following, first in meters and then in millimeters: 472 centimeters, 641 decimeters, 32 kilometers.

135. Express the following in terms of square meters: 75 square centimeters, 125 square millimeters, .025 square decimeter, .029 square kilometer.

136. Express the following in terms of cubic meters: 1772 liters, 476 liters, 4900 deciliters, .1067 deciliter, 71,234 cubic decimeters, 776 cubic decimeters.

137. No. 24 gage sheet steel is .635 millimeter thick and weighs 4.8820 kilograms per square meter. Find the thickness in decimal parts of an inch and weight per square foot in ounces (three decimal places).

138. Find the difference between $4\frac{19}{32}$ inches and 11.4 centimeters. Express answer in inches and in centimeters.

139. Convert 25 kilograms per square centimeter to pounds per square inch.

140. What is the weight of the mercury in a tube 1 square centimeter in cross section and 760 millimeters long? Mercury is 13.596 times as heavy as water. Express in kilograms. The weight of water is one gram per cubic centimeter.

141. A block of stone weighs 7143 decigrams. A cubic decimeter weighs 2.7 kilograms. What is the volume of the stone in cubic inches?

142. Change to English units:

(*a*) 42 meters (to feet)
(*b*) 25 kilograms (to pounds)
(*c*) 7 liters (to cubic inches)
(*d*) 10,000 kilometers (to miles)
(*e*) $18\frac{3}{4}$ liters (to quarts)
(*f*) 4.75 centimeters (to inches)

143. Change to metric units:

(*a*) 25 miles (to meters)
(*b*) 4.7 tons (to kilograms)
(*c*) 4 yd. 1 in. (to millimeters)
(*d*) 45 cubic feet (to cubic meters)
(*e*) 10 lb. 7 oz. (to kilograms)
(*f*) 71 sq. yd. (to square centimeters)

144. A man weighs 172 lb. He walks 3.4 miles. Express these figures in kilograms and meters.

145. The watt is the electrical unit of power. What is the size of a kilowatt?

CHAPTER VI

PERCENTAGE

47. Explanation.—Percentage is merely a kind of fraction or, rather, a particular kind of decimal fraction, of which the denominator is always 100. Instead of writing the denominator, we use the term "per cent" to indicate that the denominator is 100. When we speak of "6 per cent" we mean $\frac{6}{100}$ or .06. These all mean the same thing; namely, 6 parts out of 100. Instead of writing out the words "per cent" we more often use the sign % after the number, as, for instance, 6%, which means "6 per cent." Since per cent means hundredths of a thing, then the whole of anything is 100% of itself, meaning $\frac{100}{100}$, or the whole. If a man is getting 60 cts. an hour and gets an increase of 10%, this increase will be 10% (or $\frac{10}{100}$ or .10) of 60 cts. and this is easily seen to be 6 cts., so his new rate is 66 cts. Another way of working this would be to say that his old rate is 100% of itself and his increase is 10% of the old rate, so that altogether he is to get 110% of the old rate. Now 110% is the same as 1.10 and $1.10 \times 60 = 66$ cts., the new rate.

To find a certain per cent of anything, we multiply exactly as we would in finding a fractional part or decimal part of anything. The only difference is that before multiplying we must first change the per cent to its corresponding fraction or decimal.

Examples:

Calculate the following.

(*a*) 6% of 50.

(*b*) 12.5% of 6.4.

(*c*) $33\frac{1}{3}$% of 990.

Solutions:

(*a*) $6\% = \frac{6}{100}$ or .06

$\frac{6}{100} \times 50 = \frac{300}{100} = 3$, *Answer.*

Or we can multiply by the decimal .06

$.06 \times 50 = 3$, *Answer.*

(*b*) $12.5\% = \frac{12.5}{100} = .125 = \frac{1}{8}$

$\frac{1}{8} \times 6.4 = .8$, *Answer.*

Also $.125 \times 6.4 = .8$, *Answer.*

(c) $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = .33\frac{1}{3} = \frac{1}{3}$

$\frac{1}{3} \times 990 = 330$, *Answer.*

Also $.33\frac{1}{3} \times 990 = 330$, *Answer.*

Unless the number of per cent can be reduced to a simple fraction like $\frac{1}{3}$, $\frac{1}{8}$, $\frac{3}{4}$, it is generally easier and takes less time to convert it to a decimal instead of a fraction before making the calculation.

For example, in figuring the interest on \$1200 at the rate of $3\frac{1}{2}\%$, we would first change $3\frac{1}{2}\%$ to .035 and then multiply \$1200 by .035, which gives \$42.00.

$$\begin{array}{r} \$1200 \\ .035 \\ \hline 6000 \\ 3600 \\ \hline \end{array}$$

\$42.000, *Answer.*

Any decimal fraction may be easily changed to per cent.

$$.875 = \frac{875}{1000} = \frac{87.5}{100} = 87.5\%$$

Here we first change the decimal to a common fraction having 1000 for a denominator. Next we reduce this fraction to one whose denominator is 100. Then we drop this denominator and use, instead, the per cent sign (%) written after the numerator. This sign indicates, in this case, 87.5 parts out of 100, or

$$\frac{87.5}{100}$$

The change from a decimal to percentage can be made without changing to a common fraction as was just done. Having a decimal, move the decimal point *two* places to the right and write per cent after the new number.

$.625 = 62.5\%$ $.06 = 6\%$ $1.10 = 110\%$

A common fraction is reduced to per cent by first changing it to a decimal, and then changing the decimal to per cent by moving the decimal point 2 places to the right. We reduce $\frac{1}{4}$ to per cent as follows: $\frac{1}{4} = .25 = 25\%$. Thus, $\frac{1}{4}$ of anything is the same as 25% of it, because $25\% = .25 = \frac{25}{100} = \frac{1}{4}$. From this it is clearly evident that fractions, decimals, and per cent are all one and the same thing but expressed differently.

Thus:

$\frac{25}{100} = .25$ (the decimal point indicates the denominator)

$\frac{25}{100} = 25\%$(the % sign indicates the denominator)

$\frac{25}{100} = \frac{1}{4}$ (reduced to its lowest terms)

The following table gives a number of different per cents with the corresponding decimal fractions and common fractions:

Per cent	Decimal fraction	Common fraction	Per cent	Decimal fraction	Common fraction
1%	.01	$\frac{1}{100}$	25%	.25	$\frac{25}{100} = \frac{1}{4}$
2%	.02	$\frac{2}{100} = \frac{1}{50}$	$33\frac{1}{3}\%$	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100} = \frac{1}{3}$
$2\frac{1}{2}\%$	.025	$\frac{2\frac{1}{2}}{100} = \frac{1}{40}$	$37\frac{1}{2}\%$	.375	$\frac{37\frac{1}{2}}{100} = \frac{3}{8}$
5%	.05	$\frac{5}{100} = \frac{1}{20}$	50%	.50	$\frac{50}{100} = \frac{1}{2}$
$6\frac{1}{4}\%$	.0625	$\frac{6\frac{1}{4}}{100} = \frac{1}{16}$	75%	.75	$\frac{75}{100} = \frac{3}{4}$
10%	.10	$\frac{10}{100} = \frac{1}{10}$	90%	.90	$\frac{90}{100} = \frac{9}{10}$
$12\frac{1}{2}\%$	.125	$\frac{12\frac{1}{2}}{100} = \frac{1}{8}$	100%	1.00	$\frac{100}{100} = 1$
$16\frac{2}{3}\%$	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100} = \frac{1}{6}$	200%	2.00	$\frac{200}{100} = 2$

48. The Uses of Percentage.—In shop work, the chief use of percentage is to express loss or gain in certain quantities or to state portions or quantities that are used or unused, good or bad, finished or unfinished, etc. Very often we hear expressions like: "two out of five of those castings are bad;" or "nine out of ten

of those cutters should be replaced." If in the first illustration we wanted to talk in terms of per cent, we would say "40% of those castings are bad," because "two out of five" is the same as $\frac{2}{5} = \frac{40}{100} = 40\%$. In the second case, we would say, "90 per cent of those cutters should be replaced." Here, "nine out of ten" $= \frac{9}{10} = \frac{90}{100} = 90\%$. If a piece of work is said to be 60% completed, it means that if we divide the whole work on the job into 100 equal parts, we have already done 60 of these parts, or $\frac{60}{100}$ of the whole.

If a shop is running with 50% of its full force, it means that $\frac{50}{100}$ or $\frac{1}{2}$ of the full force is working. If the full force of men is 1300, then the present force is 50% of 1300 $= .50 \times 1300 =$ 650. If the full force were 700 men, then the 50% would be 350.

Suppose again a shop has 50 men working, which the foreman states is only 25% of his normal crew. If 50 is 25%, then 1% of his normal crew is $50 \div 25 = 2$ men, and 100%, or the normal crew, is $100 \times 2 = 200$ men. This is readily checked by the fact that 50 is $\frac{1}{4}$ of 200, and $\frac{1}{4} = 25\%$.

Another very common use of percentage is in stating the portions or quantities of the ingredients going to make up a whole. We often see formulas for brasses, bronzes, and other alloys in which the proportions of the different metals used are indicated by per cents. For example, brass usually contains about 65% copper and 35% zinc. Then, in 100 lb. of brass, there would be 65 lb. of copper and 35 lb. of zinc. Suppose, however, that instead of 100 lb. we wanted to mix a smaller amount, say 8 lb The amount of copper needed would be 65% (or .65) of 8 lb.

$.65 \times 8 = 5.20$ lb., or $5\frac{2}{10}$ lb., the copper needed

$.35 \times 8 = 2.80$ lb., or $2\frac{8}{10}$ lb., the zinc needed

Sometimes, in dealing with very small per cents, we find a decimal per cent such as used in the specifications for boiler steel, where it is stated that the sulphur in the steel shall not exceed .04%. Now this is not 4%; neither is it .04; but it is .04%, meaning four one-hundredths per cent, or *four one-hundredths of one one-hundredth.* This is $\frac{4}{100}$ of $\frac{1}{100} = \frac{4}{10,000}$, so if we write this .04% as a decimal, it will be .0004. It is a very common mistake to misunderstand these decimal per cents, and the student should be very careful in reading them. Likewise, he should be careful in changing a decimal into per cent that the decimal point is moved *two* places to the right.

49. Efficiencies.—Another common use of percentage is in stating the efficiencies of engines or machinery. The efficiency of a machine is that part of the power supplied to it, that the machine delivers. This is generally stated in per cent, meaning so many out of each hundred units. If it requires 100 hp. to drive a dynamo and the dynamo generates only 92 hp. of electricity, then the efficiency of the dynamo is $\frac{92}{100}$, or 92%. If the engine driving a machine shop delivers 250 hp. to the line shaft, but the line shaft delivers only 200 hp. to the machines, then the efficiency of the line shaft is $\frac{200}{250} = .80 = 80\%$. The other 50 hp., or 20%, is lost in the friction of the shaft in its bearings and in the slipping of the belts. The efficiencies of all machinery should be kept as high as possible because the difference between 100% and the efficiency means money lost. The large amount of power that is often lost in line shafting can be readily appreciated when we try to turn a shaft by hand and try to imagine the power that would be required to turn it 200 or 300 times a minute.

50. Discount.—In selling bolts, screws, rivets, and a great many other similar articles, manufacturers have a standard list of prices for the different sizes and lengths, and they give their customers discounts from these list prices. These discounts or reductions in price are always given in per cent. Sometimes they are very complicated, containing several per cents to be deducted one after another. Each discount, in such a case, is figured on the basis of what is left after the preceding per cents have been deducted.

Example:

The list price of $\frac{1}{4}$- by-$1\frac{1}{4}$ in. stove bolts is \$1 per hundred. If a firm gets a quotation of 75, 10, and 10% discount from list price, what would it pay for the bolts per hundred?

75% of 100 cts. = 75 cts.
100 − 75 = 25 cts.
10% of 25 cts. = $2\frac{1}{2}$ cts.
$25 - 2\frac{1}{2} = 22\frac{1}{2}$ cts.
10% of $22\frac{1}{2}$ cts. = $2\frac{1}{4}$ cts.
$22\frac{1}{2} - 2\frac{1}{4} = 20\frac{1}{4}$ cts.

Explanation: 75, 10, and 10% discount means 75% deducted from the list price, then 10% deducted from that remainder, then 10% taken from the second remainder.

Starting with 100 cts., the list price, we deduct the first discount of 75%. This leaves 25 cts. The next discount of 10% means 10% off from this balance. Deducting this leaves $22\frac{1}{2}$ cts. Next, we take 10% from this, leaving $20\frac{1}{4}$ cts, per hundred as the actual cost of these stove bolts.

51. Classes of Problems.—Nearly all problems in percentage can be divided into three classes on the same basis as explained in Art. 28. There are three items in almost any percentage problem: namely, the *whole*, the *part*, and the *per cent.* For example, suppose we have a question like this: "If 35% of the belts in a shop are worn out and need replacing, and there are 220 belts altogether, how many belts are worn out?" In this case, the *whole* is the number of belts in the shop, 220. The *part* is the number of belts to be replaced, which is the number to be calculated. The *per cent* is given as 35%.

Any two of these items may be given and we can calculate the missing one. We thus have the three cases:

1. Given the whole and the per cent, to find the part.
2. Given the part and the per cent that it is of the whole, to find the whole.
3. Given the whole and the part, to find what per cent the part is of the whole.

The principles taught under common fractions will apply equally well in working problems under these cases, the only difference being that here a per cent is used instead of a common fraction. In working problems, the per cent should always be changed to a decimal.

One difficulty in working percentage problems is in deciding just what number is the whole (or the base, as it is often called). The following illustration shows the importance of this.

If I offer a man \$2000 for his house, but he asks \$3000, then his price is 50% greater than my offer, while my offer is $33\frac{1}{3}$% less than his price. The difference is \$1000 either way; but if we take my offer as the base, it would be necessary for me to raise it $\frac{1}{2}$, or 50%, to meet his price. On the other hand, for him to meet my bid, he would only have to cut his price $\frac{1}{3}$ or $33\frac{1}{3}$%. In the first case \$2000 is the base. In the second case \$3000 is the base.

Examples of the three types of problems, before mentioned, may help somewhat in getting an understanding of the processes to be used.

Example of Case 1:

How many pounds of nickel are there in 1 ton of nickel-steel containing 2.85% nickel?

1 ton = 2000 lb.

2.85% = .0285

.0285 × 2000 = 57 lb., *Answer.*

Explanation: Here we have the whole (2000 lb.) and the per cent (2.85%) to find the part. After changing the 2.85% to a decimal fraction, the problem becomes a simple problem in multiplication of decimals.

Example of Case 2:

The machines in a small pattern shop require altogether 12 hp. and are to be driven from a line shaft by a single electric motor. If we assume that 20% of the power of the motor will be lost in the line shaft and belting, what size motor must we install?

100% − 20% = 80%
80% = .80, or .8
12 ÷ .8 = 15, *Answer.*

Explanation: Here the per cent given (20%) is based on the horsepower of the motor, which is, as yet, unknown. The horsepower of the motor is 100% of itself, and if 20% is lost, then the machines will receive 80% or .8 of the power of the motor. This is 12 hp. The whole will be 12 ÷ .8 = 15 hp. This is the size of motor to install.

Example of Case 3:

If the force in a shop is increased from 160 to 200 men, what per cent is the capacity of the shop increased?

200 ÷ 160 = 1.25
1.25 = 125%
125% − 100% = 25%, *Answer.*

Explanation: The present force is $1\frac{1}{4}$ or 1.25 of the old force, because 200 is $\frac{200}{160}$ of 160, and $\frac{200}{160} = 1\frac{1}{4}$. This is the same as 125%. The increase is, therefore, 25% of the former force.

Note.—To reduce the present force back to the old number would require a reduction of only 20%, because now the base is different on which to figure the per cent. 40 ÷ 200 = .20, or 20%.

PROBLEMS

146. Write the following per cents as common fractions:

4%, $12\frac{1}{2}$%, 25%, 75%

147. Write the following per cents as decimals:

6%, 13%, $9\frac{1}{2}$%, 43%

148. What per cent of $\frac{3}{4} \times \frac{8}{15}$ is .7?

149. What per cent of a pound is 1 oz.? (There are 16 oz. in 1 lb.)

150. A machinist turned out 47 bolts in one day, which was 7% more than he turned out the day before; how many did he turn out the day before?

151. In a manufacturer's catalogue the list price of a pyrometer is given as $75.50. Upon request for prices, however, the manufacturer quotes a discount of 30%. What price must be paid for the pyrometer?

152. If 165 lb. of clay are used in an order of 2240 lb. of paper, what percentage of the finished product is clay?

153. The power plant of Fig. 13 contains two main turbine-driven generators whose rated capacities are 7500 and 10,000 kw., respectively. If the 7500-kw. machine is shut down for repairs while the other one carries the load, what per cent of the total rated capacity of the plant is in operation?

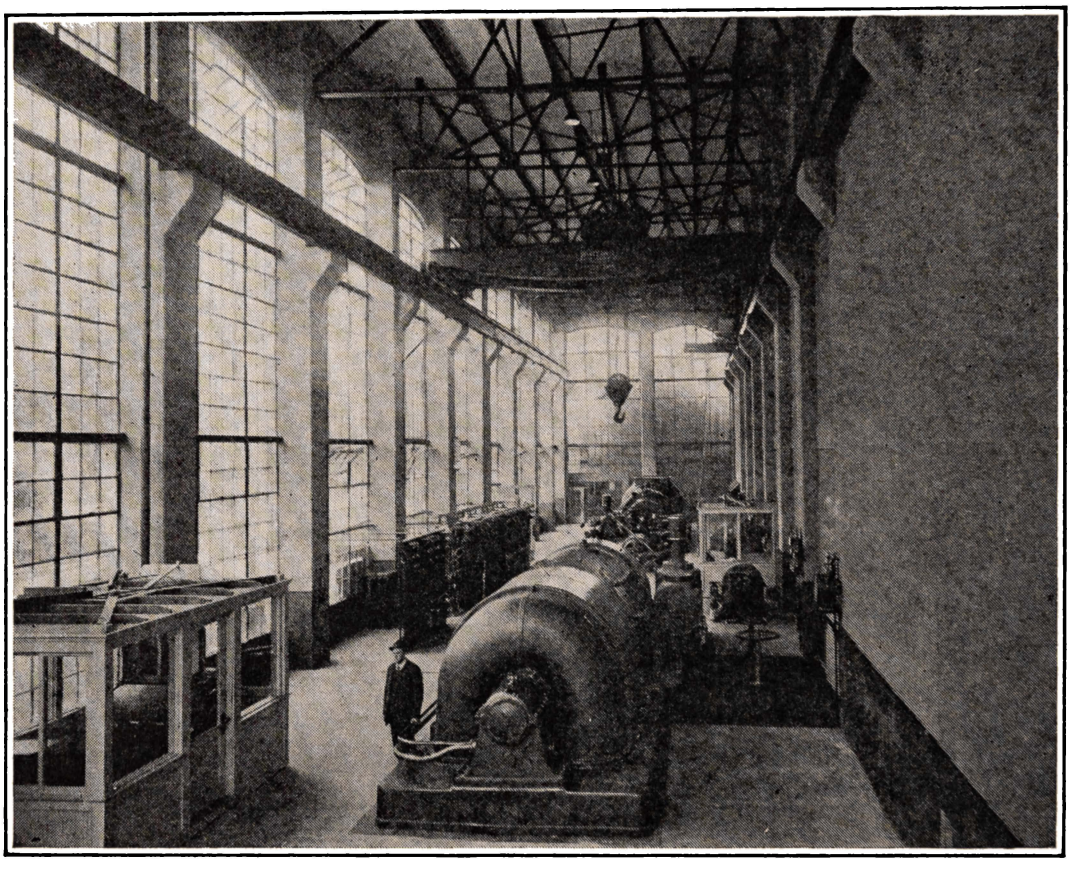


FIG. 13.—Steam turbine power plant. Illustration for Problem 153.

154. When iron pipe is cast, the usual allowance made for shrinkage is $\frac{1}{8}$ in. per foot. What per cent is this?

155. The composition of white metal is to be 4 parts by weight of copper, 87 of tin, and 9 of antimony. Write these as percentages of the whole, and find the weight of each ingredient necessary to make up 4321 lb. of the alloy.

156. If the water power in use in a country is 5,721,000 hp. and the undeveloped horsepower is 7,673,000, what per cent of the total water power is developed?

157. The cost of manufacture of a certain sized motor is $52. If the manufacturer desires to make a profit of 10%, at what price does he sell the motor?

158. Out of a lot of 650 castings 30 were spoiled through defective foundry work, and 25 were spoiled in the machine shop. What per

cent of the total number of castings was spoiled in the foundry and what per cent of the total number was spoiled in the machine shop?

159. An hydraulic power station has an available waterpower theoretically equivalent to 2000 hp. If the turbines deliver 1600 hp. to the generators, what per cent of the available waterpower is lost before reaching the generators?

160. A certain firm installed a new brass furnace in its foundry. The actual purchase price of the new furnace was \$800, which, however, was found to be only 90% of the total cost when completely installed. What was the total cost?

161. By the use of stellite, a certain machine shop reduced the cost of finishing one casting from \$1.20 to \$.90. What per cent was the cost reduced?

162. Five hundred pounds of bronze bearings are to be made. The mixture is 77% copper, 8% tin, and 15% lead. How many pounds of copper, tin, and lead are required?

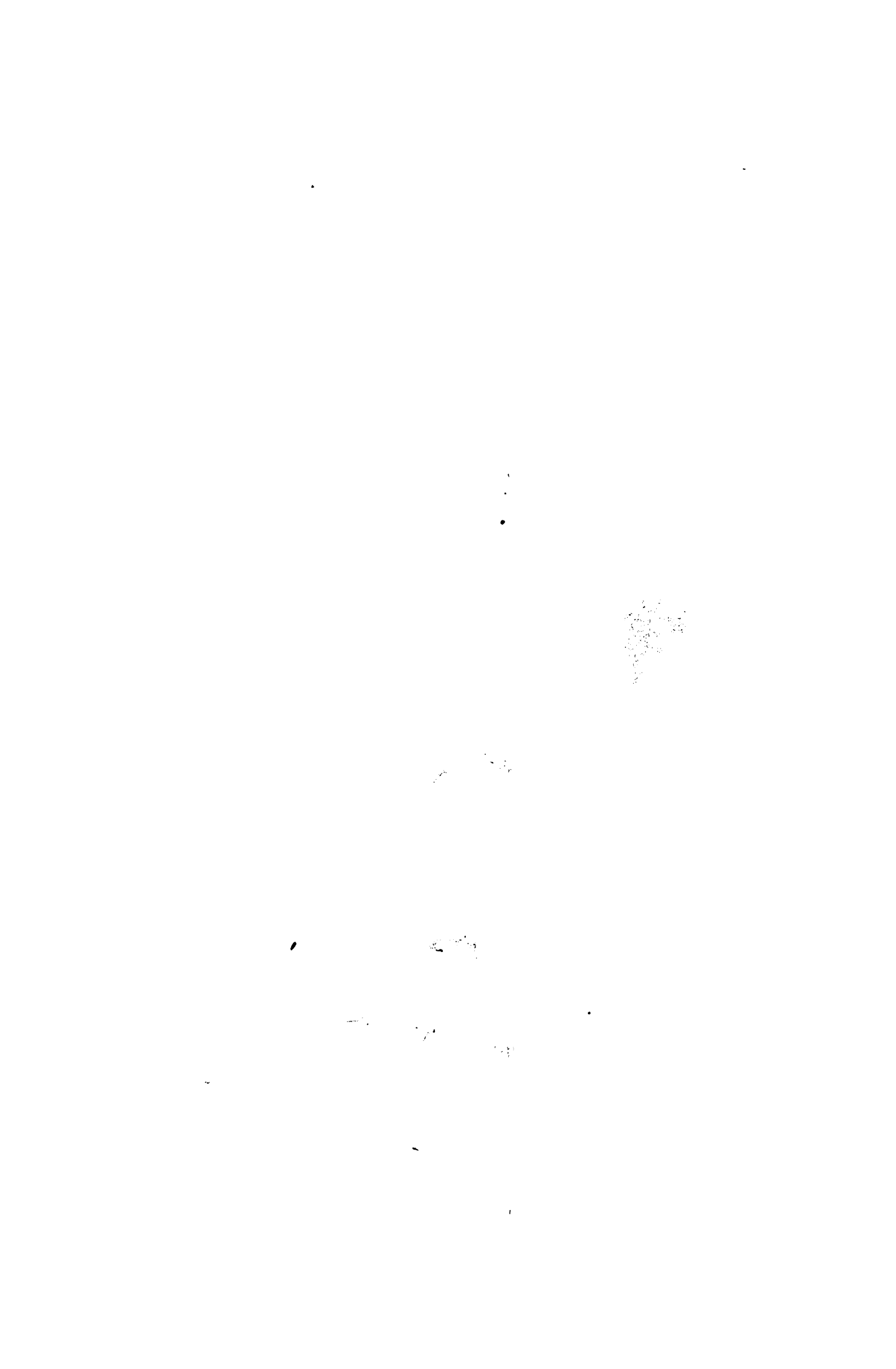
163. In testing a shop drive it was found that the machines driven by one motor required horsepower as follows:

60-in. mill	3.31 hp.
20-in. lathe	.75 hp.
48-in. lathe	2.42 hp.
42-in. by 42-in. by 12-ft. planer	4.82 hp.
16-in. shaper	.33 hp.

The total power delivered by the motor was 13.65 hp. What per cent of the total power was used in belting and line shaft? What per cent by the machines?

164. A man who has been drawing \$5.50 a day has his pay cut 10% on May 1, and the following September he is given an increase of 10% on his rate at that time. How much will he get per day after September?

165. The following weights of metals are melted to make up a solder; 18 lb. of tin, 75 lb. of bismuth, 37.5 lb. of lead, and 19.5 lb. of cadmium. What per cent of the total weight is there of each metal?



CHAPTER VII

CIRCUMFERENCES OF CIRCLES; CUTTING AND GRINDING SPEEDS

52. Shop Uses.—In the operation of almost any machine, care must be used to determine the speed which will give the best results. Lathes, milling machines, boring mills, etc. are provided with means for changing the speed, according to the nature of the work being done. Emery wheels and grindstones, however, are often set up and run at any speed which the pulleys happen to have, regardless of the diameter.

If an emery wheel of large size is put on a spindle that has been belted to drive a smaller wheel, the speed may be too great for the larger wheel and, if the difference is considerable, the large wheel may fly to pieces. Every mechanic should know how to calculate the proper sizes of pulleys to use for emery wheels or grindstones, the correct speed at which to run the work in his lathe, or the most economical speeds to use for belts and pulleys. Some data on this subject may be useful and will afford applications for arithmetical principles.

53. Circles.—To understand what has just been mentioned, it is necessary to obtain a knowledge of circles and their properties. The distance across a circle, measured straight through the center, is called the *Diameter*. Circles are generally designated by their diameters. Thus a 6-in. circle means a circle 6 in. in diameter. Sometimes the radius is used. The *Radius* is the distance from the center to the edge or circumference and is, therefore, just half the diameter. If a circle is designated by the radius, we should be careful to say so. Thus, there would be no misunderstanding if we said "a circle of 5-in. radius;" but unless the word "radius" is used, we always understand that the measurement given is the diameter. The *Circumference* is the name given to the *distance around* the circle, as indicated in Fig. 14. The circumference of any circle is always 3.1416 times the diameter. In other words, if we measure the diameter with a string and lay this off around the circle, it will take a little over three times the

length of the string to go once around the circle. This number 3.1416 is, without doubt, the most used in practical work of any figure in mathematics. In writing formulas, it is quite common to represent this decimal by the Greek letter π (pronounced "pi"), instead of writing out the whole number. For this reason, the number 3.1416 is given the name "pi."

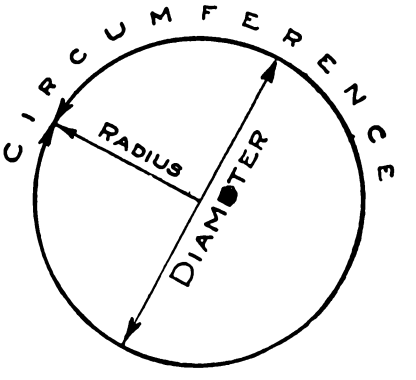


FIG. 14.—Principal measurements of circle.

Where it is more convenient and extreme accuracy is not required, the fraction $\frac{22}{7}$ may be used for π instead of the more exact value 3.1416.

$$\frac{22}{7} = 3\frac{1}{7} = 3.1429$$

It, therefore, gives values of the circumference slightly too large, but in many cases it is sufficiently accurate and saves time.

Example:

What length of steel sheet would be needed to roll into a drum 32 in. in diameter?

When the sheet is rolled up, its length will become the circumference of a 32-in. circle. The circumference must be π times 32.

$$3.1416 \times 32 = 100.5 + \text{in.}$$

The length of the sheet must, therefore, be $100\frac{1}{2}$ in.; and if it is to be lapped and riveted, we would have to add a suitable allowance of 1 in. or so for making the joint.

54. Formulas.—A formula, in mathematics, is a rule in which mathematical signs and letters are used to take the place of words. We say that "the circumference of a circle equals 3.1416 times the diameter." This is a rule. But suppose we merely write

$$C = \pi \times D$$

This is the same rule expressed as a *formula.* We have used C instead of the words "the circumference of a circle"; the sign $=$ replaces the word "equals"; the symbol π is used instead of the number 3.1416; $\times$ stands for "times"; and D stands for "the diameter."

Formulas do not save much, if any space, because it is necessary usually to explain what the letters stand for. They have, however, the great advantage that intricate mathematical operations can be shown much more clearly than if they were written out in a long sentence or statement. One can usually see in one glance at a formula just what is to be done with the numbers that are given in the problem, in order to find the quantity that is unknown.

Example:

What is the circumference in feet of a 16-in. emery wheel?
$C = \pi \times D$
$C = 3.1416 \times 16 = 50.2656$ in.
50.2656 in. $\div 12 = 4.1888$ ft., *Answer.*

Explanation: We have the diameter given and want to get the circumference. We, therefore, use the formula which says that $C = \pi \times D$. π is always 3.1416, and D in this case is 16 in. Then C, is 50.2656 in. But the problem calls for the circumference in feet. This is $\frac{1}{12}$ of the number of inches, or it is the number of inches divided by 12.

In the work of this chapter, the circumferences of circles are always used in feet, and, consequently, should always be calculated in feet. If D is in feet, we will get C in feet, while, if D is in inches, C will be in inches. If the diameter can be reduced to exact feet, it is easier to use the diameter in feet when multiplying by π, rather than to reduce to feet after multiplying.

Example:

What is the circumference of a 48-in. flywheel?
48 in. $\div 12 = 4$ ft., the diameter.
$C = \pi \times D$
$C = 3.1416 \times 4 = 12.5664$ ft., *Answer.*

This is much shorter than it would be to multiply 3.1416 by 48 and then divide the product by 12.

Frequently it is desired to find the diameter of some object, such as a tree, tank, or chimney, where it is an easy matter to

measure the circumference but quite difficult to measure the diameter. From the preceding formula we note that

$$\text{Circumference} = 3.1416 \times \text{diameter}$$

Since we multiply the diameter by 3.1416 to get the circumference, we can reverse the operation and divide the circumference by 3.1416 to get the diameter. In other words we "work the problem backwards," *dividing* instead of *multiplying*. Consider a circle having a diameter of 2 in.

$$3.1416 \times \text{Diameter} = \text{Circumference}$$
$$3.1416 \times 2 = 6.2832 \text{ in. circumference}$$

If we work backwards from the circumference, we get

$$6.2832 \div 3.1416 = 2 \text{ in., which is the diameter}$$
$$\text{Circumference} \div 3.1416 = \text{Diameter}$$

Expressed as a formula,

$$D = \frac{C}{\pi}$$

Example:

A circular steel tank measures 37 ft. $8\frac{1}{2}$ in. in circumference. What is its diameter?

$$37 \text{ ft. } 8\tfrac{1}{2} \text{ in.} = 37\ \frac{8\frac{1}{2}}{12} \text{ ft.} = 37.7+ \text{ ft.}$$

$$D = \frac{C}{\pi}$$

$$\text{Therefore, Diameter} = \frac{37.7}{3.1416} = 12 \text{ ft., } \textit{Answer.}$$

We might also change the circumference to inches.

$$37 \text{ ft. } 8\tfrac{1}{2} \text{ in.} = 452\tfrac{1}{2} \text{ in.}$$

$$\text{Diameter} = \frac{452.5}{3.1416} = 144+ \text{ in., } \textit{Answer.}$$

55. Circumferential Speeds.—When a flywheel or emery wheel or any circular object makes one complete revolution, each point on the rim or circumference travels once around the circumference and returns to its starting point. Thus any point on the rim travels a distance equal to the circumference when the wheel turns around once. When the wheel turns ten times, the point will have traveled a distance of ten times the circumference. In 1 min. it will travel a distance equal to the product of the circumference times the number of revolutions per minute. The distance, in feet per minute, traveled by a point on the circum-

ference of a wheel is called its *Circumferential Speed, Rim Speed,* or *Surface Speed.* It is also sometimes called *Peripheral Speed,* because the circumference is sometimes given the name of *periphery.* It is by the surface speed that we determine how fast to run our flywheels, belts, emery wheels, and grindstones, and what speeds to use in cutting materials in a machine. Written as a formula·

$$S = C \times N$$

where S is the surface speed
C is the circumference
N is the number of revolutions per minute (r.p.m.)

Expressed in words this formula states that the surface speed of any wheel is equal to the circumference of the wheel multiplied by the number of revolutions per minute.

Example:

What would be the rim speed of a 7-ft. flywheel when running at 210 r.p.m.?

$C = \pi \times D$

$C = \frac{22}{7} \times 7 = 22$ ft.

$S = C \times N$

$S = 22 \times 210 = 4620$

4620 ft. per min., *Answer.*

Explanation: First we find the circumference of the wheel, by multiplying the diameter by π. Here is a case where it is much easier to use $\frac{22}{7}$ for π than to use the decimal 3.1416, and the result is sufficiently accurate for our purposes. We get 22 ft. for the circumference. We can now get the rim speed, which is equal to the product of the circumference times the number of revolutions per minute; or $S = C \times N$. C being 22 ft. and N being 210 r.p.m., we find that S is 4620 ft. per min. Hence, the rim of this flywheel travels at a speed of 4620 ft. per minute.

If we have given the rim speed or surface speed of a flywheel or pulley, and we know the circumference, we can find the r.p.m. by using the method of "working backwards," as explained in Art. 54. Since r.p.m. × Circumference = Surface Speed, it is also true that Speed ÷ Circumference = r.p.m. This is just a matter of reversing the first operation. In the example just worked, if we want to give the flywheel a rim speed of 5280 ft. per minute, it requires no argument to show that the wheel will have to run at 5280 ÷ 22 = 240 r.p.m. In this case, we have the relation,

r.p.m. = Surface Speed ÷ Circumference

or $$N = \frac{S}{C}$$

This formula expresses the same relation as $S = C \times N$, but it is now rearranged to enable us to find the r.p.m. when the rim speed and the circumference are given.

Again, we frequently know the proper surface speed and the number of r.p.m. which a wheel is to run, but the question is to determine the diameter which will satisfy these conditions. This sometimes happens in the case of emery wheels. In the example above we divided the rim speed by the circumference to get the r.p.m., and in a similar manner we can divide the rim speed by the r.p.m. to get the circumference.

$$\text{Circumference} = \text{Surface Speed} \div \text{r.p.m.}$$

or

$$C = \frac{S}{N}$$

After finding the circumference, we can find the diameter by dividing by 3.1416. Here again, we have merely rearranged the formula $S = C \times N$ so as to be in more suitable form for finding the circumference when the surface speed and r.p.m. are given.

Examples:

1. A 12-in. grindstone has a surface speed of 2200 ft. per minute. How many r.p.m. is it making?

$D = 12$ in. $= 1$ ft.

$C = 3.1416 \times 1 = 3.1416$ ft. circumference

$$N = \frac{S}{C}$$

$$N = \frac{2200}{3.1416} = 700+ \text{ r.p.m., } \textit{Answer.}$$

2. A pulley is to be run at 3500 r.p.m., and its surface speed should be about 5500 ft. per minute. What diameter pulley should be used?

First we find the circumference:

$$C = \frac{S}{N}$$

$$C = \frac{5500}{3500} = 1.57 \text{ ft.}$$

From the circumference, we find the diameter:

$$D = \frac{C}{3.1416}$$

$$D = \frac{1.57}{3.1416} = .5 \text{ ft., or a 6-in. pulley would be used, } \textit{Answer.}$$

56. Grindstones and Emery Wheels.—Makers of grindstones and emery wheels usually give the proper speed for the stones in *feet per minute.* This refers to the distance that a

point on the circumference of the stone should travel in 1 min. and is called the "surface speed" or the "grinding speed."

The proper speed at which to run grindstones depends upon the kind of grinding to be done and the strength of the stones. For heavy grinding they can be run quite fast. For grinding edge tools they must be run more slowly to get smooth surfaces and to prevent heating the fine edges of the tools. The following surface speeds may be taken as representing good practice:

Grindstones:

For machinists' tools, 800 to 1000 ft. per minute.
For carpenters' tools, 550 to 600 ft. per minute.

Grindstones for very rapid grinding:

Coarse Ohio stones, 2500 ft. per minute.
Fine Huron stones, 3000 to 3400 ft. per minute.

Sometimes the rule is given for grindstones as follows: "Run at such a speed that the water just begins to fly." This is a speed of about 800 ft. per minute and would be a good average speed for sharpening all kinds of tools.

Examples:

1. A 36-in. grindstone, used for sharpening carpenters' and patternmakers' tools, is run at 60 r.p.m. Is this speed correct?

We must first find the circumference and then the surface speed to see if it falls between the allowable limits.

36 in. $\div$ 12 = 3 ft., the diameter
$C = \pi \times D$
$C = 3.1416 \times 3 = 9.4248$ ft.
$S = C \times N$
$S = 9.4248 \times 60 = 565.488$
$S = 565.488$ f.p.m.

Explanation: First we find the circumference, which is 9.4248 ft. Using this and the r.p.m., we find S to be 565 f.p.m. (feet per minute). As this lies between the allowed limits (550 to 600 f.p.m.), the speed of the stone is correct.

2. How many r.p.m. should a 50-in. Huron stone be run if it is to be used for rough grinding?

$C = \pi \times D$
$C = 3.1416 \times 50 = 157.08$ in.
$C = \frac{157.08 \text{ in.}}{12} = 13.09$ ft.
$N = \frac{S}{C}$
Take $S = 3200$ f.p.m.
$N = \frac{3200}{13} = 246.+$ r.p.m.

Explanation: First we find the circumference of the stone in feet, which is a little over 13 ft. The proper speed is given as 3000 to 3400 f.p.m. Trying 3200 we find that N is 246 r.p.m. The stone should, therefore, be belted to run about 240 or 250 r.p.m.

Emery wheels are usually run at a speed of about 5500 f.p.m. A good, ready rule, easy to remember, is a speed of *a mile a minute.* Most emery wheel arbors are fitted with two pulleys of different diameters. When the wheel is new, the larger pulley on the arbor should be used; and when the wheel becomes worn down sufficiently, the belt should be shifted to the smaller pulley. The belt on an emery wheel should not be shifted, however, without first calculating the effect on the surface speed of the wheel. Many serious accidents have been caused by emery wheels bursting as a result of being driven at too great a speed. Before putting a new wheel on an arbor the resultant surface speed should be calculated to see if the speed in r.p.m. is suitable for the size of the wheel.

Example:

What size emery wheel should be ordered to go on a spindle running 1700 r.p.m.?

$$C = \frac{S}{N}$$

$$C = \frac{5280}{1700} = 3.106 \text{ ft.}$$

$$D = \frac{C}{\pi}$$

$$D = \frac{3.106}{3.1416} = 1 \text{ ft., nearly.}$$

D = 12-in. wheel, *Answer.*

Note.—A wheel of exactly 12-in. diameter would, at 1700 r.p.m., have a surface speed of 5340 f.p.m. ($1700 \times \pi = 5340$).

57. Cutting Speeds.—Cutting speeds on lathe and boring mill work may be calculated in the same way that grinding speeds are calculated. The life of a lathe tool depends upon the rate at which it cuts the metal. This cutting speed is the speed with which the work revolves past the tool and is, therefore, obtained by multiplying the circumference of the work by the r.p.m. The same formulas are used as in the calculations for emery wheels and grindstones but, of course, the allowable speeds are much different. Tables of proper cutting speeds are given in many handbooks in f.p.m. *To find the necessary r.p.m., divide the cutting speed by the circumference of the work measured in feet.*

The cutting speeds used in shops have increased considerably with the advent of the high-speed steels. No exact figures can be given for the best speeds at which to cut different metals. The proper speed depends upon the nature of the cut, whether

finishing or roughing, on the size of the work and its ability to stand heavy cuts, the rigidity and power of the lathe, the nature of the metal being cut, and the kind of tool used. If the work is not very rigid it is, of course, best to take a light cut and run at rather high speed. On the other hand, it is generally agreed that more metal can be removed in the same time if a moderate speed is used and a heavy cut taken.

As nearly as any general rules can be given, the following table gives about the average cutting speeds.

CUTTING SPEEDS IN FEET PER MINUTE

Material to be cut	Cutting speeds	
	Carbon tool steel	High-speed tool steel
Cast iron	30 to 40	60 to 80
Soft steel or wrought iron	25 to 30	50 to 60
Tool steel	20 to 25	40 to 50
Brass	80 to 100	160 to 200

$$\frac{\text{Cutting speed per minute (in feet)}}{\text{Circumference of work (in feet)}} = \text{r.p.m.}$$

Example:

An iron casting is 30 in. in diameter. Find the number of r.p.m. necessary for a cutting speed of 40 f.p.m.

$$C = \pi \times D = 3.1416 \times 30 = 94.248 \text{ in.}$$

$$\frac{94.248}{12} = 7.854 \text{ ft., circumference}$$

$$N = \frac{S}{C} = \frac{40}{7.854} = 5.09 \text{ r.p.m., } \textit{Answer.}$$

The same principles apply to milling and drilling, except that in these cases the tool is turning instead of the work. Consequently, the cutting speeds are obtained from the product of the circumference of the tool times its r.p.m.

In calculating the cutting speed of a drill, figure the speed of the outer end of the lip, or, in other words, the speed of the drill circumference.

Example:

A $\frac{1}{2}$-in. drill is making 300 r.p.m.; what is the cutting speed?

$$3.1416 \times \frac{1}{2} = 1.5708, \text{ circumference in inches}$$

$$\frac{1.5708}{12} = .131 \text{ ft. (nearly)}$$

$$.131 \times 300 = 39.3 \text{ f.p.m., cutting speed}$$

58. Pulleys and Belts.—If the rim of a pulley is run at too great a speed, the pulley may burst. The rim speeds of pulleys are calculated in the same manner as are grinding and cutting speeds. A general rule for cast-iron pulleys is that they should not have a rim speed of over a mile a minute (5280 f.p.m.). This speed may be exceeded somewhat if care is taken that the pulley is well balanced and is sound and of good design.

The proper speeds for belts is taken up fully in a later chapter under the general subject of belting. It is well, however, to point out now that the speed at which any belt is traveling through the air is practically the same as that of the rim of either of the pulleys over which the belt runs; and if we neglect the small amount of slipping which usually occurs between a belt and its pulleys, we can say that the speed of a belt is the same as the rim speed of the pulleys. It will be seen from this that if two pulleys are connected by a belt, their rim speeds are practically the same.

PROBLEMS

166. Compute the circumferences of the following circles whose diameters are 7 in., 2 ft. 3 in., $\frac{1}{\pi}$ ft.

167. Steel hoops are used to reinforce a wooden tank whose outside diameter is 9 ft. 6 in. If each hoop is made from a flat strip of steel, bent into a circle, and riveted together at the ends, how long a strip is required for each hoop, allowing 2 in. extra for lapping the ends?

168. What is the diameter of a post over which a ring will pass whose circumference is 15 in.?

169. If a long thin shaving or "turning" is turned from a steel cylinder 8 in. in diameter and the "turning" does not break until the cylinder has made 2.3 revolutions, what is the length of the "turning"?

170. The Bridgeport Safety Emery Wheel Co., Bridgeport, Conn., makes an emery wheel 36 in. in diameter and recommends a speed of 425 to 450 r.p.m. Calculate the surface speeds of this wheel at 425 and 450 r.p.m.

171. If an engine is rated to run at 100 r.p.m. and the governor must run 120 r.p.m. to maintain that speed, what must be the size of the pulley on the drive shaft if it is belted to the governor pulley whose diameter is 8 in.?

Hint.—The circumferential speed of the two pulleys is the same.

172. The belt conveyor of Fig. 15 is driven by the pulley in the foreground, which is 2 ft. in diameter and runs at 54 r.p.m. What is the speed of the belt in f.p.m.? If approximately 249 lb. of coal are carried by every 10 ft. of belt, how many tons of coal does it convey per hour?

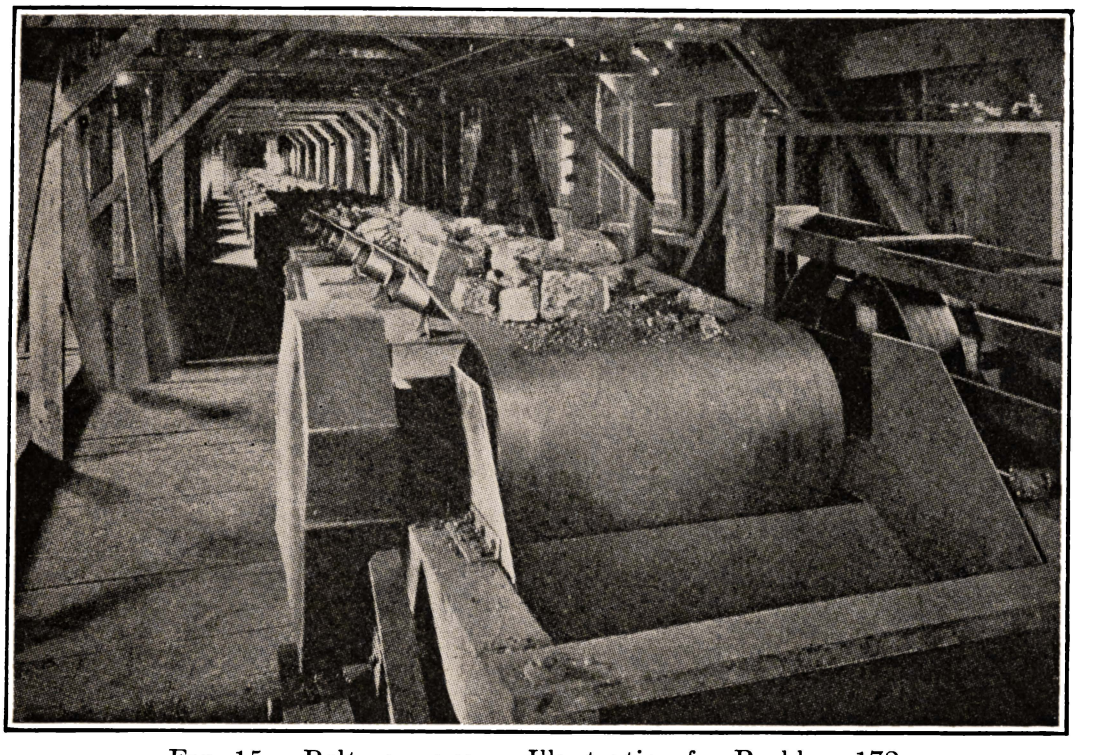


FIG. 15.—Belt conveyor. Illustration for Problem 172.

173. A line shaft rotates at 450 r.p.m. What diameter pulley is required for driving a cupola fan if a belt speed of 4250 f.p.m. is required?

174. A grindstone $3\frac{1}{2}$ ft. in diameter is used for grinding carpenters' tools. How many r.p.m. should it run?

175. At how many r.p.m. should a $1\frac{1}{4}$-in. high-speed drill be run to give a cutting speed of 80 f.p.m.

176. At how many r.p.m. should an 8-in. shaft be driven in a lathe to give a cutting speed of 60 f.p.m.?

177. A contractor purchases a 10 hp. motor having a speed of 1800 r.p.m. to drive a centrifugal pump. He knows that at belt speeds greater than 5000 f.p.m. the belt will slip. What is the diameter of the largest pulley that can be used on the motor without causing slippage?

178. In the appraisal of a certain gas company's property, it was necessary to obtain the diameter of a large gas holder. The diameter could not be measured directly, so the engineer measured

the circumference, which he found to be 550 ft. What was the diameter of the holder?

179. Over what per cent of the circumference of a 7-in. pulley can a 15-in. piece of string be stretched?

180. If a windlass drum 12.5 in. in diameter has $7\frac{1}{2}$ turns of cable around it, what is the approximate length of cable on the drum?

CHAPTER VIII

RATIO AND PROPORTION

59. Ratios.—In comparing the relative sizes of two quantities, we refer to one as being a multiple or a fraction of the other. If one casting weighs 600 lb., and another weighs 200 lb., we say that the first one is three times as heavy as the second, or that the second is one-third as heavy as the first. This *relation* between two quantities of the same kind is called a *ratio.*

In comparing the speeds of two pulleys, one of which runs 40 r.p.m. and the other one 160 r.p.m., we say that their speeds are "as 40 is to 160," or "as 1 is to 4." In this sentence, "40 is to 160" is a ratio, and so also is "1 is to 4" a ratio.

Ratios may be written in three ways. For example, the ratio of (or relation between) the diameters of two pulleys which are 12 in. and 16 in. in diameter may be written as a fraction, $\frac{12}{16}$; or, since a fraction means division, it may be written $12 \div 16$. Again, the line in the division sign is sometimes left out and it becomes 12:16. The last method, 12:16, is the one most used and will be followed here. It is read "twelve *is to* sixteen."

A ratio may be reduced to lower terms the same as a fraction, without changing the value of the ratio. If one bin in the stock room contains 1000 washers, while another bin contains 3000, then the ratio of the contents of the first bin to the contents of the second is "as 1000 is to 3000." The ratio of 1000 to 3000 can be reduced by dividing both by 1000. This leaves the ratio 1 to 3.

$$\frac{1000 \div 1000}{3000 \div 1000} = \frac{1}{3}$$

Hence, the ratio between the contents of the bins is also 1 to 3.

Likewise, the ratio 24:60 can be reduced to 2:5 by dividing both terms by 12. If we write it as a fraction we can easily see that

$$\frac{24}{60} = \frac{24 \div 12}{60 \div 12} = \frac{2}{5}$$

Therefore, 24:60 = 2:5.

The ratio of the 1000 washers to the 3000 washers is 1000:3000 or 1:3.

The ratio of 8 in. to 12 in. is 8:12 or 2:3.

The ratio of \$1 to \$1.50 is $1:1\frac{1}{2}$ or 2:3.

The ratio of 30 castings to 24 castings is 30:24 or 5:4.

60. Proportion.—When two ratios are equal, the four numbers (or terms) are said to be *in proportion.* The two ratios 2:4 and 8:16 are clearly equal, because we can reduce 8:16 to 2:4 and we can, therefore, write 2:4 = 8:16. When written thus, these four numbers form a *proportion.* When two ratios are written as proportions, the division line is sometimes left out and two dots put in its place. Thus $\frac{2}{4} = \frac{8}{16}$ may be written 2:4::8:16. This is read "2 is to 4 as 8 is to 16."

Likewise, we can say that the numbers 6, 8, 15, and 20 form a proportion because the ratio 6:8 is equal to the ratio of 15:20.

$$6:8 = 15:20$$

Now, it will be noticed that if the first and fourth terms of this proportion be multiplied together, their product will be equal to the product of the second and third terms:

(First term)		(Second term)		(Third term)		(Fourth term)
6	:	8	=	15	:	20

$$6 \times 20 = 120$$

$$8 \times 15 = 120$$

$$\text{Therefore, } 6 \times 20 = 8 \times 15$$

This is true of any proportion and forms the basis for an easy way of working practical examples, where we do not know one term of the proportion but know the other three. The first and fourth terms are called the *extremes,* and the second and third are called the *means.* Then we have the rule: "*The product of the means is equal to the product of the extremes.*"

This relation can be simply expressed as a formula.

Let a, b, c, and d represent the four terms of any proportion so that

$$a:b = c:d$$

Then, according to our rule, we have

$$a \times d = b \times c$$

Let us now see of what practical use this is. We will take this example:

If it requires 137 lb. of metal to make 19 castings, how many pounds will it take to make 13 castings from the same pattern?

Now very clearly, the ratio between the *number of castings*, "19:13," is the same as the ratio of the *weights*, or 19 *is to* 13 as the weight for 19 castings is to the weight for 13 castings.

Substituting the word "answer" in place of "weight for 13 castings," we get

$$19:13 = 137:\text{Answer}$$

Since the product of the means equals the product of the extremes, we know that

$$13 \times 137 = 19 \times \text{Answer}$$
$$\text{or } 1781 = 19 \times \text{Answer}$$

If $19 \times \text{Answer} = 1781$, then the answer must equal $\frac{1}{19}$ of 1781.

$$\text{Answer} = \frac{1781}{19}$$
$$\text{Answer} = 93.7+ \text{ lb.}$$

In using proportion, the following should be kept in mind:

Rule 1: Be sure that each ratio is made up of two like terms. Thus in the problem just solved, the first ratio is made up of 19 and 13, which are both *numbers of castings*, and the second ratio consists of two *weights*.

Rule 2: Be sure that when the first term is larger than the second the third term is also larger than the fourth, or if the first term is smaller than the second, the third is also smaller than the fourth.

Rule 3: Solve by the rule that "the product of the means equals the product of the extremes."

Examples:

1. A certain firm bought 13 used motors for a lump sum of $520. A short time later it purchased 9 more at the same rate that was paid for the first lot. What was the cost of the second lot of 9 motors?

Since they were paid for at the same rate, the costs of the two lots will be proportional to the number of motors in each lot.

By our first rule, then, one ratio will be *numbers of motors* and the other ratio will be *costs*.

Ratio of the numbers of motors is 13:9.

Ratio of the costs is 520:cost of 9 motors.

The proportion will be,

13:9 = 520:cost of 9 motors.

By rule 2, the fourth term must be smaller than the third term, since the second term, 9, is smaller than the first term, 13. The cost of 9 motors will

evidently be less than the cost of 13, so we know our proportion is correct. Substituting the word "answer" for "cost of 9 motors" we have

$$13:9 = 520:\text{Answer}$$

Multiplying the means and extremes we get:

$$13 \times \text{Answer} = 9 \times 520$$
$$13 \times \text{Answer} = 4680$$
$$\text{Answer} = \frac{1}{13} \times 4680$$
$$\text{Answer} = \$360$$

2. Two men receive the same rate of pay. One man works 9 hr. and receives \$6. If the other man works 11 hr., how much does he receive?

The ratio of *time* is 9:11

The ratio of *pay* is \$6:Answer

$$9:11 = 6:\text{Answer}$$

By multiplying means and extremes, we get

$$9 \times \text{Answer} = 11 \times 6$$
$$9 \times \text{Answer} = \$66$$
$$\text{Answer} = \frac{1}{9} \times 66$$
$$\text{Answer} = \$7.33\tfrac{1}{3}$$

The preceding examples have all been what is termed "direct proportion," since the weights increase directly as the number of castings increase; the costs increase directly as the number of motors; and the pay increases directly as the number of hours.

61. Inverse Proportion.—There are certain cases of proportion where for a given *increase* of one factor, there is a corresponding *decrease* of another factor. Two pulleys belted together are an example of this. Our own observation tells us that the larger of the two pulleys makes the less number of r.p.m., and the more we increase the diameter of either pulley, the more will its speed in r.p.m. decrease provided the belt speed is not changed.

This is called an "inverse proportion," and the number of revolutions are said to vary *inversely* as the diameter; that is, in the reverse order. The same rules are observed for inverse proportions as for direct proportions.

In direct proportions, the first and third terms correspond, and the second and fourth terms correspond. Thus in the last example, 9 hr. corresponds to a pay of \$6 and 11 hr. to a pay of \$7.33. In inverse proportions, however, the first and fourth terms correspond and the second and third correspond.

Example:

Two pulleys of diameters 9 in. and 12 in. are belted together. The 9-in. pulley makes 270 r.p.m. How many r.p.m. does the 12-in. pulley make?

The first two terms of the proportion will be the ratio of the diameters, 9:12.

The ratio of the r.p.m. will then be as *r.p.m. of 12-in. pulley:* 270, since, according to rule 2, the third term must be smaller than the fourth if the first is smaller than the second. Setting up the proportion,

$$9:12 = \text{Answer}:270$$

It will be noted that 270 r.p.m. corresponds to the 9-in. pulley, and that these two terms are extremes.

We now solve by rule 3.

$$12 \times \text{Answer} = 9 \times 270$$
$$12 \times \text{Answer} = 2430$$
$$\text{Answer} = \frac{1}{12} \times 2430$$
$$\text{Answer} = 202.5 \text{ r.p.m.}$$

62. Speeds and Diameters of Pulleys.—Problems involving speeds and diameters of pulleys can be worked by proportion, as just explained, or they can be worked by a set rule, based on proportion, without going through all the process of setting up the proportion.

In the previous example it was pointed out that the diameter and speed of one pulley form the extremes of the proportion, and the diameter and speed of the other form the means. Since the product of the means equals the product of the extremes, we obtain the following simple relation for pulleys belted together: The product of the diameter and revolutions of one pulley equals the product of the diameter and revolutions of the other. This gives us the following simple rule for working pulley problems.

Rule for Finding the Speeds or Diameters of Pulleys.—Take the pulley of which we know both the diameter and the r.p.m., and multiply these two numbers together. Then divide this product by the known number of the other pulley. The result is the desired number.

Examples:

1. A 36-in. pulley running 240 r.p.m. is belted to a 15-in. pulley. Find the r.p.m. of the 15-in. pulley.

$36 \times 240 = 8640$, the product of the known diameter and revolutions.

$8640 \div 15 = 576$, the r.p.m. of the 15-in. pulley, *Answer.*

2. A 36-in. grindstone is to be driven at a speed of 800 f.p.m. from a 6-in. pulley on the line shaft which is running 225 r.p.m. What size pulley must be put on the grindstone arbor?

$$N = \frac{S}{C}$$

$$= \frac{800}{3.1416 \times 3} = 85 \text{ r.p.m., nearly}$$

$$6 \times 225 = 1350$$

$$1350 \div 85 = 16 \text{ in., nearly}$$

Use a 16-in. pulley on the arbor

Explanation: First we must find the r.p.m. for the grindstone, as explained in Chap. VII. To get the required surface speed we find 85 r.p.m. necessary.

Now we have the r.p.m. and the size of the line shaft pulley. The product of these two numbers is 1350. Dividing this by the r.p.m. of the grindstone arbor gives 16 in. as the nearest even size of pulley; so we will use that size.

PROBLEMS

181. What is the ratio of the circumference of a circle to its diameter?

182. What is the value of A (*Answer*) in the following proportions:

(*a*) $17 : 45 = 14 : A$

(*b*) $3 : A = 5 : 25$

(*c*) $3\frac{3}{4} : 9\frac{4}{5} = 6 : A$

183. Two pulleys are belt connected. The smaller one runs at a speed of 750 r.p.m. and the larger at 200 r.p.m. What is the ratio of their speeds?

184. The diameter of the smaller pulley in Problem 183 is 4 in. What is the diameter of the larger pulley?

185. Two car repairers are paid on a piece work basis. One man completes an average of 21 pieces of work per day and the other completes $10\frac{1}{2}$ pieces per day. What is the ratio of the first man's pay to that of the second man's?

186. Three men, each operating a drill press, are able to turn out 156 pieces a day. The foreman receives a rush order for this particular piece and puts two more drill presses at this job. How many pieces can he then obtain per day?

187. A boiler uses an average of 2 tons of coal every $5\frac{1}{2}$ hr. How many tons will it use in a day of 24 hr.?

188. A high-speed engine running at 300 r.p.m. drives a generator. The pulley on the engine shaft is 4 ft. in diameter, and the generator pulley is 16 in. in diameter. At what speed will the generator run?

189. In the manufacture of a certain type of boiler an approximate ratio of heating surface to grate surface of 50:1 is used. About what heating surface would a 100-hp. boiler of this type contain if the grate area was 35 sq. ft.?

190. The line-shaft in Fig. 16 runs 250 r.p.m. Determine the size of line-shaft pulley to run the grinder at 1600 r.p.m., using the countershaft as shown in the figure.

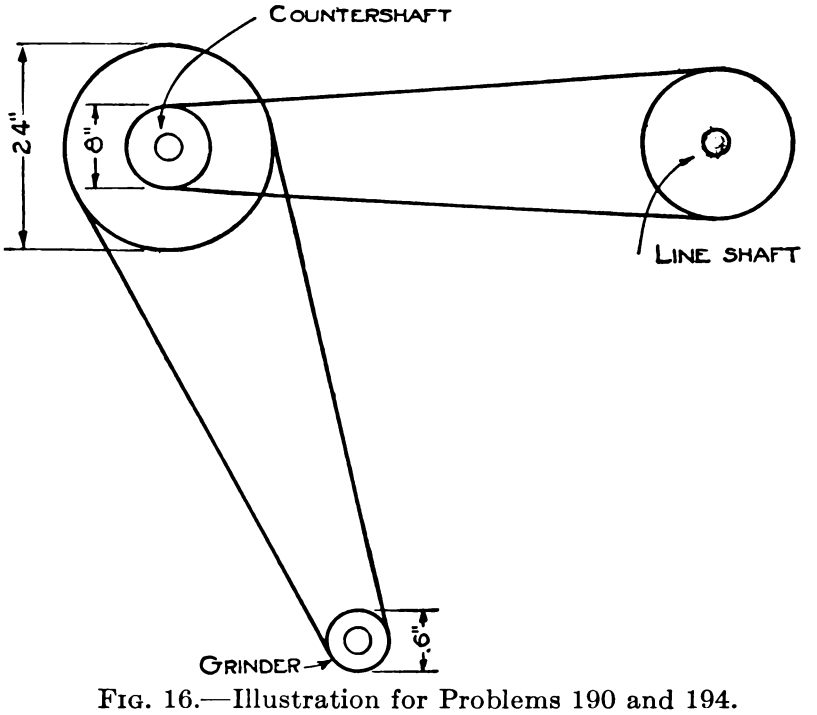


FIG. 16.—Illustration for Problems 190 and 194.

191. A train travels 378 mi. in 9 hr. Set up a proportion and determine how far the train will travel in 14 hr.

192. If the ratio of men to women in a community is 7 to 6, what is the population if there are 4200 men? How many women are there?

193. If it takes 17 min. to fill a 75 gal. tank, how long will it take to fill the tank .6 full if the rate of flow is increased 7%?

194. The grinding wheel of Fig. 16 is replaced by a 6-in. emery wheel, which must be run at a surface speed of 5000 ft. per minute. The large pulley on the countershaft is 15 in. in diameter, the line-shaft pulley is 3 ft. in diameter, and the speed of the line-shaft is 244 r.p.m. Calculate the required diameter of the small pulley on the countershaft.

195. A city with a population of 62,500 has 9000 telephones. If the ratio of telephones to population does not change, determine the number of telephones when the population reaches 100,000.

CHAPTER IX

GEAR RATIOS; GEAR TRAINS; CHANGE GEARS

63. Gear Ratios.—The same principles as are applied to pulleys can be applied to gears. If we have two gears running together as shown in Fig. 17, the product of the diameter and r.p.m. of one gear will be equal to the product of the diameter and r.p.m. of the other. In studying gearing, we do not deal with the diameters so much as we do with the numbers of teeth. We find that gears are generally designated by the numbers of teeth. For

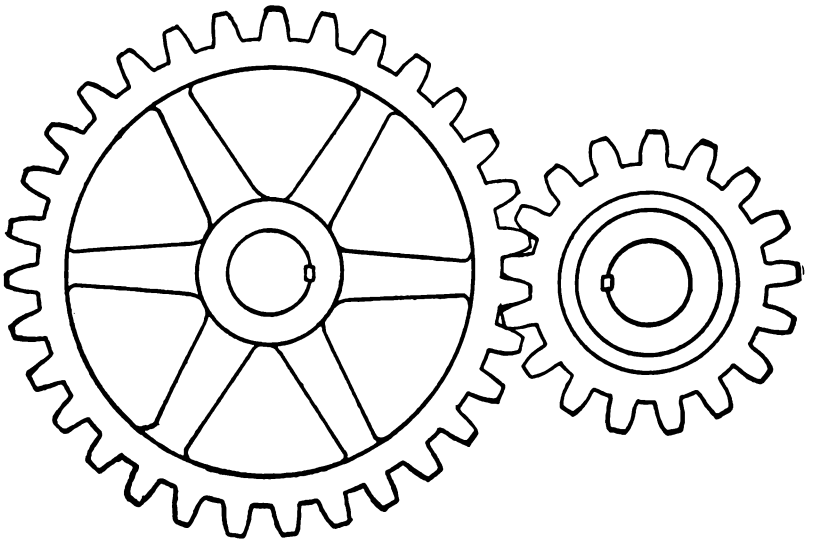


FIG. 17.—Pair of spur gears.

example, we talk of 16-tooth gears and 24-tooth gears, etc., but we seldom talk about gears of certain diameters.

In making these calculations for gears, we can use the numbers of teeth instead of the diameters. When a gear is revolving, the number of teeth that pass a certain point in one minute will be the product of the number of teeth times the r.p.m. of the gear.

If one gear is driving another, as in Fig. 17, each tooth on one gear will force along one tooth on the other one. Consequently, the product of the number of teeth times the r.p.m. of the second gear will be the same as for the first gear. This gives us our rule for the relation of the speeds and numbers of teeth of gears.

Rule for Finding the Speeds or Numbers of Teeth of Gears: Take the gear of which we know both the r.p.m. and the number of teeth and multiply these two numbers together. Divide their product by the number that is known about the other gear. The quotient will be the unknown number.

Example:

A 38-tooth gear running 360 r.p.m. is to drive another gear at 190 r.p.m. What must be the number of teeth on the other gear?

$38 \times 360 = 13{,}680$, the product of the number of teeth and revolutions of one gear.

$$190 \times \text{Answer} = 13{,}680$$

$$\text{Answer} = \frac{13{,}680}{190}$$

$$\text{Answer} = 72 \text{ teeth.}$$

64. Gear and Pulley Trains.—A gear train consists of any number of gears used to transmit motion from one point to another. Figure 18 shows the simplest form of gear train, having but two gears. Figure 19 shows the same gears *A* and *B*, as in Fig. 18, but with a third gear, usually called an intermediate or idle gear, between them. The intermediate gear *C* can be used for either of two reasons:

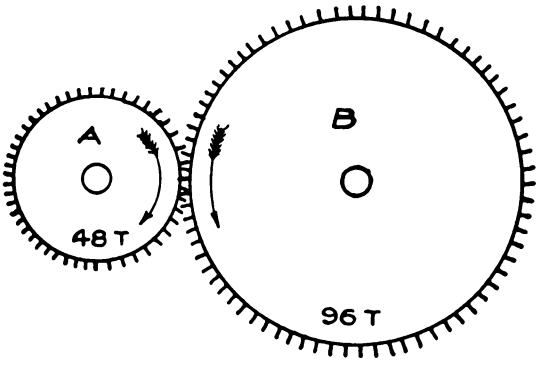


FIG. 18.—Simple form of gear train.

1. To connect *A* and *B* and thus permit of a greater distance between the centers of *A* and *B* without increasing the size of the gears; or

2. To reverse the direction of rotation of either *A* or *B*. If *A* turns in a clockwise direction, as shown in both Figs. 18 and 19, *B* in Fig. 18 will turn in the opposite, or counterclockwise direction, but in Fig. 19, *B* will turn in the same direction as *A*.

The introduction of the intermediate gear C has no effect on the speed ratio of A to B. If A has 48 teeth and B 96 teeth, the speed ratio of A to B will be 2 to 1 in either Fig. 18 or Fig. 19.

From Fig. 19 we can set up the proportion

$$\text{r.p.m. of } A : \text{r.p.m. of } B = 96 : 48$$

or writing the ratios in the form of fractions

$$\frac{\text{r.p.m. of } A}{\text{r.p.m. of } B} = \frac{96}{48} \text{ or } \frac{2}{1}$$

Thus the speed ratio of A to B is 2 to 1.

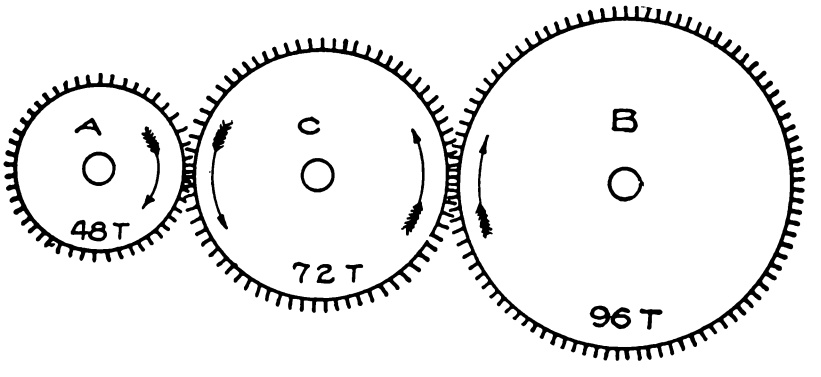


FIG. 19.—Gear train with idler.

In the case shown in Fig. 19, when A moves a distance of one tooth, the same amount of motion will be given to C, and C must at the same time move B one tooth. To move B 96 teeth, or one revolution, will require a motion of 96 teeth on A, or two revolutions of A. Hence, A will turn twice to each turn of B, or the speed ratio of A to B is 2 to 1, just as in the case of Fig. 18.

65. Compound Gear and Pulley Trains.—Quite often it is desired to make such a great change in speed that it is practically necessary to use two or more pairs of gears or pulleys to accomplish it. If a great increase or reduction of speed is made by a single pair of gears or pulleys, it means that the difference in the diameters will have to be very great. The belt drive of a lathe is an example of a compound train of pulleys, though here the train is used chiefly for other reasons. In the first step the pulley on the line shaft drives a pulley on the countershaft; then another pulley on the countershaft drives the lathe. The back gearing on a lathe is an example of compound gearing, two pairs of gears being used to make the speed reduction from the cone pulley to the spindle and face plate.

Figure 20 shows a common arrangement of compound gearing. Here A drives B and causes a certain reduction of speed. B and C are fastened together, and therefore, make the same number of revolutions per minute. A further reduction in speed is made by the two gears C and D. A and C are the driving gears of the two pairs, and B and D are the driven gears.

The safest and surest method of calculating compound gear and pulley trains is to start with the gear or pulley of known diameter

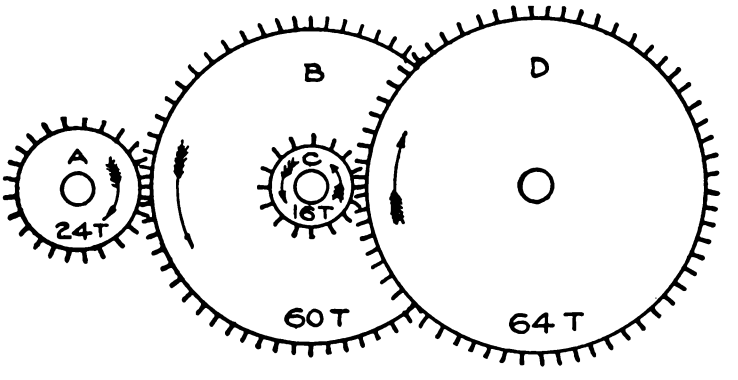


FIG. 20.—Compound gear train.

and r.p.m.; work from that pulley or gear to the next; and thus proceed from one pulley or gear to another by the simple rules for the solution of pulleys and gears given in preceding paragraphs.

Examples:

1. Let us calculate the speed ratio for the train of gears in Fig. 20. This would be the ratio of the r.p.m. of A to the r.p.m. of D.

Let N stand for the r.p.m. of A.

Now, by the rule of Art. **63.**

$$\text{r.p.m. of } B = \frac{24 \times N}{60} = .4N$$

The r.p.m. of C is the same as that of B, or $.4N$

$$\text{r.p.m. of } D = \frac{16 \times .4N}{64} = .1N, \text{ or } \frac{1}{10}N$$

Here we have used the same rule as we did for r.p.m. of B. Gear C is the known gear, and its r.p.m. is $.4N$ and its number of teeth is 16. We now find the ratio of the r.p.m. of A and D.

$$\frac{\text{r.p.m. of } A}{\text{r.p.m. of } D} = \frac{N,}{\frac{1}{10}N} = \frac{10}{1}, \textit{ Answer.}$$

The N cancels out and leaves the ratio 10 to 1.

2. Let us take the pulley train of Fig. 21 and calculate the speed ratio of the pulleys A and D.

Let N equal r.p.m. of A.

$$\text{r.p.m. of } B = \frac{36 \times N}{10} = 3.6N$$

$$\text{r.p.m. of } C = \text{r.p.m. of } B = 3.6N$$

$$\text{r.p.m. of } D = \frac{24 \times 3.6N}{12} = 7.2N$$

The speed ratio of A to D is, then,

$$\frac{\text{r.p.m. of } A}{\text{r.p.m. of } D} = \frac{N}{7.2N} = \frac{1}{7.2}, \text{ or 1 to 7.2, } \textit{Answer.}$$

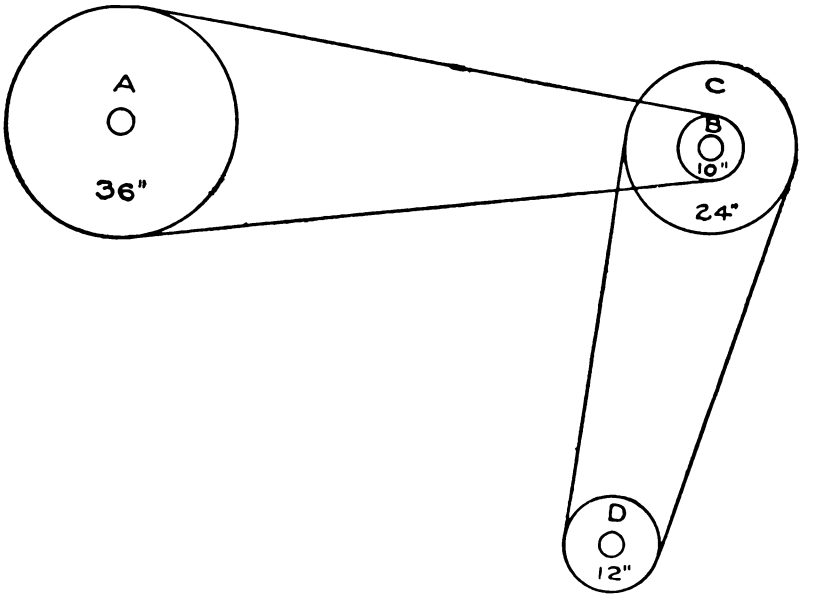


FIG. 21.—Train of pulleys.

66. Screw Cutting.—When threads are cut in a lathe, it is necessary to have a definite speed ratio between the lead screw and the work (or lathe spindle), this speed ratio depending upon the number of threads per inch which it is desired to cut. The correct speed ratio in any case is obtained by means of a train of gears between the lead screw and the spindle. In modern lathes these gears are usually mounted in a gear box at the headstock, and the proper combination for cutting any particular thread is effected by the manipulation of levers or knobs according to the directions mounted on the machine. There are, however, many lathes in use which have a set of separate "change gears" from which the machinist must select the proper combination to be used for cutting any particular thread. Such lathes are fitted with a name plate stating the gears to use for each thread. But these plates are occasionally missing from old lathes, or some of the gears have been broken or lost which makes it necessary to use a combination not listed on the plate.

In such cases the machinist must calculate the proper gear combination. In any case, a mechanic should have sufficient knowledge of gearing and arithmetic to enable him to make the necessary selection.

The lathe carriage and tool are moved by a "lead screw" having usually 2, 4, 6, or 8 threads per inch. If the lathe has a lead screw with 6 threads per inch, 6 revolutions of the lead screw will move the carriage and tool 1 in. By referring to Fig. 22 it can easily be seen that if the spindle, or work, revolves *once* while the lead screw makes 6 revolutions, the tool will cut

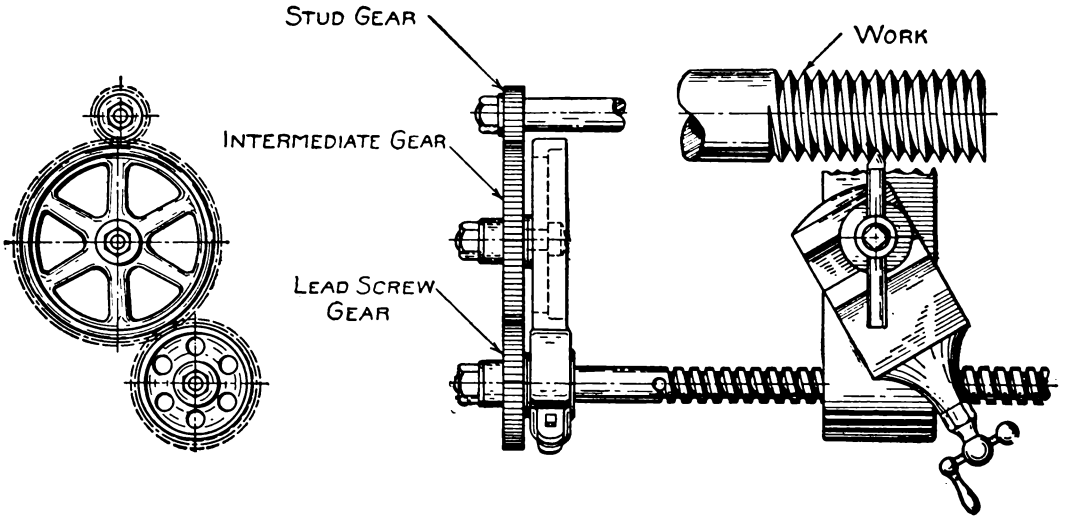


FIG. 22.—Simple gears for thread cutting.

one thread per inch; if the spindle revolves twice during this time the tool will cut 2 *threads* per inch, etc. The ratio of the spindle speed to the speed of the lead screw will, therefore, be

$$\frac{\text{Speed of spindle}}{\text{Speed of lead screw}} = \frac{\text{Threads per inch to be cut}}{\text{Threads per inch on lead screw}}$$

In the *simple-geared lathe*, the gears which are changed to obtain the proper ratio are the "stud gear" and the "lead screw gear," as shown in Fig. 22. The intermediate gear is an idler which has no effect on the speed ratio. The stud gear is connected to the spindle or work by a fixed train of gears such that the spindle may or may not have the same speed as the stud gear. The fixed-speed ratio of the spindle to the stud gear should first be determined by actually counting the number of teeth on the spindle and fixed stud gear, or by placing equal gears on the stud and lead screw and determining what thread would be cut. In the latter case the stud makes the same number of revolutions

as the lead screw, and the ratio is found by substituting "speed of stud gear" for "speed of lead screw" in the formula just given. Calling this ratio R, we can write the following formula to determine the desired change gears for a simple-geared lathe.

$$R \times \frac{\text{Threads per inch to be cut}}{\text{Threads per inch on lead screw}} = \frac{\text{Lead screw gear teeth}}{\text{Stud gear teeth}}$$

To cut 12 threads per inch with a lathe that has a 6-pitch lead screw and a fixed ratio, R, of 1 to 1, requires

$$\frac{1}{1} \times \frac{12}{6} = \frac{12}{6} \text{ or } \frac{60 \text{ teeth on lead screw gear}}{30 \text{ teeth on stud gear}}$$

Any other two gears the ratio of which is 12:6 or 2:1 may be used, provided that they are otherwise suitable.

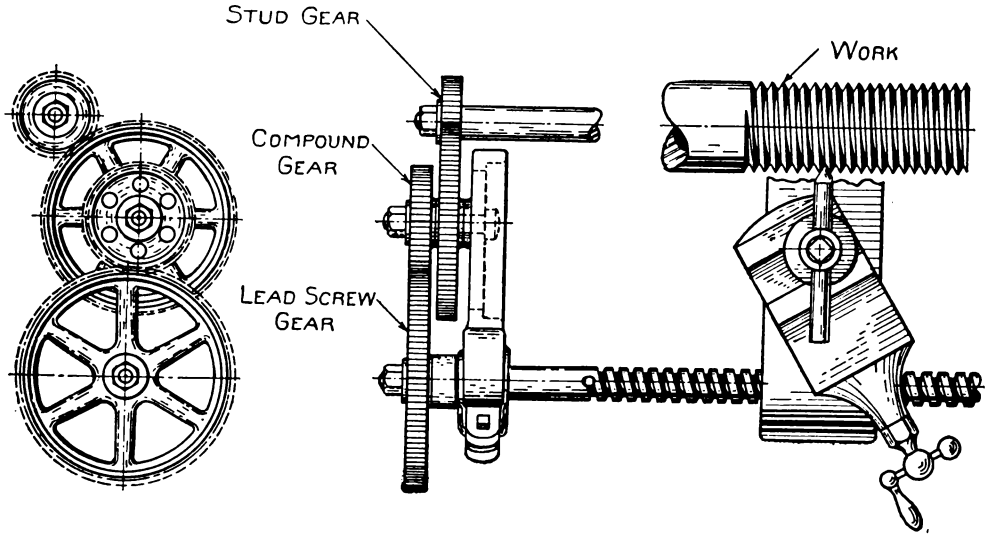


FIG. 23.—Compound gears for thread cutting.

When it is impossible or undesirable to use the gears required in simple gearing to cut a certain thread, the intermediate or idler gear is replaced by two gears, as in Fig. 23, giving what is called "compound gearing." The ratio R is obtained for compound gearing in the same manner as before, using equal gears on the stud and lead screw and also a pair of equal gears on the idler stud.

The following formula for the change gears is then used:

$$R \times \frac{\text{Threads per inch to be cut}}{\text{Threads per inch on lead screw}} = \frac{\text{Lead screw gear} \times \text{Driven compound gear}}{\text{Stud gear} \times \text{Driving compound gear}}$$

Taking the case just worked out for simple gearing, we have

$$\frac{1}{1} \times \frac{12}{6} = \frac{12}{6} \qquad \frac{12}{6} = \frac{60 \times 40}{30 \times 40}$$

This would give,

Driven compound = 60 Lead screw gear = 40
Driving compound = 30 Stud gear = 40

The combined ratio, $\frac{60 \times 40}{30 \times 40}$, must equal $\frac{12}{6}$. It can readily be seen that there are a number of combinations which would satisfy the requirements. It is simply a matter of selection, depending upon the available supply of gears.

PROBLEMS

196. In Fig. 18 if we removed the 48-tooth gear and put a 64-tooth gear in its place, what would be the speed ratio of A to B?

197. If B in Fig. 19 makes 6 revolutions, how many will C make, and how many will A make?

198. What are the gear ratios of two gears for the following number of teeth:

	A	B
(a)	6	12
(b)	6	18
(c)	7	21
(d)	34	17
(e)	1000	999

199. A machinist wishes to thread a pipe on a lathe having 2 threads per inch on the lead screw. There are to be $11\frac{1}{2}$ threads per inch on the pipe. What is the ratio of the speed to the spindle and lead screw?

200. The lead screw on the table of a milling machine has a double thread with a pitch of $\frac{1}{4}$ in. How many inches per minute is the feed if the lead screw is making 4 r.p.m.?

201. The power shear of Fig. 24 makes 20 strokes per minute at its maximum speed with the motor running at a speed of 1200 r.p.m. The small chain sprocket on the motor has 20 teeth, the large sprocket has 80 teeth, and the pinion meshing with the large gear has 10 teeth. Calculate the number of teeth required for the large gear.

Note.—The large gear revolves once every stroke, and the two sprockets can be considered as two gears in mesh.

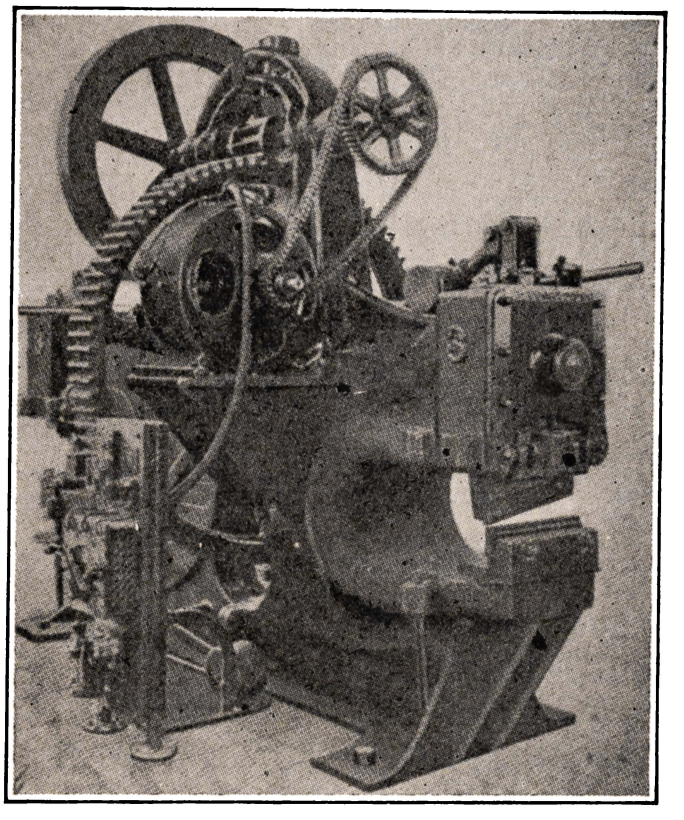


FIG. 24.—Power shear. Illustration for Problem 201.

202. Two gears are to have a speed ratio of 4.6 to 1. If the smaller gear has 15 teeth, what must be the number of teeth on the larger gear?

203. If the gear ratio of two gears is 24 to 6, and the larger gear speed is increased from 120 r.p.m. to 200, what will be the per cent increase in the speed of the smaller gear?

204. If you were to cut a 20-pitch thread on a lathe having a 4-pitch lead screw, what would be the ratio of the speeds of the spindle and the lead screw?

205. The lead screw of a lathe has 5 threads per inch. What gear ratio will be required to cut 8 threads per inch?

CHAPTER X

SQUARE ROOTS AND TABLES

67. Powers.—In taking up the calculation of square roots we must first come to an understanding of the terms "powers" and "roots."

A number is said to be "raised to a power" when it is multiplied by itself a certain number of times. The number of times which it is multiplied together determines to what power it is raised.

Thus, 2 raised to the *second power* is $2 \times 2 = 4$; 2 raised to the *third power* is $2 \times 2 \times 2 = 8$; 5 raised to the *third power* is $5 \times 5 \times 5 = 125$; 3 raised to the *fourth power* is $3 \times 3 \times 3 \times 3 = 81$; etc. Instead of writing out the multiplications as $5 \times 5 \times 5 = 125$, it is customary to write a small figure, called the "exponent," at the right of the number and slightly higher, which indicates the power it is raised to; thus, $5^3 = 125$. The small 3 indicates the third power of 5. Likewise, instead of $3 \times 3 \times 3 \times 3 = 81$, we would write $3^4 = 81$, which is read "3 to the fourth power equals 81."

The second power and the third power of any number are usually called the "square" and the "cube," respectively, and to raise a number to these powers, we "square" or "cube" the number. For instance, the square (or second power) of 6 is $6^2 = 36$; the *cube* (or third power) of 4 is $4^3 = 64$; 5 *cubed* is $5^3 = 125$; 3 *squared* is $3^2 = 9$, etc.

68. Roots.—Roots are the opposite of powers. The fourth power of 2 is $2^4 = 16$, and conversely the *fourth root* of 16 is 2. Roots are designated as *square roots, cube roots, fourth roots, fifth roots,* etc., the same as powers. To find some root of a number, such as the cube root of 27, we find a number which taken as a factor three times will give a product of 27. Thus 3 is the cube root of 27, since $3 \times 3 \times 3 = 27$; 5 is the fourth root of 625, since $5 \times 5 \times 5 \times 5 = 625$; 6 is the square root of 36, since $6 \times 6 = 36$.

The square root of a given number is, therefore, another number which when multiplied by itself will produce the given number. The square root of 4 is 2, the square root of 9 is 3, the square root of 16 is 4, etc.

Just as the small figure at the right is used to indicate the power of a number, so there is a conventional method of indicating roots. Taking the square root of a number is designated by the sign $\sqrt{\ }$ placed over the number, and this sign is called the "square root" or "radical" sign. Thus $\sqrt{4} = 2$, which is read "the square root of 4 equals 2;" $\sqrt{9} = 3$, etc. To represent a different root, such as the cube root, fourth root, etc., a small figure designating the root is placed at the upper left-hand corner of the radical sign. Thus, the cube root of 27 is written $\sqrt[3]{27} = 3$; the fourth root of 16 is written $\sqrt[4]{16} = 2$.

69. Extracting the Square Root.—Finding the square root of a given number is called *extracting the square root.* For simple numbers such as 4, 9, 16, 36, etc., the square root can be determined by inspection, but we must have some method that will apply to any number.

The actual calculation of square roots is a somewhat laborious process, but not a difficult one to learn. The first step in finding the square root of a number is to divide the number into periods or groups of two figures each, beginning at the decimal point and working both ways. The following examples indicate how this is done.

9'20'.	6'.42'97'
2'53'20'.	'21'.3
'76'30'.79'9	'67'32'.6

The number of periods to the left of the decimal point thus marked off determines the number of digits to the left of the decimal place in the square root. Thus the number 10'24'. has two groups or periods and its square root, 32, has two digits to the left of the decimal place.

The remainder of the calculation can best be explained by the solving of some examples, explaining them as we go along.

Example:

Find the square root of 186,624.

Point off into periods of two figures each (18'66'24') as explained, and it will be seen that the root must contain three digits to the left of the decimal point. The work is arranged very similarly to division.

			18'66'24'(432, *Answer.*
			16
(Trial divisor)	$20 \times 4 = 80$		2 66
		3	
(Final divisor)		83	2 49
(Trial divisor)	$20 \times 43 = 860$		17 24
		2	
(Final divisor)		862	17 24

Explanation: Find the largest integral number whose square is equal to, or less than, the first period, 18. This is 4, since 5^2 is more than 18. Write the 4 to the right for the first figure of the answer, just as the quotient is put down for division. Square the 4 and place its square, 16, under the first period, 18, and subtract, leaving a remainder of 2. Bring down the next period, 66, and annex it to the 2, giving 266 for what is called the *dividend.* Multiply that part of the root which we have already found, namely, 4, by 20 to get the *trial divisor.* Set this off to the left. Divide 266 by the trial divisor, 80, approximately. This is 3+, so the next figure of our root is probably 3. Write the 3 as the second figure of the answer, and then add it to the trial divisor 80, giving 83, which is the *final divisor.* Multiply the final divisor, 83, by the figure of the root just found, 3, giving 249. Subtract this from the dividend, 266, leaving 17. If the product of the final divisor and the last figure of the root happened to be larger than the dividend, it means that the figure last found in the root is too large and the next smaller figure should be taken.

Bring down the next period, 24, and annex it to the 17, giving a new dividend, 1724. Repeat the preceding process as follows: multiply that part of the root already found (43) by 20, giving 860 for the new trial divisor. Divide the dividend (1724) by the trial divisor, obtaining 2+ as the next figure of the answer. Write the 2 as the third figure of the answer and also add it to the trial divisor, 860, giving a final divisor of 862. Multiply this by 2 and subtract from 1724. There is no remainder, and as there are no more periods in the number, the root is complete. Therefore, $\sqrt{186624} = 432$, *Answer.*

70. Square Roots of Mixed Numbers.—If it is required to find the square root of a number composed of a whole number and a decimal, begin at the decimal point and point off periods to the right and left. If the mixed number is composed of 1 whole number and a fraction, convert the fraction to a decimal (see p. 49). Then find the root as before.

Example:

Find the square root of 257.8623.

2'57'.86'23'00(16.058+, *Answer.*

		1	
(Trial divisor)	20 × 1 = 20	1 57	
	6		
(Final divisor)	26	1 56	
(Trial divisor)	20 × 16 = 320	1 86 23	
	20 × 160 = 3200		
	5		
(Final divisor)	3205	1 60 25	
(Trial divisor)	20 × 1605 = 32100	25 98 00	
	8		
(Final divisor)	32108	25 68 64	
		29 36	

Explanation: Taking the nearest square root of the first period, 2, we get 1 for the first figure of the root. Square the 1, subtract from the first period, and bring down the next period, giving a dividend of 157. The trial divisor is then 20, which goes into 157 about 7 times. If we try 7 as the next figure of the root, the final divisor will be 27, which multiplied by 7 makes 189. This is larger than 186, the dividend; therefore, 7 is too large and we must use 6 instead of 7 for the next figure of the root. Since 57 was the last period preceding the decimal point, we then place the decimal point after the 6 in the root. The final divisor is 26. Multiply by 6 and subtract from 157, leaving a remainder of 1. Bring down the next period, 86, making a new dividend 186. The new trial divisor is 320, which we find is larger than the dividend, 186. The next figure of the root is, therefore, 0, so we place a cipher after the 6. In this case, then, simply bring down the next period, 23, and annex it to the previous dividend, making a new dividend of 18,623. We also find a new trial divisor 3200, and proceed anew. The next figure of the root is 5. After multiplying and subtracting from the dividend there is still a remainder, 2598. In case a root does not come out exactly, we can continue the process by adding additional ciphers to the number. The next period will then be two zeros and the next figure of the root turns out to be 8. There is still a remainder but we have the root to three decimal places which is quite accurate enough for ordinary work. We place a plus sign after the 8 to indicate that there are more places which have not been calculated.

71. Square Roots of Decimals.—Sometimes in the case of a decimal, one or more periods are composed entirely of ciphers. The root will then contain one cipher following the decimal point for each full period of ciphers in the number.

Example:

Take 0007856 as an example.

Beginning at the decimal point and pointing off into periods of two figures each, we have .00'07'85'60. Hence, the first figure of the root must be a cipher. To obtain the rest of the root we proceed as before.

.00'07'85'60(.0280 +, *Answer.*

	4	
40		3 85
8		
48		3 84
560		1 60

It will be noticed that the square root of a decimal will always be a decimal. If we square a fraction, we will get a smaller fraction for its square. For example, $(\frac{1}{4})^2 = \frac{1}{16}$; or as a decimal, $.25^2 = .0625$. Therefore, the opposite is true; the square root of a number entirely a decimal will be a decimal, but it will be larger than the one of which it is the square root. Notice the example just given; .0007856 is less than its square root .028.

72. Rules for Square Root.—From the preceding examples the following rules may be deduced:

1. Beginning at the decimal point separate the number into periods of two figures each. If there is no decimal point, begin with the figure farthest to the right.

2. Find the greatest whole number whose square is contained in the first or left-hand period. Write this number as the first figure in the root; subtract the square of this number from the first period, and annex the second period to the remainder.

3. Multiply the part of the root already found by 20; this gives the trial divisor. Divide the dividend by the trial divisor for the second figure of the root and add this figure to the trial divisor for the complete divisor. Multiply the complete divisor by the second figure in the root and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.)

Bring down the third period and annex it to the last remainder for the new dividend.

4. Repeat rule 3 until the last period is used, after which, if any additional decimal places are required, annex cipher periods and continue as before. If the last period in the decimal should contain but one figure, annex a cipher to make a full period.

5. If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor and bring down another period.

6. To locate the decimal point, remember that there will be as many *figures* in the root to the left of the decimal point as there were *periods* to the left of the decimal point in the original number.

73. The Law of Right Triangles.—One of the most useful laws of geometry is that relating to the sides of a right triangle. A *triangle* is a three-sided figure. A *right triangle* is one which has one square corner. Figure 25 shows such a triangle. The corner marked *C* is a "square corner." In terms of geometry, a square corner is formed when two lines meet at an angle of 90 degrees, usually called a "right angle." Thus, the two sides *a*

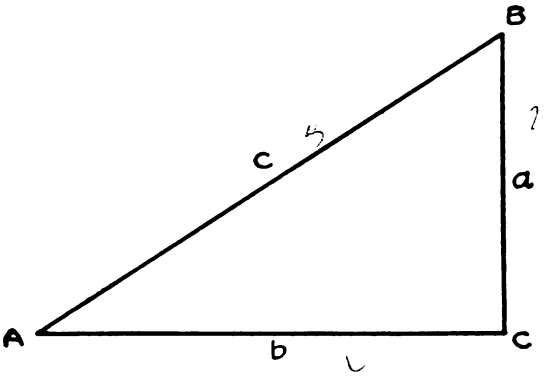


FIG. 25.—Right triangle.

and *b* make a right angle with each other, and the triangle is, therefore, called a right-angled triangle or a right triangle. The longest side, *c*, is called the *hypotenuse.*

In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. Written as a formula, this would read

$$c^2 = b^2 + a^2$$

By using this formula, we can find the square of *c*, if we know the lengths *a* and *b*. After finding the square of *c*, it is quite clear that *c* will be the square root of this number.

Thus, if the lengths of the sides of the triangle in Fig. 25 are 3, 4, and 5, the above equation expresses the relationship of the three sides.

From our knowledge of arithmetic we know that the following is true:

$$25 = 16 + 9$$
$$\text{or } 16 = 25 - 9$$
$$\text{or } 9 = 25 - 16$$

It is quite clear from this that the formula can also be written in three ways.

$$c^2 = b^2 + a^2$$
$$b^2 = c^2 - a^2$$
$$a^2 = c^2 - b^2$$

Knowing any two sides of a right triangle, the third side can be found by one of the three relations above.

Examples:

1. If the trolley pole in Fig. 26 is 24 ft. high, and the guy wire is anchored 7 ft. from the base of the pole, what is the length of the guy wire?

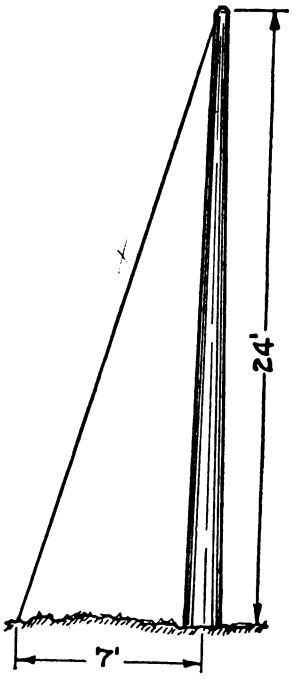


FIG. 26.—Trolley pole.

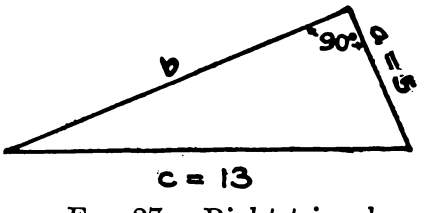


FIG. 27.—Right triangle.

The guy wire is the hypotenuse of a right triangle whose sides are 24 ft. and 7 ft.

$$c^2 = b^2 + a^2$$
$$c^2 = 24^2 + 7^2$$
$$= 576 + 49 = 625$$
$$c = \sqrt{625} = 25 \text{ ft., } \textit{Answer.}$$

2. If the triangle of Fig. 27 is a right triangle having the hypotenuse $c = 13$ in. and the side $a = 5$ in., what is the length of the side b?

$$c^2 = b^2 + a^2$$
Hence, $$b^2 = c^2 - a^2$$
$$b^2 = 13^2 - 5^2$$
$$= 169 - 25 = 144$$
$$b = \sqrt{144} = 12 \text{ in., } \textit{Answer.}$$

This property of right triangles is also useful in laying out right angles on a large scale. This is done by using three strings, wires, or chains of such lengths that when stretched they form a right triangle. A useful set of numbers that will give this are 3, 4, and 5, since $3^2 + 4^2 = 5^2$ $(9 + 16 = 25)$.

Any three other numbers having the same ratios as 3, 4, and 5 can be used if desired. Six (6), 8, and 10; 9, 12, and 15; 12, 16, and 20; 15, 20, and 25—any of these sets of numbers can be used. A surveyor will often use lengths of 15 ft., 20 ft., and 25 ft. on his chain to lay out a square corner; this method can also be used in aligning engines, shafting, etc.

74. The Value of Tables.—There are certain calculations that are made thousands of times a day by different people in different parts of the world. For example, the circumferences of circles of different diameters are being calculated every day by a great many persons. To save time that would otherwise be wasted in useless repetitions, many of the common operations have been calculated and the results collected and arranged in what are called *tables*. Thus a table of circumferences of circles would simply consist of a list of the circumferences of a great many circles, so arranged that the circumference corresponding to a certain diameter could easily be found. With such a table, it is no longer necessary to multiply 3.1416 by the diameter every time the circumference of a circle is desired. By simply referring to the tables, the circumference corresponding to the known diameter can be determined at once.

Just what tables one needs most, depends upon his occupation. The machinist has use for tables of the decimal equivalents of common fractions, tables of cutting speeds, tables of change gears to use for screw cutting, etc. The draftsman would use tables of strengths and weights of different materials, safe loads for bolts, beams, etc., tables of proportions of standard machine parts of different sizes, etc. The engineer uses tables of the properties of steam, and of the horsepower of engines, boilers, etc.

There are certain mathematical tables that are of value to nearly everyone. Among these are the tables given in this chapter: Tables of Circumferences and Areas of Circles; Tables of Squares, Cubes, Square Roots, and Cube Roots of Numbers.

75. How to Use the Tables.—The same method of arrangement has been used in all the tables given in this chapter, and one

should have no difficulty in reading all the tables after he has mastered one of them.

A table of circumferences for circles of diameters from .1 to 100 is given on pages 106 to 108. Each number in the ten main columns is a circumference, and the diameter corresponding to it is read from the column of figures from 1 to 100 at the left-hand dege and the row of figures from 0 to .9 at the top. The method of finding a circumference is quite simple. For example, a circle has a diameter of 8 in. In the small column at the left, headed "Dia.," we find the figure 8, and directly opposite the 8 in the column immediately to the right, is the circumference of the circle, 25.1327 in. In like manner, the following circumferences can be found.

Diameter 4 in.	Circumference 12.5664 in.
Diameter 21 in.	Circumference 65.9734 in.
Diameter 35 in.	Circumference 109.9557 in.

Suppose that the diameter of the foregoing circle is 8 3 in. instead of 8 in. We find the 8 in the diameter column. The circumference is on a level with the 8, but in the column beneath the figure .3 or the fourth column from the left. The circumference thus found is 26.0752 in. Likewise if the diameter is 8.7 in., the circumference is opposite the 8, but in the column beneath the .7, and its value is 27.3319 in. The student should check the following circumferences from the table to be sure he understands the method of finding them.

Diameter 20.6 in.	Circumference 64.7168 in.
Diameter 1.4 in.	Circumference 4.3982 in.
Diameter 76.9 in.	Circumference 241.5885 in.

A table of circumferences is also given on page 105 for diameters advancing by eighths instead of tenths. From this table, circumferences can be found for such diameters as $1\frac{1}{8}$, $\frac{5}{8}$, $10\frac{7}{8}$, etc. The table is arranged in the same manner, except that the columns are headed at the top with the fractions from 0 to $\frac{7}{8}$ instead of 0 to .9. The circumference of a circle of $5\frac{3}{8}$-in. diameter is opposite the 5 and in the column beneath the fraction, $\frac{3}{8}$, etc.

On pages 114 to 116 is the table of squares of numbers from 1 to 1000. The top row of figures from 0 to 9 are slightly different

from those of the circumference table in that they are not fractional or decimal parts of the number but form the last digit before the decimal point. To find the square of 12, for instance, we locate the 1 in the number column on the left edge and the 2 in the top row of figures. Its square, 144, will be found on a level with the 1 and in the column beneath the 2. Likewise the square of 25 is on a level with the 2 and in the column beneath the figure 5. Twenty-five squared (25^2) equals 625. The square of 407 is on a level with the 40 and in the column beneath the 7. Four hundred seven squared (407^2) = 165,649. Check the following squares from the table:

$$13^2 = 169$$
$$109^2 = 11{,}881$$
$$995^2 = 990{,}025$$

The method of using the tables of square roots, cubes, and cube roots, also given in this chapter, is exactly the same as explained for the table of squares.

Examples:

1. What would be the length of a steel sheet from which to make a 28-in. circular drum, allowing $1\frac{1}{4}$ in. extra for lapping and riveting the ends? The circumference of the drum is measured in the direction of the length of the sheet.

In this case the length of the sheet will be equal to the circumference of the drum, plus the amount required for lapping and riveting.

From the table, circumference = 87.9646 in.
Amount required for lap = 1.25 in.
Total length of sheet required = 89.2146 in., *Answer.*

Squares, cubes, etc., of mixed numbers can also be found from the tables by the proper manipulation of the decimal point.

2. Find the square of 83.2.

The actual figures in the square of 83.2 will be the same as in the square of 832. From the table, the square of 832 is 692,224. However, 83.2 has one decimal place, and its square will, therefore, contain two decimal places. The square of 832 is then found by shifting the decimal point two places to the left in the square of 832.

$$832^2 = 692{,}224$$
$$83.2^2 = 6922.24, \textit{ Answer.}$$

PROBLEMS

206. Extract the square root of the following numbers by the method explained in Arts. **69–72:**

(*a*) 64,516 *Answer*, 254
(*b*) 198.1369
(*c*) .571428

Note.—One answer is given so that the student can see whether he understands the operations of square root before proceeding further.

207. Extract the square root of

(*a*) 516,961
(*b*) 23,804,641
(*c*) 1159.4025
(*d*) 2 to 4 decimal places
(*e*) 3 to 4 decimal places
(*f*) .3364

208. From the tables, find the squares, cubes, square roots, and cube roots of the following numbers:

(*a*) 123
(*b*) 956
(*c*) 56.7
(*d*) 17
(*e*) 281
(*f*) 763

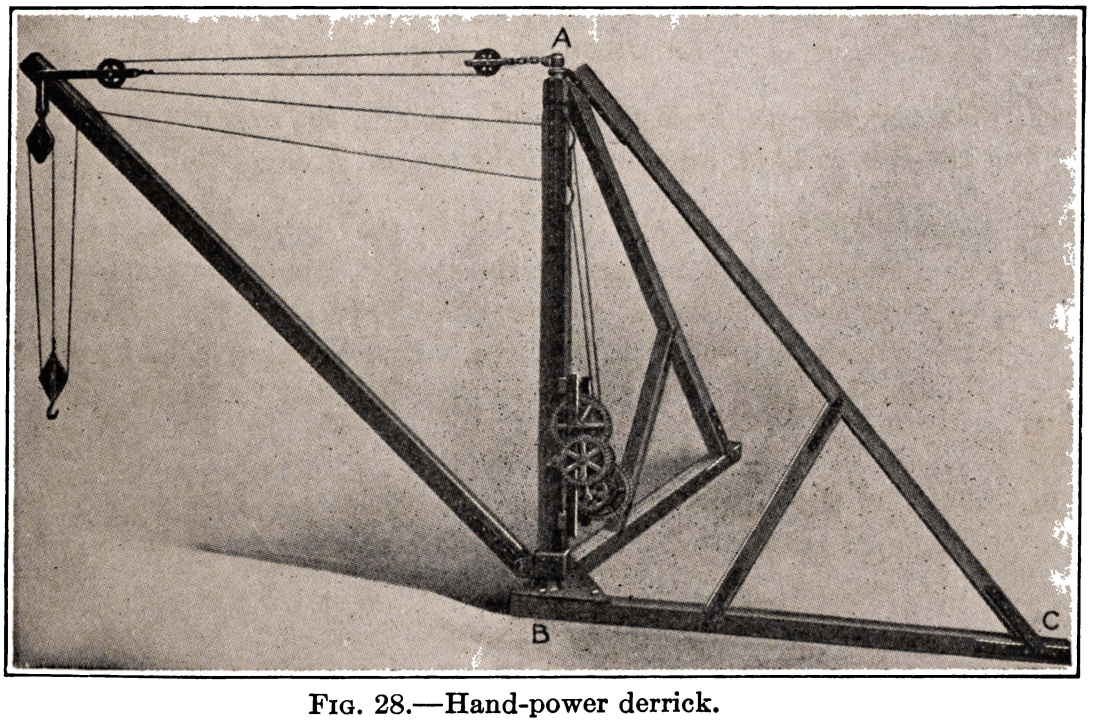


FIG. 28.—Hand-power derrick.

209. The hand-power derrick of Fig. 28 has a mast AB 15 ft. in height, and the leg BC is fastened to AC at a distance of 16 ft. from the foot of the mast AB. Calculate the length of the leg AC.

210. Figure 29 shows an end view of the roof framing for a barn. If the run is to be 12 ft. and the rise 8 ft., what rafter length is required, allowing 16 in. for the extension?

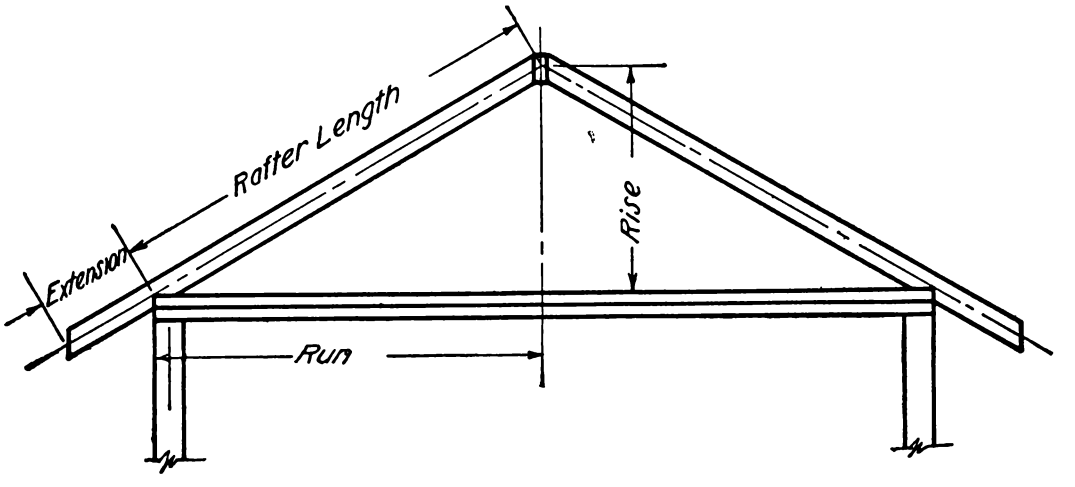


FIG. 29.—Roof framing for barn.

211. From the tables, determine the circumferences of the following circles:

(*a*) Diameter 6 in.

(*b*) Diameter $31\frac{1}{4}$ in.

(*c*) Diameter 21.7 in.

212. Find the distance from the top of a tree to the end of its shadow if the tree is 42 ft. high and the shadow 97 ft. long.

213. A steel stack 75 ft. high is supported by four guy wires fastened to a ring two-thirds of the way up the stack. The other ends of the guy wires are anchored at a distance of 50 ft. from the base of the stack and on a level with the base. How many feet of wire are required for all four guys, allowing 10% extra for fastening the ends?

214. Solve for the hypotenuse when the other two sides of a right triangle are as follows:

(*a*) 4 and 8

(*b*) .8 and .6

(*c*) 32 and 71

215. An electric hoist is designed for a nominal capacity of 7000 lb. with a rope speed of 150 f.p.m. at the drum. If the hoisting drum makes 36 r.p.m., what diameter of drum is required?

Note.—The hoisting rope is wound up on the drum.

216. A 30-in. pulley and a 24-in. pulley are keyed to the same shaft. If the belt on the larger pulley has a speed of 3000 f.p.m., what is the belt speed of the smaller pulley?

217. The following rule may be used to calculate the diameter of the cylinders of an automobile engine if the rated horsepower of the engine and the number of cylinders are known.

> Multiply the rated horsepower of the engine by 2.5. Divide by the number of cylinders. The square root of this result gives the diameter of the cylinders in inches.

Calculate the cylinder diameter of an eight-cylinder automobile engine rated at 33.8 hp.

218. A square tank has a depth of 1 ft. What must be the dimension of the sides of the tank if it is to contain 19,375 lb. of water? A cubic foot of water weighs 62.5 lb.

219. Find the sum of the following:

$$\sqrt{1{,}876{,}104} + \sqrt{20.25} + \sqrt{9.99}$$

220. Calculate the value of the following:

The square root of 278 divided by $1.2\sqrt{\frac{3}{8}}$.

TABLE 1

CIRCUMFERENCES OF CIRCLES

Diameters $\frac{1}{8}$ to 50, advancing by eighths

Dia.	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
0	.....	0.3927	0.7854	1.1781	1.5708	1.9635	2.3562	2.7489
1	3.1416	3.5343	3.9270	4.3197	4.7124	5.1051	5.4978	5.8905
2	6.2832	6.6759	7.0686	7.4613	7.8540	8.2467	8.6394	9.0321
3	9.4248	9.8175	10.2102	10.6029	10.9956	11.3883	11.7810	12.1737
4	12.5664	12.9591	13.3518	13.7445	14.1372	14.5299	14.9226	15.3153
5	15.7080	16.1007	16.4934	16.8861	17.2788	17.6715	18.0642	18.4569
6	18.8496	19.2423	19.6350	20.0277	20.4204	20.8131	21.2058	21.5984
7	21.9911	22.3838	22.7765	23.1692	23.5619	23.9546	24.3473	24.7400
8	25.1327	25.5254	25.9181	26.3108	26.7035	27.0962	27.4889	27.8816
9	28.2743	28.6670	29.0597	29.4524	29.8451	30.2378	30.6305	31.0232
10	31.4159	31.8086	32.2013	32.5940	32.9867	33.3794	33.7721	34.1648
11	34.5575	34.9502	35.3429	35.7356	36.1283	36.5210	36.9137	37.3064
12	37.6991	38.0918	38.4845	38.8772	39.2699	39.6626	40.0553	40.4480
13	40.8407	41.2334	41.6261	42.0188	42.4115	42.8042	43.1969	43.5896
14	43.9823	44.3750	44.7677	45.1604	45.5531	45.9458	46.3385	46.7312
15	47.1239	47.5166	47.9093	48.3020	48.6947	49.0874	49.4801	49.8728
16	50.2655	50.6582	51.0509	51.4436	51.8363	52.2290	52.6217	53.0144
17	53.4071	53.7998	54.1925	54.5852	54.9779	55.3706	55.7633	56.1560
18	56.5487	56.9414	57.3341	57.7268	58.1195	58.5122	58.9049	59.2976
19	59.6903	60.0830	60.4757	60.8684	61.2611	61.6538	62.0465	62.4392
20	62.8319	63.2246	63.6173	64.0100	64.4026	64.7953	65.1880	65.5807
21	65.9734	66.3661	66.7588	67.1515	67.5442	67.9369	68.3296	68.7223
22	69.1150	69.5077	69.9004	70.2931	70.6858	71.0785	71.4712	71.8639
23	72.2566	72.6493	73.0420	73.4347	73.8274	74.2201	74.6128	75.0055
24	75.3982	75.7909	76.1836	76.5763	76.9690	77.3617	77.7544	78.1471
25	78.5398	78.9325	79.3252	79.7179	80.1106	80.5033	80.8960	81.2887
26	81.6814	82.0741	82.4668	82.8595	83.2522	83.6449	84.0376	84.4303
27	84.8230	85.2157	85.6084	86.0011	86.3938	86.7865	87.1792	87.5719
28	87.9646	88.3573	88.7500	89.1427	89.5354	89.9281	90.3208	90.7135
29	91.1062	91.4989	91.8916	92.2843	92.6770	93.0697	93.4624	93.8551
30	94.2478	94.6405	95.0332	95.4259	95.8186	96.2113	96.6040	96.9967
31	97.3894	97.7821	98.1748	98.5675	98.9602	99.3529	99.7456	100.138
32	100.531	100.924	101.316	101.709	102.102	102.494	102.887	103.280
33	103.673	104.065	104.458	104.851	105.243	105.636	106.029	106.421
34	106.814	107.207	107.600	107.992	108.385	108.778	109.170	109.563
35	109.956	110.348	110.741	111.134	111.527	111.919	112.312	112.705
36	113.097	113.490	113.883	114.275	114.668	115.061	115.454	115.846
37	116.239	116.632	117.024	117.417	117.810	118.202	118.596	118.988
38	119.381	119.773	120.166	120.559	120.951	121.344	121.737	122.129
39	122.522	122.915	123.308	123.700	124.093	124.486	124.878	125.271
40	125.664	126.056	126.449	126.842	127.235	127.627	128.020	128.413
41	128.805	129.198	129.591	129.983	130.376	130.769	131.161	131.554
42	131.947	132.340	132.732	133.125	133.518	133.910	134.303	134.696
43	135.088	135.481	135.874	136.267	136.659	137.052	137.445	137.837
44	138.230	138.623	139.015	139.408	139.801	140.194	140.586	140.979
45	141.372	141.764	142.157	142.550	142.942	143.335	143.728	144.121
46	144.513	144.906	145.299	145.691	146.084	146.477	146.869	147.262
47	147.655	148.048	148.440	148.833	149.226	149.618	150.011	150.404
48	150.796	151.189	151.582	151.975	152.367	152.760	153.153	153.545
49	153.938	154.331	154.723	155.116	155.509	155.902	156.294	156.687
50	157.080	157.472	157.865	158.258	158.650	159.043	159.436	159.829

TABLE 2

CIRCUMFERENCES OF CIRCLES

Diameters .1 to 100, advancing by tenths

Dia.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.....	.31416	.62832	.94248	1.2566	1.5708	1.8850	2.1991	2.5133	2.8274
1	3.1416	3.4558	3.7699	4.0841	4.3982	4.7124	5.0265	5.3407	5.6549	5.9640
2	6.2832	6.5973	6.9115	7.2257	7.5398	7.8540	8.1681	8.4823	8.7965	9.1106
3	9.4248	9.7379	10.0531	10.3673	10.6814	10.9956	11.3097	11.6239	11.9381	12.2522
4	12.5664	12.8805	13.1947	13.5088	13.8230	14.1372	14.4513	14.7655	15.0796	15.3938
5	15.7080	16.0221	16.3363	16.6504	16.9646	17.2788	17.5929	17.9071	18.2212	18.5354
6	18.8496	19.1637	19.4779	19.7920	20.1062	20.4204	20.7345	21.0487	21.3628	21.6770
7	21.9911	22.3053	22.6195	22.9336	23.2478	23.5619	23.8761	24.1903	24.5044	24.8186
8	25.1327	25.4469	25.7611	26.0752	26.3894	26.7035	27.0177	27.3319	27.6460	27.9602
9	28.2743	28.5885	28.9027	29.2168	29.5310	29.8451	30.1593	30.4734	30.7876	31.1018
10	31.4159	31.7301	32.0442	32.3584	32.6726	32.9867	33.3009	33.6150	33.9292	34.2434
11	34.5575	34.8717	35.1858	35.5000	35.8142	36.1283	36.4425	36.7566	37.0708	37.3850
12	37.6991	38.0133	38.3274	38.6416	38.9557	39.2699	39.5841	39.8982	40.2124	40.5265
13	40.8407	41.1549	41.4690	41.7832	42.0973	42.4115	42.7257	43.0398	43.3540	43.6681
14	43.9823	44.2965	44.6106	44.9248	45.2389	45.5531	45.8673	46.1814	46.4956	46.8097
15	47.1239	47.4380	47.7522	48.0664	48.3805	48.6947	49.0088	49.3230	49.6372	49.9513
16	50.2655	50.5796	50.8938	51.2080	51.5221	51.8363	52.1504	52.4646	52.7788	53.0929
17	53.4071	53.7212	54.0354	54.3496	54.6637	54.9779	55.2920	55.6062	55.9203	56.2345
18	56.5487	56.8628	57.1770	57.4911	57.8053	58.1195	58.4336	58.7478	59.0619	59.3761
19	59.6903	60.0044	60.3186	60.6327	60.9469	61.2611	61.5752	61.8894	62.2035	62.5177
20	62.8319	63.1460	63.4602	63.7743	64.0885	64.4026	64.7168	65.0310	65.3451	65.6593
21	65.9734	66.2876	66.6018	66.9159	67.2301	67.5442	67.8584	68.1726	68.4867	68.8009
22	69.1150	69.4292	69.7434	70.0575	70.3717	70.6858	71.0000	71.3142	71.6283	71.9425
23	72.2566	72.5708	72.8849	73.1991	73.5133	73.8274	74.1416	74.4557	74.7699	75.0841
24	75.3982	75.7124	76.0265	76.3407	76.6549	76.9690	77.2832	77.5973	77.9115	78.2257
25	78.5398	78.8540	79.1681	79.4823	79.7965	80.1106	80.4248	80.7389	81.0531	81.3672
26	81.6814	81.9956	82.3097	82.6239	82.9380	83.2522	83.5664	83.8805	84.1947	84.5088
27	84.8230	85.1372	85.4513	85.7655	86.0796	86.3938	86.7080	87.0221	87.3363	87.6504
28	87.9646	88.2788	88.5929	88.9071	89.2212	89.5354	89.8495	90.1637	90.4779	90.7920
29	91.1062	91.4203	91.7345	92.0487	92.3628	92.6770	92.9911	93.3053	93.6195	93.9336
30	94.2478	94.5619	94.8761	95.1903	95.5044	95.8186	96.1327	96.4469	96.7611	97.0752
31	97.3894	97.7035	98.0177	98.3319	98.6460	98.9602	99.2743	99.5885	99.9026	100.2168
32	100.5310	100.8451	101.1593	101.4734	101.7876	102.1018	102.4159	102.7301	103.0442	103.3584
33	103.6726	103.9867	104.3009	104.6150	104.9292	105.2434	105.5575	105.8717	106.1858	106.5000
34	106.8142	107.1283	107.4425	107.7566	108.0708	108.3849	108.6991	109.0133	109.3274	109.6416
35	109.9557	110.2699	110.5841	110.8982	111.2124	111.5265	111.8407	112.1549	112.4690	112.7832

36	113.0973	113.4115	113.7257	114.0398	114.3540	114.6681	114.9823	115.2965	115.6106	115.9248
37	116.2389	116.5531	116.8672	117.1814	117.4956	117.8097	118.1239	118.4380	118.7522	119.0664
38	119.3805	119.6947	120.0088	120.3230	120.6372	120.9513	121.2655	121.5796	121.8939	122.2080
39	122.5221	122.8363	123.1504	123.4646	123.7788	124.0929	124.4071	124.7212	125.0354	125.3495
40	125.6637	125.9779	126.2920	126.6062	126.9203	127.2345	127.5487	127.8628	128.1770	128.4911
41	128.8053	129.1195	129.4336	129.7478	130.0619	130.3761	130.6903	131.0044	131.3186	131.6327
42	131.9469	132.2611	132.5752	132.8894	133.2035	133.5177	133.8318	134.1460	134.4602	134.7743
43	135.0885	135.4026	135.7168	136.0310	136.3451	136.6593	136.9734	137.2876	137.6018	137.9159
44	138.2301	138.5442	138.8584	139.1726	139.4867	139.8009	140.1150	140.4292	140.7434	141.0575
45	141.3717	141.6858	142.0000	142.3141	142.6283	142.9425	143.2566	143.5708	143.8849	144.1991
46	144.5133	144.8274	145.1416	145.4557	145.7699	146.0841	146.3982	146.7124	147.0265	147.3407
47	147.6549	147.9690	148.2832	148.5973	148.9115	149.2257	149.5398	149.8540	150.1681	150.4823
48	150.7964	151.1106	151.4248	151.7389	152.0531	152.3682	152.6814	152.9956	153.3097	153.6239
49	153.9380	154.2522	154.5664	154.8805	155.1947	155.5088	155.8230	156.1372	156.4513	156.7655
50	157.0796	157.3938	157.7080	158.0221	158.3363	158.6504	158.9646	159.2787	159.5929	159.9071
51	160.2212	160.5354	160.8495	161.1637	161.4779	161.7920	162.1062	162.4203	162.7345	163.0487
52	163.3628	163.6770	163.9911	164.3053	164.6195	164.9336	165.2478	165.5619	165.8761	166.1903
53	166.5044	166.8186	167.1327	167.4469	167.7610	168.0752	168.3894	168.7035	169.0177	169.3318
54	169.6460	169.9602	170.2743	170.5885	170.9026	171.2168	171.5310	171.8451	172.1593	172.4734
55	172.7876	173.1018	173.4159	173.7301	174.0442	174.3584	174.6726	174.9867	175.3009	175.6150
56	175.9292	176.2433	176.5575	176.8718	177.1858	177.5000	177.8141	178.1283	178.4425	178.7566
57	179.0708	179.3849	179.6991	180.0133	180.3274	180.6416	180.9557	181.2699	181.5841	181.8982
58	182.2124	182.5265	182.8407	183.1549	183.4690	183.7832	184.0973	184.4115	184.7256	185.0398
59	185.3540	185.6681	185.9823	186.2964	186.6106	186.9248	187.2389	187.5531	187.8672	188.1814
60	188.4956	188.8097	189.1239	189.4380	189.7522	190.0664	190.3805	190.6947	191.0088	191.3230
61	191.6372	191.9513	192.2655	192.5796	192.8938	193.2079	193.5221	193.8363	194.1504	194.4646
62	194.7787	195.0929	195.4071	195.7212	196.0354	196.3495	196.6637	196.9779	197.2920	197.6062
63	197.9203	198.2345	198.5487	198.8628	199.1770	199.4911	199.8053	200.1195	200.4346	200.7478
64	201.0620	201.3761	201.6902	202.0044	202.3186	202.6327	202.9469	203.2610	203.5752	203.8894
65	204.2035	204.5177	204.8318	205.1460	205.4602	205.7743	206.0885	206.4026	206.7168	207.0310
66	207.3451	207.6593	207.9734	208.2876	208.6017	208.9159	209.2301	209.5442	209.8584	210.1725
67	210.4867	210.8009	211.1150	211.4292	211.7433	212.0575	212.3717	212.6858	213.0000	213.3141
68	213.6283	213.9425	214.2566	214.5708	214.8849	215.1991	215.5133	215.8274	216.1416	216.4556
69	216.7699	217.0841	217.3982	217.7124	218.0265	218.3407	218.6548	218.9690	219.2832	219.5973
70	219.9115	220.2256	220.5398	220.8540	221.1681	221.4823	221.7964	222.1106	220.4248	222.7389
71	223.0531	223.3672	223.6814	223.9956	224.3097	224.6239	224.9380	225.2522	225.5664	225.8805
72	226.1947	226.5088	226.8230	227.1371	227.4513	227.7655	228.0796	228.3938	228.7079	229.0221
73	229.3363	229.6504	229.9646	230.2787	230.5929	230.9071	231.2212	231.5354	231.8495	232.1637
74	232.4779	232.7920	233.1062	233.4203	233.7345	234.0487	234.3628	234.6770	234.9911	235.3053
75	235.6194	235.9336	236.2478	236.5619	236.8761	237.1902	237.5044	237.8186	238.1327	238.4469
76	238.7610	239.0752	239.3894	239.7035	240.0177	240.3318	240.6460	240.9602	241.2743	241.5885
77	241.9026	242.2168	242.5310	242.8451	243.1592	243.4734	243.7876	244.1017	244.4159	244.7301
78	245.0442	245.3584	245.6725	245.9867	246.3009	246.6150	246.9292	247.2433	247.5575	247.8717
79	248.1858	248.5000	248.8141	249.1283	249.4425	249.7566	250.0708	250.3849	250.6991	251.0133
80	251.3274	251.6416	251.9557	252.2699	252.5840	252.8982	253.2124	253.5265	253.8407	254.1548
81	254.4690	254.7832	255.0973	255.4115	255.7256	256.0398	256.3540	256.6681	256.9823	257.2964

TABLE 2.—(*Continued*)

CIRCUMFERENCES OF CIRCLES

Diameters .1 to 100, advancing by tenths

Dia.	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
82	257.6106	257.9248	258.2389	258.5531	258.8672	259.1814	259.4956	259.8097	260.1239	260.4380
83	260.7522	261.0663	261.3805	261.6947	262.0088	262.3230	262.6371	262.9513	263.2655	263.5796
84	263.8938	264.2079	264.5221	264.8363	265.1504	265.4646	265.7787	266.0929	266.4071	266.7212
85	267.0354	267.3495	267.6637	267.9779	268.2920	268.6062	268.9203	269.2345	269.5486	269.8628
86	270.1770	270.4911	270.8053	271.1194	271.4336	271.7478	272.0619	272.3761	272.6902	273.0044
87	273.3186	273.6327	273.9469	274.2610	274.5752	274.8894	275.2035	275.5177	275.8318	276.1460
88	276.4602	276.7743	277.0885	277.4026	277.7168	278.0309	278.3451	278.6593	278.9734	279.2876
89	279.6017	279.9159	280.2301	280.5442	280.8584	281.1725	281.4867	281.8009	282.1150	282.4292
90	282.7433	283.0575	283.3717	283.6858	284.0000	284.3141	284.6283	284.9425	285.2566	285.5708
91	285.8849	286.1991	286.5132	286.8274	287.1416	287.4557	287.7699	288.0840	288.3982	288.7124
92	289.0265	289.3407	289.6548	289.9690	290.2832	290.5973	290.9115	291.2256	291.5398	291.8540
93	292.1681	292.4823	292.7964	293.1106	293.4248	293.7389	294.0531	294.3672	294.6814	294.9956
94	295.3097	295.6239	295.9380	296.2522	296.5663	296.8805	297.1947	297.5088	297.8230	298.1371
95	298.4513	298.7655	299.0796	299.3938	299.7079	300.0221	300.3363	300.6504	300.9646	301.2787
96	301.5929	301.9071	302.2212	302.5354	302.8495	303.1637	303.4779	303.7920	304.1062	304.4203
97	304.7345	305.0486	305.3628	305.6770	305.9911	306.3053	306.6194	306.9336	307.2478	307.5619
98	307.8761	308.1902	308.5044	308.8186	309.1327	309.4469	309.7610	310.0752	310.3894	310.7035
99	311.0177	311.3318	311.6460	311.9602	312.2743	312.5885	312.9026	313.2168	313.5309	313.8451
100	314.1593									

TABLE 3

AREAS OF CIRCLES

Diameters $\frac{1}{8}$ to 50, advancing by eighths

Dia.	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
0	.....	0.0123	0.0491	0.1105	0.1964	0.3068	0.4418	0.6013
1	0.7854	0.9940	1.2272	1.4849	1.7671	2.0739	2.4053	2.7612
2	3.1416	3.5466	3.9761	4.4301	4.9087	5.4119	5.9396	6.4918
3	7.0686	7.6699	8.2958	8.9462	9.6211	10.321	11.045	11.793
4	12.566	13.364	14.186	15.033	15.904	16.800	17.721	18.665
5	19.635	20.629	21.648	22.691	23.758	24.850	25.967	27.109
6	28.274	29.465	30.680	31.919	33.183	34.472	35.785	37.122
7	38.485	39.871	41.282	42.718	44.179	45.664	47.173	48.707
8	50.265	51.849	53.456	55.088	56.745	58.426	60.132	61.862
9	63.617	65.397	67.201	69.029	70.882	72.760	74.662	76.589
10	78.540	80.516	82.516	84.541	86.590	88.664	90.763	92.886
11	95.033	97.205	99.402	101.62	103.87	106.14	108.43	110.75
12	113.10	115.47	117.86	120.28	122.72	125.19	127.68	130.19
13	132.73	135.30	137.89	140.50	143.14	145.80	148.49	151.20
14	153.94	156.70	159.48	162.30	165.13	167.99	170.87	173.78
15	176.71	179 67	182.65	185.66	188.69	191.75	194.83	197.93
16	201.06	204.22	207.39	210.60	213.82	217.08	220.35	223.65
17	226.98	230.33	233.71	237.10	240.53	243.98	247.45	250.95
18	254.47	258.02	261.59	265.18	268.80	272.45	276.12	279.81
19	283.53	287.27	291.04	294.83	298.65	302.49	306.35	310.24
20	314.16	318.10	322.06	326.05	330.06	334.10	338.16	342.25
21	346.36	350.50	354.66	358.84	363.05	367.28	371.54	375.83
22	380.13	384.46	388.82	393.20	397.61	402.04	406.49	410.97
23	415.48	420.00	424.56	429.13	433.74	438.36	443.01	447.69
24	452.39	457.11	461.86	466.64	471.44	476.26	481.11	485.98
25	490.87	495.79	500.74	505.71	510.71	515.72	520.77	525.84

TABLE 3.—(*Continued*)

AREAS OF CIRCLES

Diameters $\frac{1}{8}$ to 50, advancing by eighths

Dia.	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
26	530.93	536.05	541.19	546.35	551.55	556.76	562.00	567.27
27	572.56	577.87	583.21	588.57	593.96	599.37	604.81	610.27
28	615.75	621.26	626.80	632.36	637.94	643.55	649.18	654.84
29	660.52	666.23	671.96	677.71	683.49	689.30	695.13	700.98
30	706.86	712.76	718.69	724.64	730.62	736.62	742.64	748.69
31	754.77	760.87	766.99	773.14	779.31	785.51	791.73	797.98
32	804.25	810.54	816.86	823.21	829.58	835.97	842.39	848.83
33	855.30	861.79	868.31	874.85	881.41	888.00	894.62	901.26
34	907.92	914.61	921.32	928.06	934.82	941.61	948.42	955.25
35	962.11	969.00	975.91	982.84	989.80	996.78	1003.8	1010.8
36	1017.9	1025.0	1032.1	1039.2	1046.3	1053.5	1060.7	1068.0
37	1075.2	1082.5	1089.8	1097.1	1104.5	1111.8	1119.2	1126.7
38	1134.1	1141.6	1149.1	1156.6	1164.2	1171.7	1179.3	1186.9
39	1194.6	1202.3	1210.0	1217.7	1225.4	1233.2	1241.0	1248.8
40	1256.6	1264.5	1272.4	1280.3	1288.2	1296.2	1304.2	1312.2
41	1320.3	1328.3	1336.4	1344.5	1352.7	1360.8	1369.0	1377.2
42	1385.4	1393.7	1402.0	1410.3	1418.6	1427.0	1435.4	1443.8
43	1452.2	1460.7	1469.1	1477.6	1486.2	1494.7	1503.3	1511.9
44	1520.5	1529.2	1537.9	1546.6	1555.3	1564.0	1572.8	1581.6
45	1590.4	1599.3	1608.2	1617.0	1626.0	1634.9	1643.9	1652.9
46	1661.9	1670.9	1680.0	1689.1	1698.2	1707.4	1716.5	1725.7
47	1734.9	1744.2	1753.5	1762.7	1772.1	1781.4	1790.8	1800.1
48	1809.6	1819.0	1828.5	1837.9	1847.5	1857.0	1866.5	1876.1
49	1885.7	1895.4	1905.0	1914.7	1924.4	1934.2	1943.9	1953.7
50	1963.5	1973.3	1983.2	1993.1	2003.0	2012.9	2022.8	2032.8

TABLE 4

AREAS OF CIRCLES

Diameters .1 to 100, advancing by tenths

Dia.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	.....	.007854	.031416	.070686	.12566	.19636	.28274	.38485	.50265	.63617
1	.7854	.9503	1.1310	1.3273	1.5394	1.7671	2.0106	2.2698	2.5447	2.8353
2	3.1416	3.4636	3.8013	4.1548	4.5239	4.9087	5.3093	5.7256	6.1575	6.6052
3	7.0686	7.5477	8.0425	8.5530	9.0792	9.6211	10.1788	10.7521	11.3411	11.9459
4	12.5664	13.2025	13.8544	14.5220	15.2053	15.9043	16.6190	17.3494	18.0956	18.8574
5	19.6350	20.4282	21.2372	22.0618	22.9022	23.7583	24.6301	25.5176	26.4208	27.3397
6	28.2743	29.2247	30.1907	31.1725	32.1699	33.1831	34.2119	35.2565	36.3168	37.3928
7	38.4845	39.5919	40.7150	41.8539	43.0084	44.1786	45.3646	46.5663	47.7836	49.0167
8	50.2655	51.5300	52.8102	54.1061	55.4177	56.7450	58.0880	59.4468	60.8212	62.2114
9	63.6173	65.0388	66.4761	67.9291	69.3978	70.8822	72.3823	73.8981	75.4296	76.9769
10	78.5398	80.1185	81.7128	83.3229	84.9487	86.5901	88.2473	89.9202	91.6088	93.3132
11	95.0332	96.7689	98.5203	100.2875	102.0703	103.8689	105.6832	107.5132	109.3588	111.2202
12	113.0973	114.9901	116.8987	118.8229	120.7628	122.7185	124.6898	126.6769	128.6796	130.6981
13	132.7323	134.7822	136.8478	138.9291	141.0261	143.1388	145.2672	147.4114	149.5712	151.7468
14	153.9380	156.1450	158.3677	160.6061	162.8602	165.1300	167.4155	169.7167	172.0336	174.3662
15	176.7146	179.0786	181.4584	183.8539	186.2650	188.6919	191.1345	193.5928	196.0668	198.5565
16	201.0619	203.5831	206.1199	208.6724	211.2407	213.8246	216.4243	219.0397	221.6708	224.3176
17	226.9801	229.6583	232.3522	235.0618	237.7871	240.5282	243.2849	246.0574	248.8456	251.6494
18	254.4690	257.3043	260.1553	263.0220	265.9044	268.8025	271.7163	274.6459	277.5911	280.5521
19	283.5287	286.5211	289.5292	292.5530	295.5925	298.6477	301.7186	304.8052	307.9075	311.0255
20	314.1593	317.3087	320.4739	323.6547	326.8513	330.0636	333.2916	336.5353	339.7947	343.0698
21	346.3606	349.6671	352.9893	356.3273	359.6809	363.0503	366.4354	369.8361	373.2526	376.6848
22	380.1327	383.5963	387.0756	390.5707	394.0814	397.6078	401.1500	404.7078	408.2814	411.8706
23	415.4756	419.0963	422.7327	426.3848	430.0526	433.7361	437.4354	441.1503	444.8809	448.6273
24	452.3893	456.1671	459.9606	463.7698	467.5946	471.4352	475.2916	479.1636	483.0513	486.9547
25	490.8739	494.8087	498.7592	502.7255	506.7075	510.7052	514.7185	518.7476	522.7924	526.8529
26	530.9292	535.0211	539.1287	543.2521	547.3911	551.5459	555.7163	559.9025	564.1044	568.3220
27	572.5553	576.8043	581.0690	585.3494	589.6455	593.9574	598.2849	602.6282	606.9871	611.3618
28	615.7522	620.1582	624.5800	629.0175	633.4707	637.9397	642.4243	646.9246	651.4406	655.9724
29	660.5199	665.0830	669.6619	674.2565	678.8668	683.4927	688.1345	692.7919	697.4650	702.1538
30	706.8583	711.5786	716.3145	721.0662	725.8336	730.6167	735.4154	740.2299	745.0601	749.9060
31	754.7676	759.6450	764.5380	769.4467	774.3712	779.3113	784.2672	789.2388	794.2260	799.2290
32	804.2477	809.2821	814.3322	819.3980	824.4796	829.5768	834.6897	839.8184	844.9628	850.1229
33	855.2986	860.4902	865.6973	870.9202	876.1588	881.4131	886.6831	891.9688	897.2703	902.5874
34	907.9203	913.2688	918.6331	924.0131	929.4088	934.8202	940.2473	945.6901	951.1486	956.6228
35	962.1127	967.6184	973.1397	978.6768	984.2296	989.7980	995.3822	1000.9821	1006.5977	1012.2290

TABLE 4.—(*Continued*)

AREAS OF CIRCLES

Diameters .1 to 100, advancing by tenths

Dia.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
36	1017.8760	1023.5387	1029.2172	1034.9113	1040.6211	1046.3467	1052.0880	1057.8449	1063.6176	1069.4060
37	1075.2101	1081.0299	1086.8654	1092.7166	1098.5835	1104.4622	1110.3645	1116.2786	1122.2083	1128.1538
38	1134.1149	1140.0918	1146.0844	1152.0927	1158.1167	1164.1564	1170.2118	1176.2830	1182.3698	1188.4723
39	1194.5906	1200.7246	1206.8742	1213.0396	1219.2207	1225.4175	1231.6300	1237.8582	1244.1021	1250.3617
40	1256.6371	1262.9281	1269.2348	1275.5573	1281.8955	1288.2493	1294.6189	1301.0042	1307.4052	1313.8219
41	1320.2543	1326.7024	1333.1663	1339.6458	1346.1410	1352.6520	1359.1786	1365.7210	1372.2791	1378.8529
42	1385.4424	1392.0476	1398.6685	1405.3051	1411.9574	1418.6254	1425.3092	1432.0086	1438.7238	1445.4546
43	1452.2012	1458.9635	1465.7415	1472.5352	1479.3446	1486.1697	1493.0105	1499.8670	1506.7392	1513.6272
44	1520.5308	1527.4502	1534.3853	1541.3360	1548.3025	1555.2847	1562.2826	1569.2962	1576.3255	1583.3705
45	1590.4313	1597.5077	1604.5999	1611.7077	1618.8313	1625.9705	1633.1255	1640.2962	1647.4826	1654.6847
46	1661.9025	1669.1360	1676.3852	1683.6502	1690.9308	1698.2272	1705.5392	1712.8670	1720.2105	1727.5696
47	1734.9445	1742.3351	1749.7414	1757.1634	1764.6012	1772.0546	1779.5237	1787.0086	1794.5091	1802.0254
48	1809.5574	1817.1050	1824.6684	1832.2475	1839.8423	1847.4528	1855.0790	1862.7210	1870.3786	1878.0519
49	1885.7410	1893.4457	1901.1662	1908.9024	1916.6543	1924.4218	1932.2051	1940.0041	1947.8189	1955.6493
50	1963.4954	1971.3572	1979.2348	1987.1280	1995.0370	2002.9716	2010.9020	2018.8581	2026.8299	2034.8174
51	2042.8206	2050.8395	2058.8742	2066.9245	2074.9905	2083.0723	2091.1697	2099.2829	2107.4118	2115.5563
52	2123.7166	2131.8926	2140.0843	2148.2917	2156.5149	2164.7537	2173.0082	2181.2785	2189.5644	2197.8661
53	2206.1834	2214.5165	2222.8653	2231.2298	2239.6100	2248.0059	2256.4175	2264.8448	2273.2879	2281.7466
54	2290.2210	2298.7112	2307.2171	2315.7386	2324.2759	2332.8289	2341.3976	2349.9820	2358.5821	2367.1979
55	2375.8294	2384.4767	2393.1396	2401.8183	2410.5126	2419.2227	2427.9485	2436.6899	2445.4471	2454.2200
56	2463.0086	2471.8129	2480.6330	2489.4687	2498.3201	2507.1873	2516.0701	2524.9687	2533.8830	2542.8129
57	2551.7586	2560.7200	2569.6971	2578.6899	2587.6984	2596.7227	2605.7626	2614.8182	2623.8896	2632.9766
58	2642.0794	2651.1979	2660.3321	2669.4820	2678.6475	2687.8289	2697.0259	2706.2386	2715.4670	2724.7112
59	2733.9710	2743.2465	2752.5378	2761.8448	2771.1675	2780.5058	2789.8599	2799.2297	2808.6152	2818.0165
60	2827.4334	2836.8660	2846.3143	2855.7784	2865.2582	2874.7536	2884.2648	2893.7917	2903.3343	2912.8925
61	2922.4666	2932.0563	2941.6617	2951.2828	2960.9196	2970.5722	2980.2404	2989.9244	2999.6241	3009.3394
62	3019.0705	3028.8173	3038.5798	3048.3580	3058.1519	3067.9616	3077.7869	3087.6279	3097.4847	3107.3571
63	3117.2453	3127.1492	3137.0687	3147.0040	3156.9550	3166.9217	3176.9041	3186.9023	3196.9161	3206.9456
64	3216.9909	3227.0518	3237.1285	3247.2208	3257.3289	3267.4527	3277.5922	3287.7474	3297.9183	3308.1049
65	3318.3072	3328.5253	3338.7590	3349.0084	3359.2736	3369.5545	3379.8510	3390.1633	3400.4913	3410.8350
66	3421.1944	3431.5695	3441.9603	3452.3668	3462.7891	3473.2270	3483.6807	3494.1500	3504.6351	3515.1359
67	3525.6523	3536.1845	3546.7324	3557.2960	3567.8753	3578.4704	3589.0811	3599.7075	3610.3497	3621.0075
68	3631.6811	3642.3704	3653.0753	3663.7960	3674.5324	3685.2845	3696.0523	3706.8358	3717.6351	3728.4500
69	3739.2807	3750.1270	3760.9890	3771.8668	3782.7603	3793.6695	3804.5944	3815.5349	3826.4913	3837.4633
70	3848.4510	3859.4544	3870.4735	3881.5084	3892.5589	3903.6252	3914.7072	3925.8048	3936.9182	3948.0473
71	3959.1921	3970.3526	3981.5288	3992.7208	4003.9284	4015.1517	4026.3908	4037.6455	4048.9160	4060.2022

72	4071.5041	4082.8216	4094.1549	4105.5039	4116.8687	4128.2491	4139.6452	4151.0570	4162.4846	4173.9278
73	4185.3868	4196.8615	4208.3518	4219.8579	4231.3797	4242.9172	4254.4704	4266.0393	4277.6240	4289.2243
74	4300.8403	4312.4721	4324.1195	4335.7827	4347.4616	4359.1562	4370.8664	4382.5924	4394.3341	4406.0915
75	4417.8647	4429.6535	4441.4580	4453.2783	4465.1142	4476.9659	4488.8332	4500.7163	4512.6151	4524.5296
76	4536.4598	4548.4057	4560.3673	4572.3446	4584.3376	4596.3464	4608.3708	4620.4110	4632.4668	4644.5384
77	4656.5257	4668.7287	4680.8474	4692.9818	4705.1319	4717.2977	4729.4792	4741.6765	4753.8894	4766.1180
78	4778.3624	4790.6225	4802.8982	4815.1897	4827.4969	4839.8198	4852.1584	4864.5127	4876.8828	4889.2685
79	4901.6699	4914.0871	4926.5199	4938.9685	4951.4328	4963.9127	4976.4084	4988.9198	5001.4469	5013.9897
80	5026.5482	5039.1224	5051.7124	5064.3180	5076.9394	5089.5764	5102.2292	5114.8977	5127.5818	5140.2817
81	5152.9973	5165.7286	5178.4756	5191.2384	5204.0168	5216.8109	5229.6208	5242.4463	5255.2876	5268.1446
82	5281.0172	5293.9056	5306.8097	5319.7295	5332.6650	5345.6162	5358.5832	5371.5658	5384.5641	5397.5782
83	5410.6079	5423.6534	5436.7146	5449.7914	5462.8840	5475.9923	5489.1163	5502.2560	5515.4115	5528.5826
84	5541.7694	5554.9720	5568.1902	5581.4242	5594.6738	5607.9392	5561.2203	5634.5171	5647.8296	5661.1578
85	5674.5017	5687.8613	5701.2367	5714.6277	5728.0344	5741.4569	5754.8951	5768.3489	5781.8185	5795.3038
86	5808.8048	5822.3215	5835.8539	5849.4020	5862.9659	5876.5454	5890.1406	5903.7516	5917.3782	5931.0206
87	5944.6787	5958.3525	5972.0419	5985.7471	5999.4680	6013.2047	6026.9570	6040.7250	6054.5088	6068.3082
88	6082.1234	6095.9542	6109.8008	6123.6631	6137.5410	6151.4347	6165.3441	6179.2692	6193.2101	6207.1666
89	6221.1388	6235.1268	6249.1304	6263.1498	6277.1848	6291.2356	6305.3021	6319.3843	6333.4822	6347.5958
90	6361.7251	6375.8701	6390.0308	6404.2073	6418.3994	6432.6073	6446.8308	6461.0701	6475.3251	6489.5958
91	6503.8822	6518.1843	6532.5021	6546.8356	6561.1848	6575.5497	6589.9304	6604.3267	6618.7388	6633.1666
92	6647.6100	6662.0692	6676.5441	6691.0347	6705.5410	6720.0630	6734.6007	6749.1542	6763.7233	6778.3081
93	6792.9087	6807.5249	6822.1569	6836.8046	6541.4680	6866.1471	6880.8419	6895.5524	6910.2786	6925.0205
94	6939.7781	6954.5515	6969.3405	6984.1453	6998.9657	7013.8019	7028.6538	7043.5214	7058.4047	7073.3037
95	7088.2184	7103.1488	7118.0949	7133.0568	7148.0343	7163.0276	7178.0365	7193.0612	7208.1016	7223.1577
96	7238.2294	7253.3169	7268.4201	7283.5391	7298.6737	7313.8240	7328.9901	7344.1718	7359.3693	7374.5824
97	7389.8113	7405.0559	7420.3162	7435.5921	7450.8838	7466.1913	7481.5144	7496.8532	7512.2077	7527.5780
98	7542.9639	7558.3656	7573.7830	7589.2161	7604.6648	7620.1293	7635.6095	7651.1054	7666.6170	7682.1443
99	7697.6874	7713.2461	7728.8205	7744.4107	7760.0166	7775.6381	7791.2754	7806.9284	7822.5971	7838.2815
100	7853.9816									

TABLE 5

SQUARES OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
0	.....	1	4	9	16	25	36	49	64	81
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801
10	10000	10201	10404	10609	10816	11025	11236	11449	11664	11881
11	12100	12321	12544	12769	12996	13225	13456	13689	13924	14161
12	14400	14641	14884	15129	15376	15625	15876	16129	16384	16641
13	16900	17161	17424	17689	17956	18225	18496	18769	19044	19321
14	19600	19881	20164	20449	20736	21025	21316	21609	21904	22201
15	22500	22801	23104	23409	23716	24025	24336	24649	24964	25281
16	25600	25921	26244	26569	26896	27225	27556	27889	28224	28561
17	28900	29241	29584	29929	30276	30625	30976	31329	31684	32041
18	32400	32761	33124	33489	33856	34225	34596	34969	35344	35721
19	36100	36481	36864	37249	37636	38025	38416	38809	39204	39601
20	40000	40401	40804	41209	41616	42025	42436	42849	43264	43681
21	44100	44521	44944	45369	45796	46225	46656	47089	47524	47961
22	48400	48841	49284	49729	50176	50625	51076	51529	51984	52441
23	52900	53361	53824	54289	54756	55225	55696	56169	56644	57121
24	57600	58081	58564	59049	59536	60025	60516	61009	61504	62001
25	62500	63001	63504	64009	64516	65025	65536	66049	66564	67081
26	67600	68121	68644	69169	69696	70225	70756	71289	71824	72361
27	72900	73441	73984	74529	75076	75625	76176	76729	77284	77841
28	78400	78961	79524	80089	80656	81225	81796	82369	82944	83521
29	84100	84681	85264	85849	86436	87025	87616	88209	88804	89401
30	90000	90601	91204	91809	92416	93025	93636	94249	94864	95481
31	96100	96721	97344	97969	98596	99225	99856	100489	101124	101761
32	102400	103041	103681	104329	104976	105625	106276	106929	107584	108241
33	108900	109561	110224	110889	111556	112225	112896	113569	114244	114921
34	115600	116281	116964	117649	118336	119025	119716	120409	121104	121801
35	122500	123201	123904	124609	125316	126025	126736	127449	128164	128881
36	129600	130321	131044	131769	132496	133225	133956	134689	135424	136161
37	136900	137641	138384	139129	139876	140625	141376	142129	142884	143641

38	144400	145161	145924	146689	147456	148225	148996	149769	150544	151321
39	152100	152881	153664	154449	155236	156025	156816	157609	158404	159201
40	160000	160801	161604	162409	163216	164025	164836	165649	166464	167281
41	168100	168921	169744	170569	171396	172225	173056	173889	174724	175561
42	176400	177241	178084	178929	179776	180625	181476	182329	183184	184041
43	184900	185761	186624	187489	188356	189225	190096	190969	191884	192721
44	193600	194481	195364	196249	197136	198025	198916	199809	200704	201601
45	202500	203401	204304	205209	206116	207025	207936	208849	209764	210681
46	211600	212521	213444	214369	215296	216225	217156	218089	219024	219961
47	220900	221841	222784	223729	224676	225625	226576	227529	228484	229441
48	230400	231361	232324	233289	234256	235225	236196	237169	238144	239121
49	240100	241081	242064	243049	244036	245025	246016	247009	248004	249001
50	250000	251001	252004	253009	254016	255025	256036	257049	258064	259081
51	260100	261121	262144	263169	264196	265225	266256	267289	268324	269361
52	270400	271441	272484	273529	274576	275625	276676	277729	278784	279841
53	280900	281961	283024	284089	285156	286225	287296	288369	289444	290521
54	291600	292681	293764	294849	295936	297025	298116	299209	300304	301401
55	302500	303601	304704	305809	306916	308025	309136	310249	311364	312481
56	313600	314721	315844	316969	318096	319225	320356	321489	322624	323761
57	324900	326041	327184	328329	329476	330625	331776	332929	334084	335241
58	336400	337561	338724	339889	341056	342225	343396	344569	345744	346921
59	348100	349281	350464	351649	352836	354025	355216	356409	357604	358801
60	360000	361201	362404	363609	364816	366025	367236	368449	369664	370881
61	372100	373321	374544	375769	376996	378225	379456	380689	381924	383161
62	384400	385641	386884	388129	389376	390625	391876	393129	394384	395641
63	396900	398161	399424	400689	401956	403225	404496	405769	407044	408321
64	409600	410881	412164	413449	414736	416025	417316	418609	419904	421201
65	422500	423801	425104	426409	427716	429025	430336	431649	432964	434281
66	435600	436921	438244	439569	440896	442225	443556	444889	446224	447561
67	448900	450241	451584	452929	454276	455625	456976	458329	459684	461041
68	462400	463761	465124	466489	467856	469225	470596	471969	473344	474721
69	476100	477481	478864	480249	481636	483025	484416	485809	487204	488601
70	490000	491401	492804	494209	495616	497025	498436	499849	501264	502681
71	504100	505521	506944	508369	509796	511225	512656	514089	515524	516961
72	518400	519841	521284	522729	524176	525625	527076	524529	529984	531441
73	532900	534361	535824	537289	538756	540225	541696	554169	544644	546121
74	547600	549081	550564	552049	553536	555025	556516	558009	559504	561001
75	562500	564001	565504	567009	568516	570025	571536	573049	574564	576081
76	577600	579121	580644	582169	583696	585225	586756	588289	589824	591361
77	592900	594441	595984	597529	599076	600625	602176	603729	605284	606841
78	608400	609961	611524	613089	614656	616225	617796	619369	620944	622521
79	624100	625681	627264	628849	630436	632025	633616	635209	636804	638401
80	640000	641601	643204	644809	646416	648025	649636	651249	652864	654481
81	656100	657721	659344	660969	662596	664225	665856	667489	669124	670761
82	672400	674041	675684	677329	678976	680625	682276	683929	685584	687241

TABLE 5.—(*Continued*)

SQUARES OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
83	688900	690561	692224	693889	695556	697225	698896	700569	702244	703921
84	705600	707281	708964	710649	712336	714025	715716	717409	719104	720801
85	722500	724201	725904	727609	729316	731025	732736	734449	736164	737881
86	739600	741321	743044	744769	746496	748225	749956	751689	753424	755161
87	756900	758641	760384	762129	763876	765625	767376	769129	770884	772641
88	774400	776161	777924	779689	781456	783225	784996	786769	788544	790321
89	792100	793881	795664	797449	799236	801025	802816	804609	806404	808201
90	810000	811801	813604	815409	817216	819025	820836	822649	824464	826281
91	828100	829921	831744	833569	835396	837225	839056	840889	842724	844561
92	846400	848241	850084	851929	853776	855625	857476	859329	861184	863041
93	864900	866761	868624	870489	872356	874225	876096	877969	879844	881721
94	883600	885481	887364	889249	891136	893025	894916	896809	898704	900601
95	902500	904401	906304	908209	910116	912025	913936	915849	917764	919681
96	921600	923521	925444	927369	929296	931225	933156	935089	937024	938961
97	940900	942841	944784	946729	948676	950625	952576	954529	956484	958441
98	960400	962361	964324	966289	968256	970225	972196	974169	976144	978121
99	980100	982081	984064	986049	988036	990025	992016	994009	996004	998001
100	1000000									

TABLE 6

CUBES OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
0	0	1	8	27	64	125	216	343	512	729
1	1000	1331	1728	2197	2744	3375	4096	4913	5832	6859
2	8000	9261	10648	12167	13824	15625	17576	19683	21952	24389
3	27000	29791	32768	35937	39304	42875	46656	50653	54872	59319
4	64000	68921	74088	79507	85184	91125	97336	103823	110592	117649
5	125000	132651	140608	148877	157464	166375	175616	185193	195112	205379
6	216000	226981	238328	250047	262144	274625	287496	300763	314432	328509
7	343000	357911	373248	389017	405224	421875	438976	456533	474552	493039
8	512000	531441	551368	571787	592704	614125	636056	658503	681472	704969
9	729000	753571	778688	804357	830584	857375	884736	912673	941192	970299
10	1000000	1030301	1061208	1092727	1124864	1157625	1191016	1225043	1259712	1295029
11	1331000	1367631	1404928	1442897	1481544	1520875	1560896	1601613	1643032	1685159
12	1728000	1771561	1815848	1860867	1906624	1953125	2000376	2048383	2097152	2146689
13	2197000	2248091	2299968	2352637	2406104	2460375	2515456	2571353	2628072	2685619
14	2744000	2803221	2863288	2924207	2985984	3048625	3112136	3176523	3241792	3307949
15	3375000	3442951	3511808	3581577	3652264	3723875	3796416	3869893	3944312	4019679
16	4096000	4173281	4251528	4330747	4410944	4492125	4574296	4657463	4741632	4826809
17	4913000	5000211	5088448	5177717	5268024	5359375	5451776	5545233	5639752	5735339
18	5832000	5929741	6028568	6128487	6229504	6331625	6434856	6539203	6644672	6751269
19	6859000	6967871	7077888	7189057	7301384	7414875	7529536	7645373	7762392	7880599
20	8000000	8120601	8242408	8365427	8489664	8615125	8741816	8869743	8998912	9129329
21	9261000	9393931	9528128	9663597	9800344	9938375	10077696	10218313	10360232	10503459
22	10648000	10793861	10941048	11089567	11239424	11390625	11543176	11697083	11852352	12008989
23	12167000	12326391	12487168	12649337	12812904	12977875	13144256	13312053	13481272	13651919
24	13824000	13997521	14172488	14348907	14526784	14706125	14886936	15069223	15252992	15438249
25	15625000	15813251	16003008	16194277	16387064	16581375	16777216	16974593	17173512	17373979
26	17576000	17779581	17984728	18191447	18399744	18609625	18821096	19034163	19248832	19465109
27	19683000	19902511	20123648	20346417	20570824	20796875	21024576	21253933	21484952	21717639
28	21952000	22188041	22425768	22665187	22906304	23149125	23393656	23639903	23887872	24137569
29	24389000	24642171	24897088	25153757	25412184	25672375	25934336	26198073	26463592	26730899
30	27000000	27270901	27543608	27818127	28094464	28372625	28652616	28934443	29218112	29503629
31	29791000	30080231	30371328	30664297	30959144	31255875	31554496	31855013	32157432	32461759
32	32768000	33076161	33386248	33698267	34012224	34328125	34645976	34965783	35287552	35611289
33	35937000	36264691	36594368	36926037	37259704	37595375	37933056	38272753	38614472	38958219
34	39304000	39651821	40001688	40353607	40707584	41063625	41421736	41781923	42144192	42508549
35	42875000	43243551	43614208	43986977	44361864	44738875	45118016	45499293	45882712	46268279
36	46656000	47045881	47437928	47832147	48228544	48627125	49027896	49430863	49836032	50243409
37	50653000	51064811	51478848	51895117	52313624	52734375	53157376	53582633	54010152	54439939

TABLE 6.—(*Continued*)

CUBES OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
38	54872000	55306341	55742968	56181887	56623104	57066625	57512456	57960603	58411072	58863869
39	59319000	59776471	60236288	60698457	61162984	61629875	62099136	62570773	63044792	63521199
40	64000000	64481201	64964808	65450827	65939264	66430125	66923416	67419143	67917312	68417929
41	68921000	69426531	69934528	70444997	70957944	71473375	71991296	72511713	73034632	73560059
42	74088000	74618461	75151448	75686967	76225024	76765625	77308776	77854483	78402752	78953589
43	79507000	80062991	80621568	81182737	81746504	82312875	82881856	83453453	84027672	84604519
44	85184000	85766121	86350888	86938307	87528384	88121125	88716536	89314623	89915392	90518849
45	91125000	91733851	92345408	92959677	93576664	94196375	94818816	95443993	96071912	96702579
46	97336000	97972181	98611128	99252847	99897344	100544625	101194696	101847563	102503232	103161709
47	103823000	104487111	105154048	105823817	106496424	107171875	107850176	108531333	109215352	109902239
48	110592000	111284641	111980168	112678587	113379904	114084125	114791256	115501303	116214272	116930169
49	117649000	118370771	119095488	119823157	120553784	121287375	122023936	122763473	123505992	124251499
50	125000000	125751501	126506008	127263527	128024064	128787625	129554216	130323843	131096512	131872229
51	132651000	133432831	134217728	135005697	135796744	136590875	137388096	138188413	138991832	139798359
52	140608000	141420761	142236648	143055667	143877824	144703125	145513576	146363183	147197952	148035889
53	148877000	149721291	150568768	151419437	152273304	153130375	153990656	154854153	155720872	156590819
54	157464000	158340421	159220088	160103007	160989184	161878625	162771336	163667323	164566592	165469149
55	166375000	167284151	168196608	169112377	170031464	170953875	171879616	172808693	173741112	174676879
56	175616000	176558481	177504328	178453547	179406144	180362125	181321496	182284263	183250432	184220009
57	185193000	186169411	187149248	188132517	189119224	190109375	191102976	192100033	193100552	194104539
58	195112000	196122941	197137368	198155287	199176704	200201625	201230056	202262003	203297472	204336469
59	205379000	206425071	207474688	208527857	209584584	210644875	211708736	212776173	213847192	214921799
60	216000000	217081801	218167208	219256227	220348864	221445125	222545016	223648543	224755712	225866529
61	226981000	228099131	229220928	230346397	231475544	232608375	233744896	234885113	236029032	237176659
62	238328000	239483061	240641848	241804367	242970624	244140625	245314376	246491883	247673152	248858189
63	250047000	251239591	252435968	253636137	254840104	256047875	257259456	258474853	259694072	260917119
64	262144000	263374721	264609288	265847707	267089984	268336125	269586136	270840023	272097792	273359449
65	274625000	275894451	277167808	278445077	279726264	281011375	282300416	283593393	284890312	286191179
66	287496000	288804781	290117528	291434247	292754944	294079625	295408296	296740963	298077632	299418309
67	300763000	302111711	303464448	304821217	306182024	307546875	308915776	310288733	311665752	313046839
68	314432000	315821241	317214568	318611987	320013504	321419125	322828856	324242703	325660672	327082769
69	328509000	329929371	331373888	332812557	334255384	335702375	337153536	338608873	340068392	341532099
70	343000000	344472101	345948408	347428927	348913664	350402625	351895816	353393243	354894912	356400829
71	357911000	359425431	360944128	362467097	363994344	365525875	367061696	328601813	370146232	371694959
72	373248000	374805361	376367048	377933067	379503424	381078125	382657176	384240583	385828352	387420489
73	389017000	390617891	392223168	393832837	395446904	397065375	398688256	400315553	401947272	403583419
74	405224000	406869021	408518488	410172407	411830784	413493625	415160936	416832723	418508992	420189749
75	421875000	423564751	425259008	426957777	428661064	430368875	432081216	433798093	435519512	437245479
76	438976000	440711081	442450728	444194947	445943744	447697125	449455096	451217663	452984832	454756609

77	456533000	458314011	460099648	461889917	463684824	465484375	467288576	469097433	470910952	472729139
78	474552000	476379541	478211768	480048687	481890304	483736625	485587656	487443403	489303872	491169069
79	493039000	494913671	496793088	498677257	500566184	502459875	504358336	506261573	508169592	510082399
80	512000000	513922401	515849608	517781627	519718464	521660125	523606616	525557943	527514112	529475129
81	531441000	533411731	535387328	537367797	539353144	541345375	543338496	545338513	547343432	549353259
82	551368000	553387661	555412248	557441767	559476224	561515625	563559976	565609283	567663552	569722789
83	571787000	573856191	575930368	578009537	580093704	582182875	584277056	586376253	588480472	590589719
84	592704000	594823321	596947688	599077107	601211584	603351125	605495736	607645423	609800192	611960049
85	614125000	616295051	618470208	620650477	622835864	625026375	627222016	629422793	631628712	633839779
86	636056000	638277381	640503928	642735647	644972544	647214625	649461896	651714363	653972032	656234909
87	658503000	660776311	663054848	665338617	667627624	669921875	672221376	674526133	676836152	679151439
88	681472000	683797841	686128968	688465387	690807104	693154125	695506456	697864103	700227072	702595369
89	704969000	707347971	709732288	712121957	714516984	716917375	719323136	721734273	724150792	726572699
90	729000000	731432701	733870808	736314327	738763264	741217625	743677416	746142643	748613312	751089429
91	753571000	756058031	758550528	761048497	763551944	766060875	768575296	771095213	773620632	776151559
92	778688000	781229961	783777448	786330467	788889024	791453125	794022776	796597983	799178752	801765089
93	804357000	806954491	809557568	812166237	814780504	817400375	820025856	822656953	825293672	827936019
94	830584000	833237621	835896888	838561807	841232384	843908625	846590536	849278123	851971392	854670349
95	857375000	860085351	862801408	865523177	868250664	870983875	873722816	876467493	879217912	881974079
96	884736000	887503681	890277128	893056347	895841344	898632125	901428696	904231063	907039232	909853209
97	912673000	915498611	918330048	921167317	924010424	926859375	929714176	932574833	935441352	938313739
98	941192000	944076141	946966168	949862087	952763904	955671625	958585256	961504803	964430272	967361669
99	970299000	973242271	976191488	979146657	982107784	985074875	988047936	991026973	994011992	997002999
100	1000000000									

TABLE 7

SQUARE ROOTS OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
0	.....	1.	1.4142	1.7321	2.	2.2361	2.4495	2.6458	2.8284	3.
1	3.1623	3.3166	3.4641	3.6056	3.7417	3.8730	4.	4.1231	4.2426	4.3589
2	4.4721	4.5826	4.6904	4.7958	4.8990	5.	5.0990	5.1962	5.2915	5.3852
3	5.4772	5.5678	5.6569	5.7446	5.8310	5.9161	6.	6.0828	6.1644	6.2450
4	6.3246	6.4031	6.4807	6.5574	6.6332	6.7082	6.7823	6.8557	6.9282	7.
5	7.0711	7.1414	7.2111	7.2801	7.3485	7.4162	7.4833	7.5498	7.6158	7.6811
6	7.7460	7.8102	7.8740	7.9373	8.	8.0623	8.1240	8.1854	8.2462	8.3066
7	8.3666	8.4261	8.4853	8.5440	8.6023	8.6603	8.7178	8.7750	8.8318	8.8882
8	8.9443	9.	9.0554	9.1104	9.1652	9.2195	9.2736	9.3274	9.3808	9.4340
9	9.4868	9.5394	9.5917	9.6437	9.6954	9.7468	9.7980	9.8489	9.8995	9.9499
10	10.	10.0499	10.0995	10.1489	10.1980	10.2470	10.2956	10.3441	10.3923	10.4403
11	10.4881	10.5357	10.5830	10.6301	10.6771	10.7238	10.7703	10.8167	10.8628	10.9087
12	10.9545	11.	11.0454	11.0905	11.1355	11.1803	11.2250	11.2694	11.3137	11.3578
13	11.4018	11.4455	11.4891	11.5326	11.5758	11.6190	11.6619	11.7047	11.7473	11.7898
14	11.8322	11.8743	11.9164	11.9583	12.	12.0416	12.0830	12.1244	12.1655	12.2066
15	12.2474	12.2882	12.3288	12.3693	12.4097	12.4499	12.4900	12.5300	12.5698	12.6095
16	12.6491	12.6886	12.7279	12.7671	12.8062	12.8452	12.8841	12.9228	12.9615	13.
17	13.0384	13.0767	13.1149	13.1529	13.1909	13.2288	13.2665	13.3041	13.3417	13.3791
18	13.4164	13.4536	13.4907	13.5277	13.5647	13.6015	13.6382	13.6748	13.7113	13.7477
19	13.7840	13.8203	13.8564	13.8924	13.9284	13.9642	14.	14.0357	14.0712	14.1067
20	14.1421	14.1774	14.2127	14.2478	14.2829	14.3178	14.3527	14.3875	14.4222	14.4568
21	14.4914	14.5258	14.5602	14.5945	14.6287	14.6629	14.6969	14.7309	14.7648	14.7986
22	14.8324	14.8661	14.8997	14.9332	14.9666	15.	15.0333	15.0665	15.0997	15.1327
23	15.1658	14.1987	14.2315	15.2643	15.2971	15.3297	15.3623	15.3948	15.4272	15.4596
24	15.4919	15.5242	15.5563	15.5885	15.6205	15.6525	15.6844	15.7162	15.7480	15.7797
25	15.8114	15.8430	15.8745	15.9060	15.9374	15.9687	16.	16.0312	16.0624	16.0935
26	16.1245	16.1555	16.1864	16.2173	16.2481	16.2788	16.3095	16.3401	16.3707	16.4012
27	16.4317	16.4621	16.4924	16.5227	16.5529	16.5831	16.6132	16.6433	16.6733	16.7033
28	16.7332	16.7631	16.7929	16.8226	16.8523	16.8819	16.9115	16.9411	16.9706	17.
29	17.0294	17.0587	17.0880	17.1172	17.1464	17.1756	17.2047	17.2337	17.2627	17.2916
30	17.3205	17.3494	17.3781	17.4069	17.4356	17.4642	17.4929	17.5214	17.5499	17.5784
31	17.6068	17.6352	17.6635	17.6918	17.7200	17.7482	17.7764	17.8045	17.8326	17.8606
32	17.8885	17.9165	17.9444	17.9722	18.	18.0278	18.0555	18.0831	18.1108	18.1384
33	18.1659	18.1934	18.2209	18.2483	18.2757	18.3030	18.3303	18.3576	18.3848	18.4120
34	18.4391	18.4662	18.4932	18.5203	18.5472	18.5742	18.6011	18.6279	18.6548	18.6815
35	18.7083	18.7350	18.7617	18.7883	18.8149	18.8414	18.8680	18.8944	18.9209	18.9473
36	18.9737	19.	19.0263	19.0526	19.0788	19.1050	19.1311	19.1572	19.1833	19.2094
37	19.2354	19.2614	19.2873	19.3132	19.3391	19.3649	19.3907	19.4165	19.4422	19.4679

38	19.4936	19.5192	19.5448	19.5704	19.5959	19.6214	19.6469	19.6723	19.6977	19.7231
39	19.7484	19.7737	19.7990	19.8242	19.8494	19.8746	19.8997	19.9249	19.9499	19.9750
40	20.	20.0250	20.0499	20.0749	20.0998	20.1246	20.1494	20.1742	20.1990	20.2237
41	20.2485	20.2731	20.2978	20.3224	20.3470	20.3715	20.3961	20.4206	20.4450	20.4695
42	20.4939	20.5183	20.5426	20.5670	20.5913	20.6155	20.6398	20.6640	20.6882	20.7123
43	20.7364	20.7605	20.7846	20.8087	20.8327	20.8567	20.8806	20.9045	20.9284	20.9523
44	20.9762	21.	21.0238	21.0476	21.0713	21.0950	21.1187	21.1424	21.1660	21.1896
45	21.2132	21.2368	21.2603	21.2838	21.3073	21.3307	21.3542	21.3776	21.4009	21.4243
46	21.4476	21.4709	21.4942	21.5174	21.5407	21.5639	21.5870	21.6102	21.6333	21.6564
47	21.6795	21.7025	21.7256	21.7486	21.7715	21.7945	21.8174	21.8403	21.8632	21.8861
48	21.9089	21.9317	21.9545	21.9773	22.	22.0227	22.0454	22.0681	22.0970	22.1133
49	22.1359	22.1585	22.1811	22.2036	22.2261	22.2486	22.2711	22.2935	22.3159	22.3383
50	22.3607	22.3830	22.4054	22.4277	22.4499	22.4722	22.4944	22.5167	22.5389	22.5610
51	22.5832	22.6053	22.6274	22.6495	22.6716	22.6936	22.7156	22.7376	22.7596	22.7816
52	22.8035	22.8254	22.8473	22.8692	22.8910	22.9129	22.9347	22.9565	22.9783	23.
53	23.0217	23.0434	23.0651	23.0868	23.1084	23.1301	23.1517	23.1733	23.1948	23.2164
54	23.2379	23.2594	23.2809	23.3024	23.3238	23.3452	23.3666	23.3880	23.4094	23.4307
55	23.4521	23.4734	23.4947	23.5160	23.5372	23.5584	23.5797	23.6008	23.6220	23.6432
56	23.6643	23.6854	23.7065	23.7276	23.7487	23.7697	23.7908	23.8118	23.8328	23.8537
57	23.8747	23.8956	23.9165	23.9374	23.9583	23.9792	24.	24.0208	24.0416	24.0624
58	24.0832	24.1039	24.1247	24.1454	24.1661	24.1868	24.2074	24.2281	24.2487	24.2693
59	24.2899	24.3105	24.3311	24.3516	24.3721	24.3926	24.4131	24.4336	24.4540	24.4745
60	24.4949	24.5153	24.5357	24.5561	24.5764	24.5967	24.6171	24.6374	24.6577	24.6779
61	24.6982	24.7184	24.7386	24.7588	24.7790	24.7992	24.8193	24.8395	24.8596	24.8797
62	24.8998	24.9199	24.9399	24.9600	24.9800	25.	25.0200	25.0400	25.0599	25.0799
63	25.0998	25.1197	25.1396	25.1595	25.1794	25.1992	25.2190	25.2389	25.2587	25.2784
64	25.2982	25.3180	25.3377	25.3574	25.3772	25.3969	25.4165	25.4362	25.4558	25.4755
65	25.4951	25.5147	25.5343	25.5539	25.5734	25.5930	25.6125	25.6320	25.6515	25.6710
66	25.6905	25.7099	25.7294	25.7488	25.7682	25.7876	25.8070	25.8263	25.8457	25.8650
67	25.8844	25.9037	25.9230	25.9422	25.9615	25.9808	26.	26.0192	26.0384	26.0576
68	26.0768	26.0960	26.1151	26.1343	26.1534	26.1725	26.1916	26.2107	26.2298	26.2488
69	26.2679	26.2869	26.3059	26.3249	26.3439	26.3629	26.3818	26.4008	26.4197	26.4386
70	26.4575	26.4764	26.4953	26.5141	26.5330	26.5518	26.5707	26.5895	26.6083	26.6271
71	26.6458	26.6646	26.6833	26.7021	26.7208	26.7395	26.7582	26.7769	26.7955	26.8142
72	26.8328	26.8514	26.8701	26.8887	26.9072	26.9258	26.9444	26.9629	26.9815	27.
73	27.0185	27.0370	27.0555	27.0740	27.0924	27.1109	27.1293	27.1477	27.1662	27.1846
74	27.2029	27.2213	27.2397	27.2580	27.2764	27.2947	27.3130	27.3313	27.3496	27.3679
75	27.3861	27.4044	27.4226	27.4408	27.4591	27.4773	27.4955	27.5136	27.5318	27.5500
76	27.5681	27.5862	27.6043	27.6225	27.6405	27.6586	27.6767	27.6948	27.7128	27.7308
77	27.7489	27.7669	27.7849	27.8029	27.8209	27.8388	27.8568	27.8747	27.8927	27.9106
78	27.9285	27.9464	27.9643	27.9821	28.	28.0179	28.0357	28.0535	28.0713	28.0891
79	28.1069	28.1247	28.1425	28.1603	28.1780	28.1957	28.2135	28.2312	28.2489	28.2666
80	28.2843	28.3019	28.3196	28.3373	28.3549	28.3725	28.3901	28.4077	28.4253	28.4429
81	28.4605	28.4781	28.4956	28.5132	28.5307	28.5482	28.5657	28.5832	28.6007	28.6182
82	28.6356	28.6531	28.6705	28.6880	28.7054	28.7228	28.7402	28.7576	28.7750	28.7924
83	28.8097	28.8271	28.8444	28.8617	28.8791	28.8964	28.9137	28.9310	28.9482	28.9655
84	28.9828	29.	29.0172	29.0345	29.0517	29.0689	29.0861	29.1033	29.1204	29.1376

TABLE 7.—(*Continued*)

SQUARE ROOTS OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
85	29.1548	29.1719	29.1890	29.2062	29.2233	29.2404	29.2575	29.2746	29.2916	29.3087
86	29.3258	29.3428	29.3598	29.3769	29.3939	29.4109	29.4279	29.4449	29.4618	29.4788
87	29.4958	29.5127	29.5296	29.5466	29.5635	29.5804	29.5973	29.6142	29.6311	29.6479
88	29.6648	29.6816	29.6985	29.7153	29.7321	29.7489	29.7658	29.7825	29.7993	29.8161
89	29.8329	29.8496	29.8664	29.8831	29.8998	29.9166	29.9333	29.9500	29.9666	29.9833
90	30.	30.0167	30.0333	30.0500	30.0666	30.0832	30.0998	30.1164	30.1330	30.1496
91	30.1662	30.1828	30.1993	30.2159	30.2324	30.2490	30.2655	30.2820	30.2985	30.3150
92	30.3315	30.3480	30.3645	30.3809	30.3974	30.4138	30.4302	30.4467	30.4631	30.4795
93	30.4959	30.5123	30.5287	30.5450	30.5614	30.5778	30.5941	30.6105	30.6268	30.6431
94	30.6594	30.6757	30.6920	30.7083	30.7246	30.7409	30.7571	30.7734	30.7896	30.8058
95	30.8221	30.8383	30.8545	30.8707	30.8869	30.9031	30.9192	30.9354	30.9516	30.9677
96	30.9839	31.	31.0161	31.0322	31.0483	31.0644	31.0805	31.0966	31.1127	31.1288
97	31.1448	31.1609	31.1769	31.1929	31.2090	31.2250	31.2410	31.2570	31.2730	31.2890
98	31.3050	31.3209	31.3369	31.3528	31.3688	31.3847	31.4006	31.4166	31.4325	31.4484
99	31.4643	31.4802	31.4960	31.5119	31.5278	31.5436	31.5595	31.5753	31.5911	31.6070
100	31.6228									

TABLE 8

CUBE ROOTS OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
0	.....	1.	1.2599	1.4422	1.5874	1.7100	1.8171	1.9129	2.	2.0801
1	2.1544	2.2240	2.2894	2.3513	2.4101	2.4662	2.5198	2.5713	2.6207	2.6684
2	2.7144	2.7589	2.8020	2.8439	2.8845	2.9240	2.9625	3.	3.0366	3.0723
3	3.1072	3.1414	3.1748	3.2075	3.2396	3.2711	3.3019	3.3322	3.3620	3.3912
4	3.4200	3.4482	3.4760	3.5034	3.5303	3.5569	3.5830	3.6088	3.6342	3.6593
5	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259	3.8485	3.8709	3.8930
6	3.9149	3.9365	3.9579	3.9791	4.	4.0207	4.0412	4.0615	4.0817	4.1016
7	4.1213	4.1408	4.1602	4.1793	4.1983	4.2172	4.2358	4.2543	4.2727	4.2908
8	4.3089	4.3267	4.3445	4.3621	4.3795	4.3968	4.4140	4.4310	4.4480	4.4647
9	4.4814	4.4979	4.5144	4.5307	4.5468	4.5629	4.5789	4.5947	4.6104	4.6261
10	4.6416	4.6570	4.6723	4.6875	4.7027	4.7177	4.7326	4.7475	4.7622	4.7769
11	4.7914	4.8059	4.8203	4.8346	4.8488	4.8629	4.8770	4.8910	4.9049	4.9187
12	4.9324	4.9461	4.9597	4.9732	4.9866	5.	5.0133	5.0265	5.0397	5.0528
13	5.0658	5.0788	5.0916	5.1045	5.1172	5.1299	5.1426	5.1551	5.1676	5.1801
14	5.1925	5.2048	5.2171	5.2293	5.2415	5.2536	5.2656	5.2776	5.2896	5.3015
15	5.3133	5.3251	5.3368	5.3485	5.3601	5.3717	5.3832	5.3947	5.4061	5.4175
16	5.4288	5.4401	5.4514	5.4626	5.4737	5.4848	5.4959	5.5069	5.5178	5.5288
17	5.5397	5.5505	5.5613	5.5721	5.5828	5.5934	5.6041	5.6147	5.6252	5.6357
18	5.6462	5.6567	5.6671	5.6774	5.6877	5.6980	5.7083	5.7185	5.7287	5.7388
19	5.7489	5.7590	5.7690	5.7790	5.7890	5.7989	5.8088	5.8186	5.8285	5.8383
20	5.8480	5.8578	5.8675	5.8771	5.8868	5.8964	5.9059	5.9155	5.9250	5.9345
21	5.9439	5.9533	5.9627	5.9721	5.9814	5.9907	6.	6.0092	6.0185	6.0277
22	6.0368	6.0459	6.0550	6.0641	6.0732	6.0822	6.0912	6.1002	6.1091	6.1180
23	6.1269	6.1358	6.1446	6.1534	6.1622	6.1710	6.1797	6.1885	6.1972	6.2058
24	6.2145	6.2231	6.2317	6.2403	6.2488	6.2573	6.2658	6.2743	6.2828	6.2912
25	6.2996	6.3080	6.3164	6.3247	6.3330	6.3413	6.3496	6.3579	6.3661	6.3743
26	6.3825	6.3907	6.3988	6.4070	6.4151	6.4232	6.4312	6.4393	6.4473	6.4553
27	6.4633	6.4713	6.4792	6.4872	6.4951	6.5030	6.5108	6.5187	6.5265	6.5343
28	6.5421	6.5499	6.5577	6.5654	6.5731	6.5808	6.5885	6.5962	6.6039	6.6115
29	6.6191	6.6267	6.6343	6.6419	6.6494	6.6569	6.6644	6.6719	6.6794	6.6869
30	6.6943	6.7018	6.7092	6.7166	6.7240	6.7313	6.7387	6.7460	6.7533	6.7606
31	6.7679	6.7752	6.7824	6.7897	6.7969	6.8041	6.8113	6.8185	6.8256	6.8328
32	6.8399	6.8470	6.8541	6.8612	6.8683	6.8753	6.8824	6.8894	6.8964	6.9034
33	6.9104	6.9174	6.9244	6.9313	6.9382	6.9451	6.9521	6.9589	6.9658	6.9727
34	6.9795	6.9864	6.9932	7.	7.0068	7.0136	7.0203	7.0271	7.0338	7.0406
35	7.0473	7.0540	7.0607	7.0674	7.0740	7.0807	7.0873	0.0940	7.1006	7.1072
36	7.1138	7.1204	7.1269	7.1335	7.1400	7.1466	7.1531	7.1596	7.1661	7.1726
37	7.1791	7.1855	7.1920	7.1984	7.2048	7.2112	7.2177	7.2240	7.2304	7.2368

TABLE 8.—(*Continued*)

CUBE ROOTS OF NUMBERS FROM 1 TO 1000

No.	0	1	2	3	4	5	6	7	8	9
38	7.2432	7.2495	7.2558	7.2622	7.2685	7.2748	7.2811	7.2874	7.2936	7.2999
39	7.3061	7.3124	7.3186	7.3248	7.3310	7.3372	7.3434	7.3496	7.3558	7.3619
40	7.3681	7.3742	7.3803	7.3864	7.3925	7.3986	7.4047	7.4108	7.4169	7.4229
41	7.4290	7.4350	7.4410	7.4470	7.4530	7.4590	7.4650	7.4710	7.4770	7.4829
42	7.4889	7.4948	7.5007	7.5067	7.5126	7.5185	7.5244	7.5302	7.5361	7.5420
43	7.5478	7.5537	7.5595	7.5654	7.5712	7.5770	7.5828	7.5886	7.5944	7.6001
44	7.6059	7.6117	7.6174	7.6232	7.6289	7.6346	7.6403	7.6460	7.6517	7.6574
45	7.6631	7.6688	7.6744	7.6801	7.6857	7.6914	7.6970	7.7026	7.7082	7.7138
46	7.7194	7.7250	7.7306	7.7362	7.7418	7.7473	7.7529	7.7584	7.7639	7.7695
47	7.7750	7.7805	7.7860	7.7915	7.7970	7.8025	7.8079	7.8134	7.8188	7.8243
48	7.8297	7.8352	7.8406	7.8460	7.8514	7.8568	7.8622	7.8676	7.8730	7.8784
49	7.8837	7.8891	7.8944	7.8998	7.9051	7.9105	7.9158	7.9211	7.9264	7.9317
50	7.9370	7.9423	7.9476	7.9528	7.9581	7.9634	7.9686	7.9739	7.9791	7.9843
51	7.9896	7.9948	8.	8.0052	8.0104	8.0156	8.0208	8.0260	8.0311	8.0363
52	8.0415	8.0466	8.0517	8.0569	8.0620	8.0671	8.0723	8.0774	8.0825	8.0876
53	8.0927	8.0978	8.1028	8.1079	8.1130	8.1180	8.1231	8.1281	8.1332	8.1382
54	8.1433	8.1483	8.1533	8.1583	8.1633	8.1683	8.1733	8.1783	8.1833	8.1882
55	8.1932	8.1982	8.2031	8.2081	8.2130	8.2180	8.2229	8.2278	8.2327	8.2377
56	8.2426	8.2475	8.2524	8.2573	8.2621	8.2670	8.2719	8.2768	8.2816	8.2865
57	8.2913	8.2962	8.3010	8.3059	8.3107	8.3155	8.3203	8.3251	8.3300	8.3348
58	8.3396	8.3443	8.3491	8.3539	8.3587	8.3634	8.3682	8.3730	8.3777	8.3825
59	8.3872	8.3919	8.3967	8.4014	8.4061	8.4108	8.4155	8.4202	8.4249	8.4296
60	8.4343	8.4390	8.4437	8.4484	8.4530	8.4577	8.4623	8.4670	8.4716	8.4763
61	8.4809	8.4856	8.4902	8.4948	8.4994	8.5040	8.5086	8.5132	8.5178	8.5224
62	8.5270	8.5316	8.5362	8.5408	8.5453	8.5499	8.5544	8.5590	8.5635	8.5681
63	8.5726	8.5772	8.5817	8.5862	8.5907	8.5952	8.5997	8.6043	8.6088	8.6132
64	8.6177	8.6222	8.6267	8.6312	8.6357	8.6401	8.6446	8.6490	8.6535	8.6579
65	8.6624	8.6668	8.6713	8.6757	8.6801	8.6845	8.6890	8.6934	8.6978	8.7022
66	8.7066	8.7110	8.7154	8.7198	8.7241	8.7285	8.7329	8.7373	8.7416	8.7460
67	8.7503	8.7547	8.7590	8.7634	8.7677	8.7721	8.7764	8.7807	8.7850	8.7893
68	8.7937	8.7980	8.8023	8.8066	8.8109	8.8152	8.8194	8.8237	8.8280	8.8323
69	8.8366	8.8408	8.8451	8.8493	8.8536	8.8578	8.8621	8.8663	8.8706	8.8748
70	8.8790	8.8833	8.8875	8.8917	8.8959	8.9001	8.9043	8.9085	8.9127	8.9169
71	8.9211	8.9253	8.9295	8.9337	8.9378	8.9420	8.9462	8.9503	8.9545	8.9587
72	8.9628	8.9670	8.9711	8.9752	8.9794	8.9835	8.9876	8.9918	8.9959	9.
73	9.0041	9.0082	9.0123	9.0164	9.0205	9.0246	9.0287	9.0328	9.0369	9.0410
74	9.0450	9.0491	9.0532	9.0572	9.0613	9.0654	9.0694	9.0735	9.0775	9.0816
75	9.0856	9.0896	9.0937	9.0977	9.1017	9.1057	9.1098	9.1138	9.1178	9.1218

76	9.1258	9.1298	9.1338	9.1378	9.1418	9.1458	9.1498	9.1537	9.1577	9.1617
77	9.1657	9.1696	9.1736	9.1775	9.1815	9.1855	9.1894	9.1933	9.1973	9.2012
78	9.2052	9.2091	9.2130	9.2170	9.2209	9.2248	9.2287	9.2326	9.2365	9.2404
79	9.2443	9.2482	9.2521	9.2560	9.2599	9.2638	9.2677	9.2716	9.2754	9.2793
80	9.2832	9.2870	9.2909	9.2948	9.2986	9.3025	9.3063	9.3102	9.3140	9.3179
81	9.3217	9.3255	9.3294	9.3332	9.3370	9.3408	9.3447	9.3485	9.3523	9.3561
82	9.3599	9.3637	9.3675	9.3713	9.3751	9.3789	9.3827	9.3865	9.3902	9.3940
83	9.3978	9.4016	9.4053	9.4091	9.4129	9.4166	9.4204	9.4241	9.4279	9.4316
84	9.4354	9.4391	9.4429	9.4466	9.4503	9.4541	9.4578	9.4615	9.4652	9.4690
85	9.4727	9.4764	9.4801	9.4838	9.4875	9.4912	9.4949	9.4986	9.5023	9.5060
86	9.5097	9.5134	9.5171	9.5207	9.5244	9.5281	9.5317	9.5354	9.5391	9.5427
87	9.5464	9.5501	9.5537	9.5574	9.5610	9.5647	9.5683	9.5719	9.5756	9.5792
88	9.5828	9.5865	9.5901	9.5937	9.5973	9.6010	9.6046	9.6082	9.6118	9.6154
89	9.6190	9.6226	9.6262	9.6298	9.6334	9.6370	9.6406	9.6442	9.6477	9.6513
90	9.6549	9.6585	9.6620	9.6656	9.6692	9.6727	9.6763	9.6799	9.6834	9.6870
91	9.6905	9.6941	9.6976	9.7012	9.7047	9.7082	9.7118	9.7153	9.7188	9.7224
92	9.7259	9.7294	9.7329	9.7364	9.7400	9.7435	9.7470	9.7505	9.7540	9.7575
93	9.7610	9.7645	9.7680	9.7715	9.7750	9.7785	9.7819	9.7854	9.7889	9.7924
94	9.7959	9.7993	9.8028	9.8063	9.8097	9.8132	9.8167	9.8201	9.8236	9.8270
95	9.8305	9.8339	9.8374	9.8408	9.8443	9.8477	9.8511	9.8546	9.8580	9.8614
96	9.8648	9.8683	8.8717	8.8751	9.8785	9.8819	9.8854	9.8888	9.8922	9.8956
97	9.8990	9.9024	9.9058	9.9092	9.9126	9.9160	9.9194	9.9227	9.9261	9.9295
98	9.9329	9.9363	9.9396	9.9430	9.9464	9.9497	9.9531	9.9565	9.9598	9.9632
99	9.9666	9.9699	9.9733	9.9766	9.9800	9.9833	9.9866	9.9900	9.9933	9.9967
100	10.									

CHAPTER XI

AREAS AND VOLUMES OF SIMPLE FIGURES

76. Square Measure.—One of the many applications for squares and cubes of numbers is in the measurement of *area* and *volume.* As most of us know, area is a measure of surface, or "flat space," and it is measured in terms of the *square unit.* Thus, a room has so many square feet of floor space, a roof requires a certain number of shingles per 100 sq. ft. of surface, etc. In this case the area is measured by the unit known as the "square foot." A square foot is equal to the amount of surface bounded by a square having each side 1 ft. in length.

The base of a milling machine is 3 ft. wide and 5 ft. long, and the floor space covered by the base is 15 sq. ft., as shown in Fig.

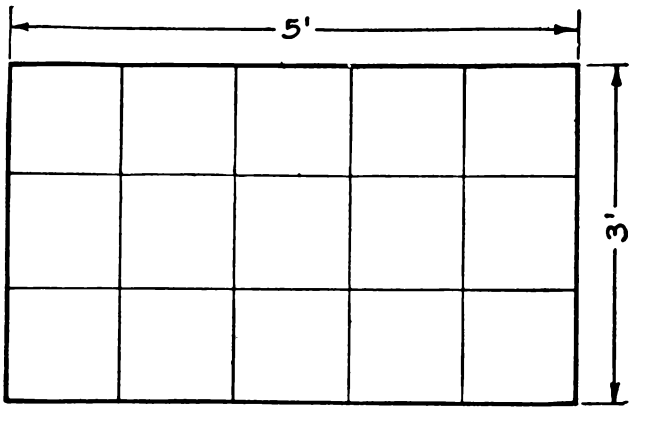


FIG. 30.—Diagram of milling machine base.

30. It can readily be seen that the area is also equal to the product of the length times the width, since $5 \times 3 = 15$. The same is true of any rectangle, that is, any figure bounded by four sides and all four corners of which are right angles or square corners.

Rule.—To find the area of a rectangle, multiply the length by the width.

The grate in a firebox is 8 ft. long and 6 ft. wide. What is the area of grate surface?

To get the area, we multiply the length by the width.

$$8 \text{ ft.} \times 6 \text{ ft.} = \text{Area.}$$

It must be emphasized here that we are not multiplying just "8 × 6," but rather "8 ft. × 6 ft." We see at once that the numerical result is 48, but what becomes of the "feet?" Since we have multiplied *feet* by *feet* the answer must be the *square of feet*, or, as it is called, "square feet"; therefore,

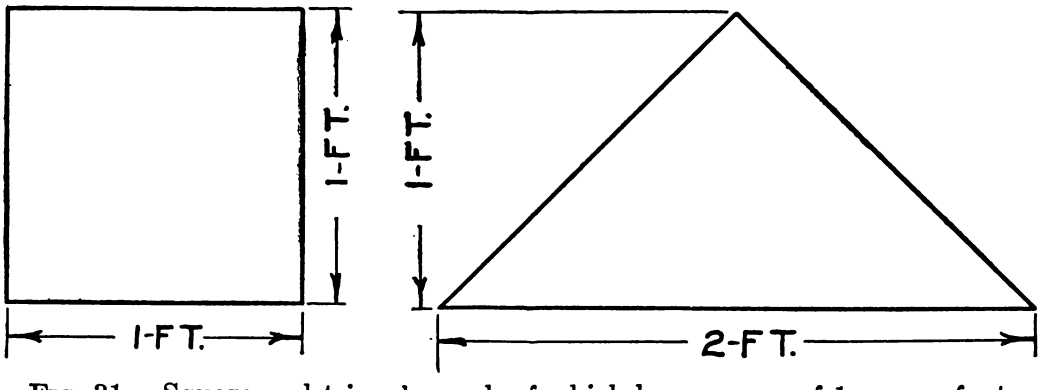


FIG. 31.—Square and triangle, each of which has an area of 1 square foot.

$$8 \text{ ft.} \times 6 \text{ ft.} = 48 \text{ sq. ft.}, \textit{Answer.}$$

A square foot of area does not necessarily have to be in the shape of a square, but may have any shape as long as it contains the required amount of area. Figure 31 shows a triangle and a square, each of which has an area of 1 sq. ft.

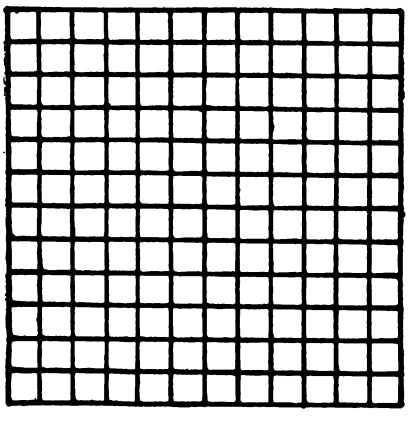


FIG. 32.—Diagram of 1 foot square divided into square inches.

The square inch is another common unit of area. This is much smaller than the square foot, being only one-twelfth as great each way. If a square foot is divided into square inches, it will be seen to contain 12 × 12, or 144 sq. in., Fig. 32. It will be readily seen that the area of any square is equal to the product of the side of the square by itself. In other words, the area of a square equals the side "squared" (referring to the process explained in Art. 67).

Care must be taken to use the same units for the length and width. Thus, you cannot multiply *inches* by *feet* and call the product *square inches* or *square feet.* Both length and width must be in the same unit, that is, both in feet, or both in inches, etc. If the unit of length is feet, the product will be square feet; if inches, the product will be square inches.

As mentioned before, 1 sq. ft. is the area of a square 1 ft. on each side, and if divided into square inches, will be found to contain 12^2, or 144 sq. in. Likewise, a square yard is 3 ft. on each side and, therefore, contains $3^2 = 9$ sq. ft. The following table gives the relation between the units ordinarily used in measuring areas:

MEASURES OF AREA (SQUARE MEASURE)

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre
640 acres = 1 square mile (sq. mi.)

Often we know the area of a rectangle and the length of one side and we wish to find the other side. This is another case of reversing the operation, or working the problem backward. Hence, we merely divide the area by the known side to obtain the unknown side.

Example:

A storage room has 1000 sq. ft. of floor space. Because of its contents it is only possible to measure the length, which is found to be 50 ft. What is the width?

1000 sq. ft. ÷ 50 ft. = 20 ft., *Answer.*

(*Check.*—50 ft. × 20 ft. = 1000 sq. ft.)

In the case of a square, it is only necessary to know the area in order to find the length of the side. Since the area of a square equals the side squared, the length of the side is simply the square root of the area.

77. Area of a Circle.—If we take a circular piece of cardboard, divide it into triangular sections as in Fig. 33, and cut one of the small sections in two equal pieces, we can then place the sections together as in Fig. 34. The figure thus found is practically a rectangle having a length equal to one-half the circumference of the circle and a width equal to one-half the diameter. Hence, the length equals $\frac{1}{2} \times 3.1416 \times$ *diameter*, and the width equals

$\frac{1}{2} \times$ *diameter.* Since the area equals length times width, we have

$$\text{Area} = \left(\frac{1}{2} \times 3.1416 \times D\right) \times \left(\frac{1}{2} \times D\right)$$

in which D is the diameter.

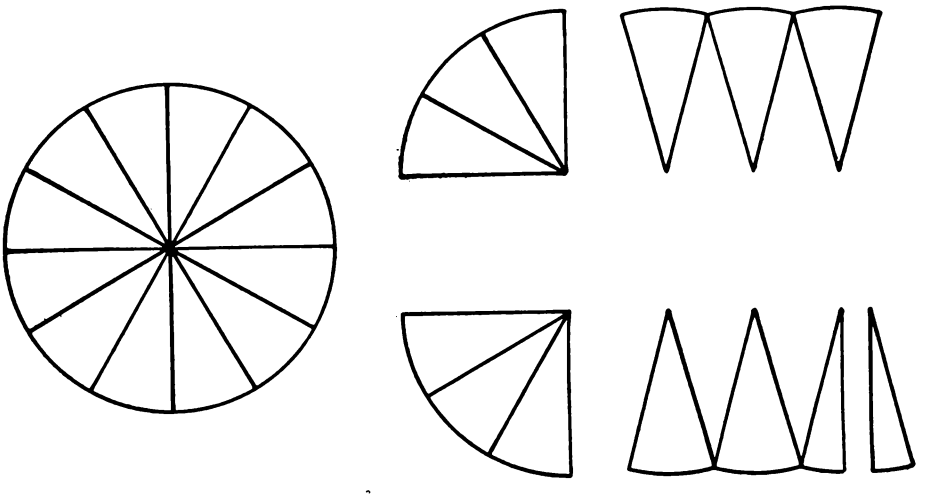


FIG. 33.—Division of circle into triangular sections.

Multiplying the two fractions together we get

$$\text{Area} = \frac{1}{4} \times 3.1416 \times D \times D$$

$$\text{Area} = \frac{3.1416}{4} \times D^2$$

which reduces to

$$\text{Area} = .7854 \times D^2$$

The area of this rectangle is also the area of the circle, because dividing the circle into parts does not change its area. Therefore,

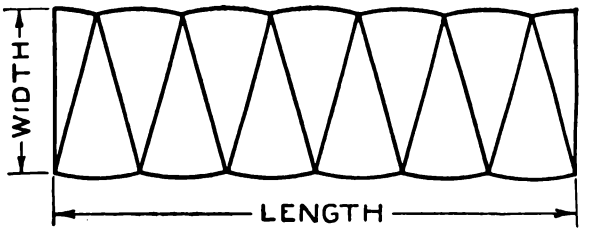


FIG. 34.—Triangular sections of circle placed together.

the area of any circle is found by squaring the diameter and multiplying the square by .7854.

$$\text{Area} = .7854D^2$$

As we saw on page 141, the radius is equal to $\frac{1}{2}$ the diameter; or, if we call the radius r, we may write $r = \frac{1}{2}D$, or $D = 2r$. Then if we substitute $2r$ for D, we get Area $= (3.1416 \times \frac{1}{2} \times 2r)$ $(\frac{1}{2} \times 2r) = 3.1416r^2 = \pi r^2$.

Examples:

1. Find the area of a circle 3 in. in diameter.

$$\begin{aligned} \text{Area} &= .7854\ D^2 \\ &= .7854 \times 3^2 \\ &= .7854 \times 9 \\ &= 7.0686 \text{ sq. in., } \textit{Answer.} \end{aligned}$$

From the formula, it is evident that the ratio of the areas of any two circles will equal the ratio of the squares of their diameters. Consider two circles of diameter d and D respectively.

$$\text{Area of first} = .7854d^2$$

$$\text{Area of second} = .7854D^2$$

$$\text{Ratio of areas} = \frac{a}{A} = \frac{.7854\ d^2}{.7854\ D^2}$$

The constant .7854 cancels out, leaving

$$\frac{a}{A} = \frac{d^2}{D^2}$$

2. In a certain pump the ratio of the piston area on the steam end to that on the water end is as 4 is to 2.25 and the diameter of the piston on the water end is 6 in. Find the diameter of the piston on the steam end.

Given:

$$\frac{a}{A} = \frac{4}{2.25}$$

Then,

$$\frac{4}{2.25} = \frac{d^2}{D^2}$$

Since D is given as 6 in.

$$\frac{4}{2.25} = \frac{d^2}{36}$$

Since d^2 divided by 36 equals $\frac{4}{2.25}$, then

$$d^2 = 36 \times \frac{4}{2.25}$$

$$d^2 = \frac{144}{2.25} = 64$$

$$d = \sqrt{64} = 8 \text{ in., } \textit{Answer.}$$

Finding the Diameter from the Area.—Very frequently the area of a circle is known and it is desired to find the diameter. Just as we worked backward in previous cases, so we can in this case. To find the area of a circle we first *square the diameter*, and second, *multiply by* .7854. To find the diameter from the area, we reverse the process and work backward from the last operation:

First, *divide by* .7854.

Second, *extract the square root.*

Consider a circle having a diameter of 8 in.

$$\text{Square of diameter} = 64$$
$$\text{Area} = .7854 \times 64 = 50.265 \text{ sq. in.}$$

Reversing the process to find the diameter, we have

$$50.265 \div .7854 = 64 \text{ sq. in.}$$
$$\text{Diameter} = \sqrt{64} = 8 \text{ in., } \textit{Answer.}$$

Example:

To get the required velocity of flow in a turbine penstock requires a cross-sectional area of 19.5 sq. ft. What diameter of penstock is needed?

Given:

$$\text{Area} = 19.5 \text{ sq. ft.}$$
$$19.5 \div .7854 = 24.8 + \text{sq. ft.}$$
$$\text{Diameter} = \sqrt{24.8}$$
$$\text{Diameter} = 4.98 + \text{ft., } \textit{Answer.}$$

A diameter of 5 ft. would be used.

78. Measure of Volume.—To measure length we use a unit of length, such as the inch, foot, yard, etc., and to measure area we use a unit of surface measure, such as the square inch, the square foot, etc. There is still another measure of size known as the measure of *volume*, or cubical contents. The measure of volume is expressed in the *cubic unit*, such as the cubic inch, cubic foot, cubic yard, etc.

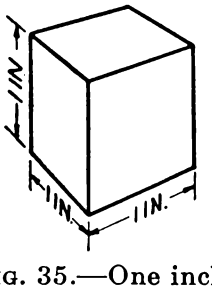


FIG. 35.—One inch cube.

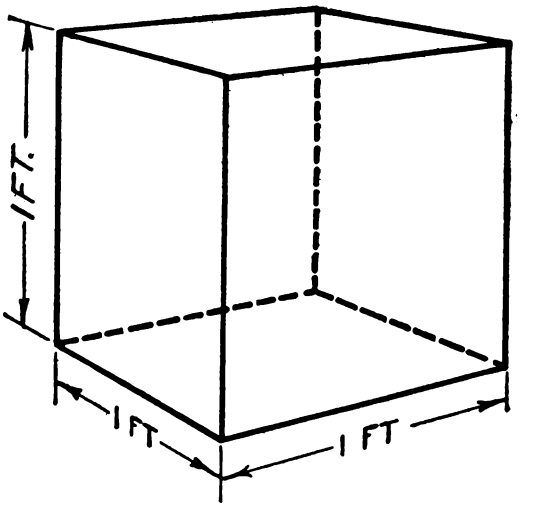


FIG. 36.—One foot cube.

The measure of length takes account of but one dimension, length; that of area considers two dimensions, length and width; but the measure of volume embraces both length and width and a third dimension, thickness.

Figure 35 shows a small block or cube each side of which measures 1 in. The volume of this block is equal to the unit

of volume called the "cubic inch." Similarly a cube, such as Fig. 36, having each side 1 ft. long, equals the unit of volume called the "cubic foot."

Suppose that we cut a slice 1 in. thick from the bottom of the block in Fig. 36, and divide this slice into 1-in. cubes, as illustrated in Fig. 37. Each small cube then contains 1 cu. in. and there will clearly be 144 cubes; therefore, the volume of the slice

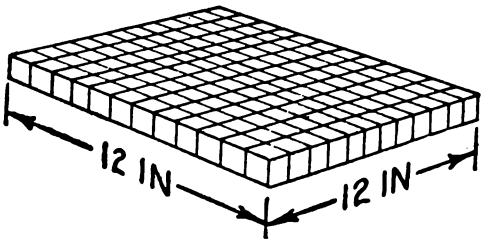


FIG. 37.—Block containing 144 cubic inches.

is 144 cu. in. As the slice cut off is but 1 in. thick, it is apparent that the block of Fig. 36 could be cut into 12 such slices, each 1 in. thick and each containing 144 cu. in. From this we see that the total volume of the block is $12 \times 144 = 1728$ cu. in. Since the volume of the block is also 1 cu. ft., it is quite evident that 1 cu. ft. must contain 1728 cu. in.

79. Finding the Volume.—A rectangular block is shown in Fig. 38, 10 in. long, 4 in. wide, and 3 in. thick. At one end of

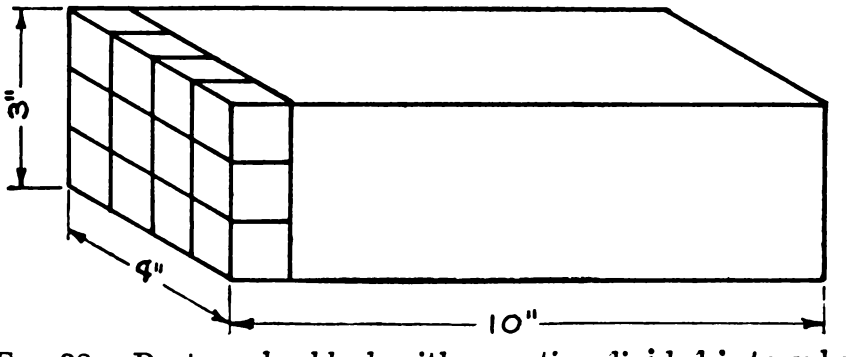


FIG. 38.—Rectangular block with a portion divided into cubes.

the block is indicated an imaginary slice 1 in. thick and cut up into 1-in. cubes. By actual count there are 12 cu. in. in this slice. We at once note the fact, also, that the area of the end surface of the block is $3 \times 4 = 12$ sq. in., or the same as the number of cubic inches. Since the block is 10 in. long, it could be cut into 10 such slices and its volume is, therefore, $10 \times 12 = 120$ cu. in.

It is quite plain to be seen that if we multiply the length by the area of the end, we will obtain the same result; thus,

$$10 \text{ in.} \times 12 \text{ sq. in.} = 120 \text{ cu. in., } \textit{Answer}$$
$$\text{Length} \times \text{Area} = \text{Volume}$$

The same thing holds true for any block or bar having a constant cross-section; that is, any block or bar that could be cut up into equal slices as illustrated in Fig. 39. It can easily be seen that

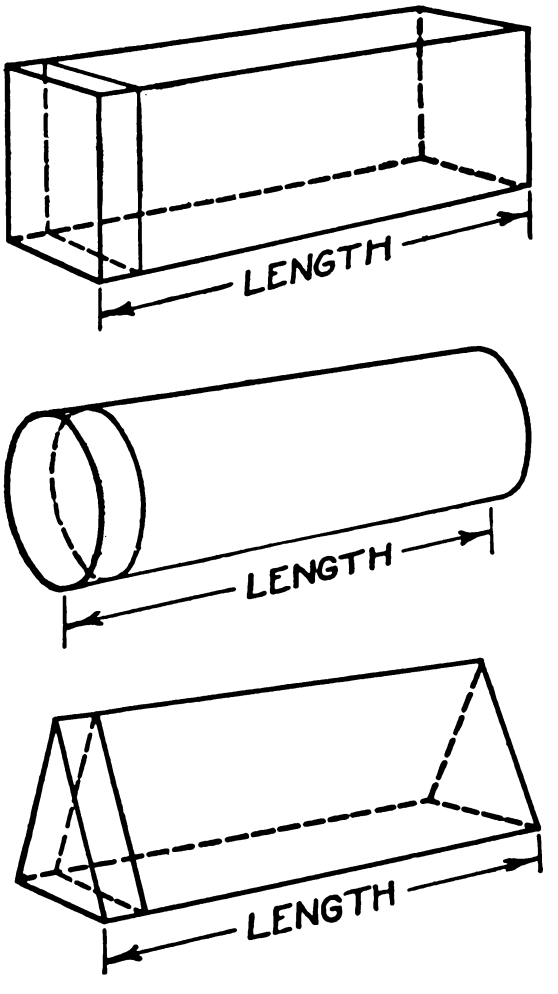


FIG. 39.—Comparative volumes of solids of equal lengths.

the area at the end of each bar in Fig. 39 equals the number of cubic units in a slice of unit thickness. The total volume, then, is the product of this amount times the length of the bar, or

$$\text{Volume} = \text{Area of end} \times \text{Length}$$

For rectangular bars, such as the top bar of Fig. 39, the area of the end equals the product of the breadth times the thickness,

and the volume thus equals the product of length times breadth times thickness. If L, B, and T represent length, breadth, and thickness, then for rectangular blocks

$$\text{Volume} = L \times B \times T$$

In finding the volume it is necessary that the proper units be used. When multiplying length times breadth times thickness,

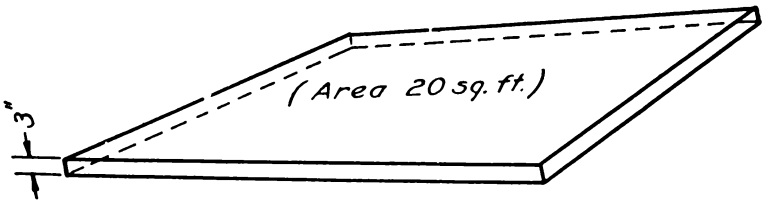


FIG. 40.—Steel plate.

all of these dimensions must be in the same unit, that is, all in inches, or all in feet, etc., and the volume will be cubic inches, cubic feet, etc., respectively. In the same manner, if the volume

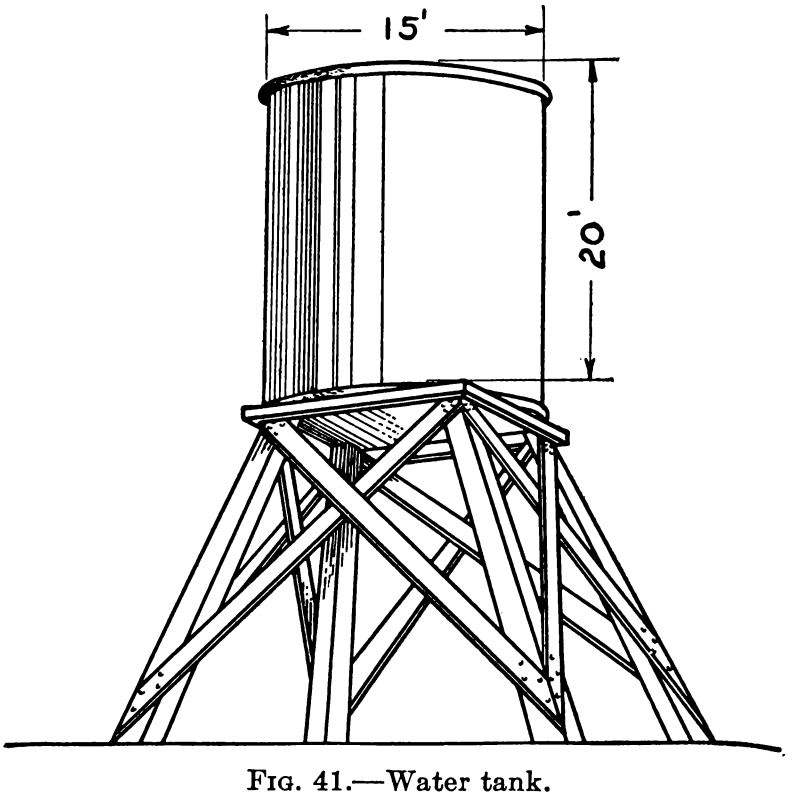


FIG. 41.—Water tank.

is found by multiplying the area by the length, the unit of area must be square inches when the length is inches, or square feet when the length is feet, etc.

Examples:

1. Find the volume of the steel plate in Fig. 40 if the area of the top surface is 20 sq. ft. and the plate is 3 in. thick.

In the case of a rectangular block, the area of any side may be considered the area of the end, and the length will be the dimension at right angles to that side.

Given:

$$\text{Area of end} = 20 \text{ sq. ft.}$$

$$\text{Length} = 3 \text{ in., or } \frac{1}{4} \text{ ft.}$$

$$\text{Volume} = \frac{1}{4} \times 20 = 5 \text{ cu. ft., } \textit{Answer.}$$

In the case of bodies which have a cylindrical shape (such as a piece of shafting) where the end is circular in shape, the area of the end is found by the rule for the area of a circle.

2. The water tank of Fig. 41 is 15 ft. in diameter and 20 ft. high. How many cubic feet of water will it hold when full? (We find the volume here exactly as for a solid body.)

$$\text{Area of end} = .7854 \times D^2$$

$$\text{Area of end} = .7854 \times 225 = 176.71 \text{ sq. ft.}$$

$$\text{Volume} = \text{Area} \times \text{Length}$$

$$\text{Volume} = 176.71 \times 20 = 3534.20 \text{ cu. ft., } \textit{Answer.}$$

80. Weights of Metals.—The chief uses in the shop for calculations of volume are in finding the amount of material needed to make some object, in finding the weight of some object that cannot be conveniently weighed, or in finding the capacity of some bin or other receptacle. Having obtained the volume of an object, it is only necessary to multiply the volume by the known weight of a unit volume of the material to get the weight of the object. In the case of the plate of which we just obtained the volume, 1 cu. ft. will weigh about 489 lb., so the total weight of the plate will be

$$5 \times 489 = 2445 \text{ lb.}$$

The weight of the water in the tank will be the number of cubic feet times 62.4, since 1 cu. ft. of water weighs 62.4 lb.

$$3534.2 \times 62.4 = 22{,}053+ \text{ lb.}$$

The following table gives the weights per cubic inch and per cubic foot for the most common metals and also for water:

WEIGHTS OF MATERIALS (Pounds)

Material	1 cu. in.	1 cu. ft.
Cast iron	.260 lb.	450 lb.
Wrought iron	.278 lb.	480 lb.
Steel	.283 lb.	489 lb.
Brass	.301 lb.	520 lb.
Copper	.318 lb.	550 lb.
Lead	.411 lb.	711 lb.
Aluminum	.094 lb.	162 lb.
Water	.036 lb.	62.4 lb.

81. Short Rule for Plates.—A flat wrought-iron plate $\frac{1}{8}$ in. thick and 1 ft. square will weigh 5 lb., since $12 \times 12 \times \frac{1}{8} = 18$ cu. in., and $18 \times .278 = 5$ lb. The rule obtained from this is very easy to remember and is very useful for plates the dimensions of which are in exact feet.

Rule.—Weight of flat iron plates = area in square feet × number of eighths of an inch in thickness × 5. This rule can also be used for steel plates by adding 2% to the result calculated from the above rule.

Example:

Find the weight of a steel plate 30 in. × 96 in. × $\frac{3}{8}$ in.

30 in. = $2\frac{1}{2}$ ft., 96 in. = 8 ft., $\frac{3}{8}$ in. = 3 eighths.

$2\frac{1}{2} \times 8 \times 3 \times 5 = 300$ lb. (weight if it were of wrought iron).

2% of 300 = 6 lb.

300 + 6 = 306 lb.; weight of steel plate, *Answer.*

If this weight is calculated by first getting the cubic inches of steel, we get:

$30 \times 96 \times \frac{3}{8} = 1080$ cu. in.

$1080 \times .283 = 305.64$ lb., weight of steel plate, *Answer.*

We see that the results check as closely as could be expected and, in fact, different plates of supposedly the same size would differ as much as this because of differences in rolling.

82. Weight of Casting from Pattern.—In foundry work, it is often desired to get the approximate weight of a casting in order to calculate the amount of metal needed to make it. The probable weight of the casting can be obtained closely enough by weighing the pattern and multiplying this weight by the proper

number from the following table. In case the pattern contains core prints, the weight of these prints should be calculated and subtracted from the pattern weight before multiplying; or the total pattern weight can be multiplied first and then the weight of metal which would occupy the same volume as the core print be subtracted from the product.

PROPORTIONATE WEIGHT OF CASTINGS TO WEIGHT OF WOOD PATTERNS

For each 1 lb. weight of pattern (less weight of core prints) when made of	Casting will weigh, in pounds, if made of			
	Cast iron	Copper or bronze	Brass	Aluminum
White pine	16.0	19.6	18.5	5.9
Mahogany	12.0	14.7	14.0	4.5
Pear wood	10.2	12.5	11.7	3.8
Birch	10.6	13.0	12.3	3.9
Brass	0.84	1.0	0.95	0.32
Aluminum	2.6	3.2	3.1	0.95
Cast iron	0.95	1.3	1.2	0.38

83. Finding Dimensions from Weights or Volumes.—Sometimes we have the weight or volume of a body, but not all of the dimensions are known, and it is desired to find the unknown dimensions.

By a method of working backward we can find the volume from the total weight by dividing by the weight per unit volume. Then, if the length is known, we can find the area of the end by dividing the volume by the length; or if the area is known, the length can be found by dividing the volume by the area. The rest of the calculation can be performed as explained in previous paragraphs of this chapter.

Example:

The superintendent of a small factory desires to store 100 tons of coal in addition to its usual supply. The only available space in which to put it is a small shed 20 ft. wide by 40 ft. long. As the roof is rather low it is necessary to find out how high the coal will be in the shed, assuming that it is piled level. (The weight of 1 cu. ft. of coal is about 43 lb.)

1 ton of coal = 2000 lb.
Volume of 1 ton = 2000 ÷ 43 = 46.5 cu. ft.
Volume of 100 tons = 100 × 46.5 = 4650 cu. ft.

Floor area of shed = 20 ft. × 40 ft. = 800 sq. ft.
Height of coal = Volume ÷ Area
Height of coal = 4650 ÷ 800 = 5.8+ ft., *Answer.*

PROBLEMS

221. The diameter of a $3\frac{1}{2}$-in. bolt at the bottom of the threads is 3.1 in. What is the cross-sectional area of the bolt at the bottom of the threads? (Use tables.)

222. How many square feet of roofing paper will be required to cover a flat roof 40 ft. wide and 70 ft. long, allowing 10% extra for lap and waste?

223. Find the area of the desk top illustrated in Fig. 41*a*.

224. What is the area of the end as shown in Fig. 41*a*.

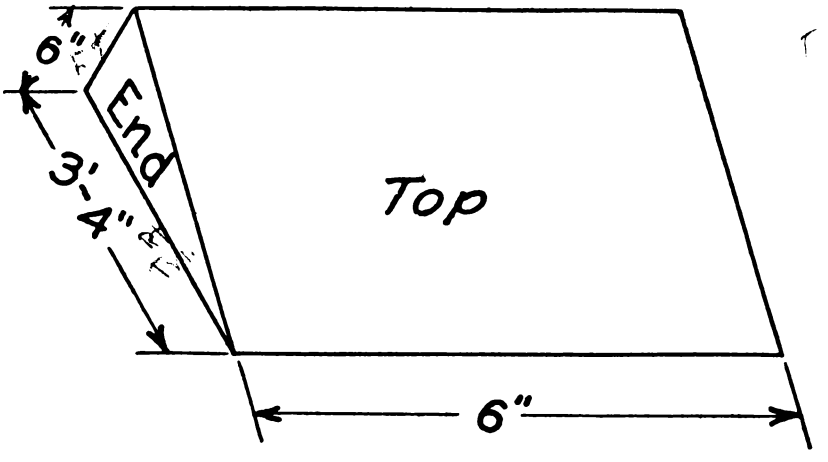


FIG. 41*a*.

225. Find the area of the diagram in Fig. 41*b*.

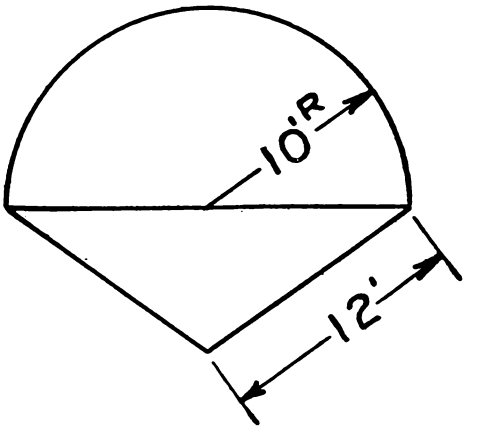


FIG. 41*b*.

226. Find the area of the diagram in Fig. 41*c*.

227. In the design of a hot air heating system it was determined that the air pipes (or leaders) from the furnace to the second floor risers

should have a cross-sectional area of about 50 sq. in. What should be the diameter of the pipes used?

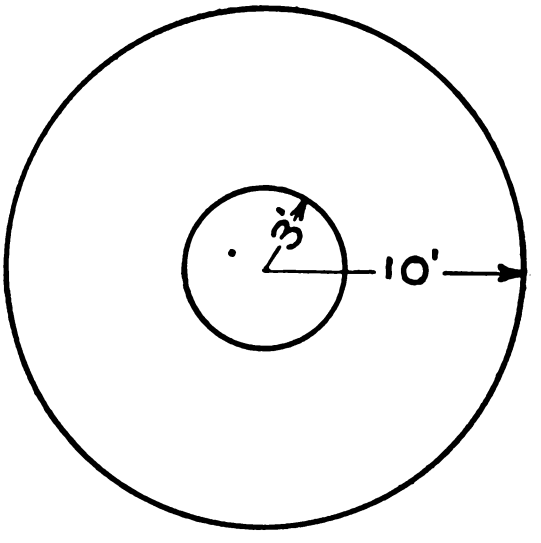


FIG. 41*c*.

228. The riser pipes in the above problem are rectangular in shape and have the same cross-sectional area as the leaders. The short dimension or depth of the riser is limited by the wall space and in this case is $3\frac{1}{2}$ in. How wide must the risers be made?

229. The piston of a steam engine is 8 in. in diameter. If the steam pressure in the cylinder is 100 lb. per square inch at the beginning of the stroke, what is the total pressure acting on the piston at that time?

230. A 6-in. pipe and an 8-in. pipe both discharge into a single header. Find the diameter of the header so that it will have an area equal to that of both pipes.

231. How many cubic feet of water will be contained in a cistern 6 ft. deep, 8 ft. wide, and 10 ft. long?

232. What is the weight of a steel shaft 6 in. in diameter and 20 ft. long?

233. A foundry is to cast 10 hollow, round, cast-iron columns, each 15 ft. long, 12 in. outside diameter, and 9 in. inside diameter. How many pounds of iron will be required to cast all the columns, allowing 15% extra for waste?

234. A paint shop wants a tank 4 ft. deep with a square bottom built to hold, when full, 400 gal. of varnish. What will be the diameter of the tank? (There are 231 cu. in. in a gallon.)

235. A copper billet 2 in. by 8 in. by 24 in. is rolled out into a plate of No. 10 B. & S. gage. The thickness of this gage is .1019 in. What would be the probable area of this plate in square feet?

CHAPTER XII

LEVERS

84. Types of Machines.—All machines consist of one or more of the three fundamental types of machines—the *Lever*, the *Cord*, and the *Inclined Plane* or *Wedge*. Any piece of mechanism can be proved to be of one or more of these types. Pulleys, gears, and cranks will be shown to be forms of levers; belts and chains come under the type called the cord; while screws, worms, and cams are forms of inclined planes. They are all used to transmit power from one place to another and to modify it, as desired.

85. The Lever.—The lever is probably the most used and the simplest type of machine. We are all familiar with it in its

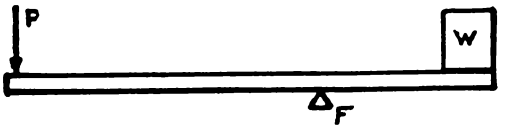


FIG. 42.—Simple lever.

simplest forms, such as crowbars, shears, pliers, tongs, and the numerous simple levers found on machine tools.

A lever is a rigid rod or bar so arranged as to be capable of turning about a fixed point. This fixed point about which the lever turns is called the *Fulcrum*. In Fig. 42 the fulcrum is represented by the small triangular block F. The position of this fulcrum determines the effect which the force P applied at one end has toward lifting the weight W at the other end. If F is close to W, a comparatively small force P may be able to raise the weight W, but if F is moved away from W and placed close to P, then a greater force will be required at P. If F is in the middle, P and W will be just equal.

In every lever there are two opposing tendencies as may be seen from Fig. 42; first, that of the load or weight W tending to descend; and second, that of the force P tending to raise W. The ability of W to descend or to resist being lifted depends upon two things—its weight and its distance from the fulcrum F.

The product of these two is the measure of the tendency of W to descend. This product is called, in books on mechanics, the *Moment.* Likewise, the force P has a moment, which is the product of the force P and the distance from P to the fulcrum F. If the force and the weight just balance each other, their moments are equal.

The length from P to F is called the *force arm* and the length from W to F, the *weight arm.* Then, for balance, we can say that,

$$\text{Force} \times \text{Force arm} = \text{Weight} \times \text{Weight arm}$$

If we let P stand for the force
a stand for the force arm
W stand for the weight
and b stand for the weight arm

as shown in Figs. 45, 46, and 47, we will have the formula

$$P \times a = W \times b$$

Although the force and weight are really *balanced* when this formula is fulfilled, still we use the formula for calculating the

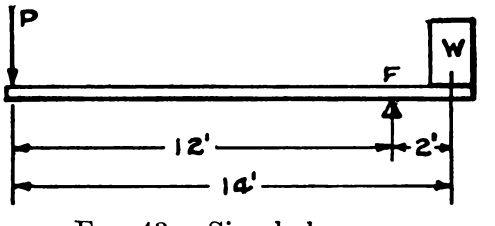


FIG. 43.—Simple lever.

forces necessary to *lift* weights. The very slightest increase in the force above that necessary for balance will cause W to rise and, therefore, we can say practically that P will lift W if

$$P \times a = W \times b$$

In Fig. 43 is shown a lever with the force P at one end, the weight W at the other, and the fulcrum F between them. From the figure, the force arm a is 12 ft. and the weight arm b is 2 ft. What force at P will lift a load W of 1800 lb.?

$$\text{Force} \times \text{Force arm} = \text{Weight} \times \text{Weight arm}$$

$$\text{or} \qquad P \times a = W \times b$$

$$\text{Therefore,} \quad P \times 12 = 1800 \times 2$$

$$P \times 12 = 3600$$

Then if P times 12 is 3600, P will be 3600 divided by 12

$$P = \frac{3600}{12}$$

$$P = 300 \text{ lb., } \textit{Answer.}$$

A simple type of lever known as a "pinch bar" is illustrated in Fig. 44. As shown here it is used chiefly for moving railway cars. A force of 2400 lb. is required at the point of the bar in order to move a certain car, and a man applies a force of 100 lb.

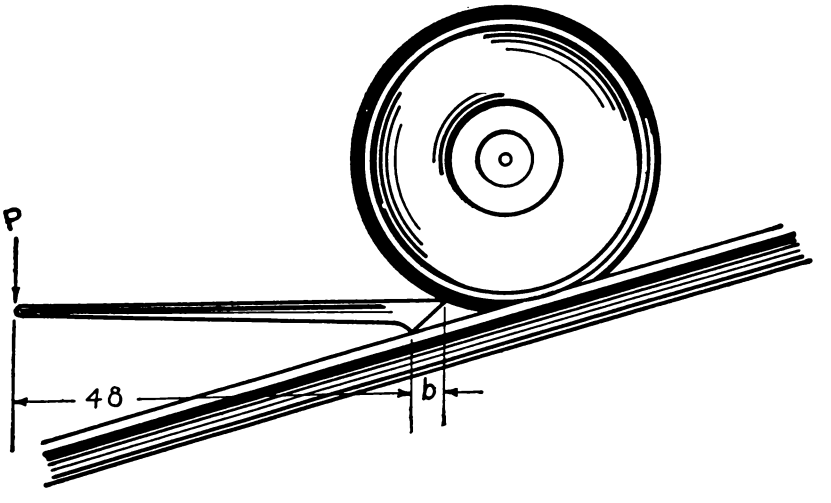


FIG. 44.—Application of lever as used for pinch bar.

at P. If the applied force is just sufficient to move the car, how long is the weight arm b? In this case the force is 100 lb., the force arm 48 in., the weight 2400 lb., and the weight arm, b.

$$\begin{aligned} P \times a &= W \times b \\ 100 \times 48 &= 2400 \times b \\ 4800 &= 2400 \times b, \text{ which is the same as} \\ 2400 \times b &= 4800 \end{aligned}$$

If 2400 times b equals 4800, then b equals 4800 divided by 2400,

$$b = \frac{4800}{2400}$$

$$b = 2 \text{ in., } \textit{Answer.}$$

It will be seen that the relation between the force, weight, force arm, and weight arm can be written as an inverse proportion.

$$\text{Force} : \text{Weight} = \text{Weight arm} : \text{Force arm}$$

$$P : W = b : a$$

$$\text{or } \frac{P}{W} = \frac{b}{a}$$

This form of expressing the relation is not generally so useful as the other form $P \times a = W \times b$. It is very useful, however, in cases where neither the force arm nor the weight arm is known.

Example:

If a man wanted to lift a 750-lb. weight by means of a 12-ft. timber used as a lever, where would he place the fulcrum so that his whole weight of 150 lb. would just raise it?

$$\frac{P}{W} = \frac{b}{a}$$
$$\frac{150}{750} = \frac{b}{a}$$
$$\text{or } \frac{b}{a} = \frac{150}{750}$$

This reduces to
$$\frac{b}{a} = \frac{1}{5}$$

The total length of the timber (12 ft.) equals the sum of b and a (Fig. 45) From the ratio $\frac{b}{a} = \frac{1}{5}$, which reads "$b$ is to a as 1 is to 5," it is clear that b is one part of the length, a is 5 parts, and the whole length is 6 parts. Five parts, or a will be $\frac{5}{6}$ of the length and one part, or b will, therefore, be $\frac{1}{6}$ of the length.

$a = \frac{5}{6} \times 12$
$a = 10$ ft., *Answer.*
$b = \frac{1}{6} \times 12$
$b = 2$ ft., *Answer.*

86. Three Classes of Levers.—Levers are divided into three kinds or classes according to the relative positions of the force, fulcrum, and weight.

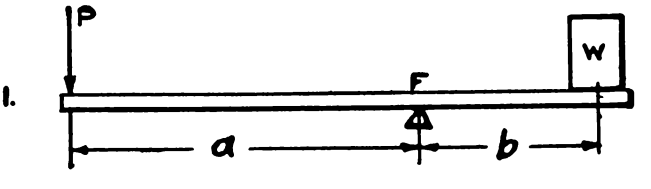


FIG. 45.—Lever of the first class.

Those shown so far are of the first class, Fig. 45, in which the fulcrum is between the force and the weight. The weight is lifted by pushing down at P.

In the second class, Fig. 46, the weight is between the fulcrum and the force, and the weight is lifted by pulling up at P.

In the third class, Fig. 47, the force P is between the weight and the fulcrum and, therefore, P must be greater than the load that it lifts. The weight is lifted by an upward force at P.

In all these types the same rule holds that:

$$\text{Force} \times \text{Force arm} = \text{Weight} \times \text{Weight arm}$$

or

$$P \times a = W \times b$$

Particular attention should be given to the fact that *the force arm and weight arm are always measured from the fulcrum.* In levers of class 2, the force arm is the entire length of the lever.

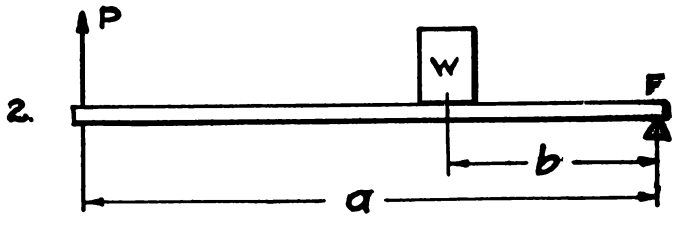


FIG. 46.—Lever of the second class.

In class 3, the force arm is shorter than the weight arm. This type may be seen on the safety valves of some boilers and is used so that a small weight can balance a considerable pressure at *P*.

Quite often there appear to be two weights, or two forces, on a lever, and it is difficult to decide which to designate as the force

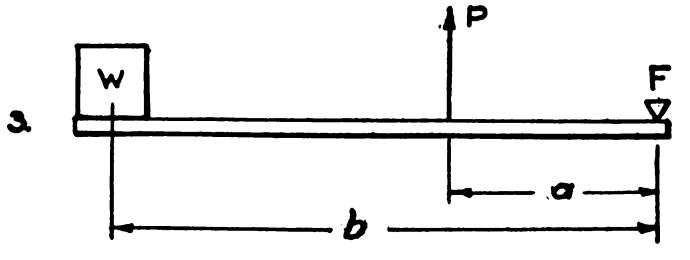


FIG. 47.—Lever of the third class.

and which as the weight. It really makes no difference which we call the force and which the weight; the relations between them would be the same in any case.

87. Compound Levers.—Compound levers are frequently used; but problems concerning them are easily reduced to repeated cases of single levers, the force of one lever corresponding to the weight of the next, etc. To illustrate this we will solve the following example:

Example:

We wish to lift 8000 lb. with a compound lever as shown in Fig. 48, the first one being 10 ft. long with the weight 2 ft from the end; the

second 16 ft. long with the fulcrum 4 ft. from the end; what will be the necessary force, P_2?

$P \times a = W \times b$

$P_1 \times 10 = 8000 \times 2$

$P_1 = \frac{16{,}000}{10} = 1600$ lb.

$W_2 = P_1 = 1600$ lb.

$P_2 \times a = W_2 \times b$

$P_2 \times 12 = 1600 \times 4$

$P_2 = \frac{6400}{12} = 533\frac{1}{3}$ lb.. *Answer.*

Explanation: Taking the first, or lower lever, we find it to be an example of the second class. W has a weight arm b of 2 ft. The force has an arm equal to the whole length of the lever, or 10 ft. The necessary force on the end of this lever we find to be 1600 lb.

The second lever must pull upward through the connection with a force of 1600 lb. In other words, the 8000-lb. weight on the first lever is equivalent to a 1600-lb. weight on the short end of the second lever. The first lever pulls downward the same amount that the

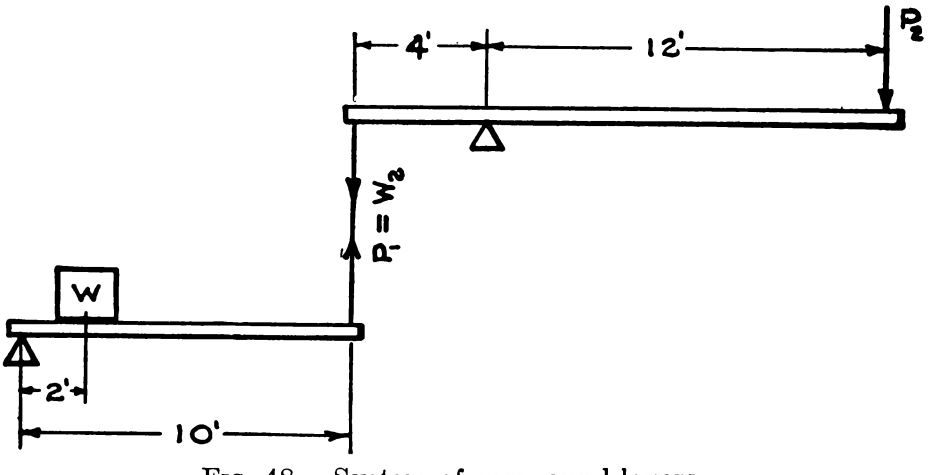


FIG. 48.—System of compound levers.

second pulls up, or $P_1 = W_2$. Having this 1600 as the weight, we find that a force of $533\frac{1}{3}$ lb. is needed on the end of the second lever.

88. Mechanical Advantage.—The ratio of the weight to the force is often called the *Mechanical Advantage* of the lever. Thus, if a man exerting a force of 50 lb. on a lever can raise a weight of 500 lb., the lever has a mechanical advantage of $\frac{500}{50} = 10$. In simple words, by such a lever every pound of force the man exerts can raise a weight of 10 lb. This means that a 1-lb. force will lift 10 lb., a 2-lb. force will lift 20 lb., a 10-lb. force will lift 100 lb., etc. The force multiplied by the M.A. (mechanical advantage) gives the weight that can be lifted, or the weight divided by the M.A. gives the force required.

From a previous paragraph, we found that the ratio of the weight to the force equals the ratio of the force arm to the weight arm, or $\frac{W}{P} = \frac{a}{b}$. Since the M.A. equals $\frac{W}{P}$, it must also equal $\frac{a}{b}$,

or the ratio of force arm to weight arm. If the M.A. equals 10, we know that $\frac{W}{P}$ equals 10, and, therefore, $\frac{a}{b}$ equals 10.

In a compound lever the M.A. of the system of levers equals the product of the M.A.'s of all the separate single levers. The M.A. of the first lever of Fig. 48 equals $\frac{10}{2}$, or 5, and of the second, $\frac{12}{4}$, or 3; hence, the M.A. of the compound lever of Fig. 48 is $5 \times 3 = 15$. The load W is 8000 lb., so that the force required at P_2 equals the weight divided by the M.A., or $8000 \div 15 = 533\frac{1}{3}$ lb.

If the mechanical advantage of a lever is 10, then 1 lb. will lift 10 lb., or 800 lb. will lift 8000 lb., etc.; but it must be remembered that the 1 lb. or the 800 lb. must travel 10 times as far as the 10 lb. or the 8000 lb.

If a lever has a mechanical advantage of 10, the force must travel 10 times as far as it lifts the weight, and, consequently, a lever effects no saving in *work*. Work is the product of force, or weight, times the distance moved, and is the same for either end of the lever. It is similar to carrying a lot of castings to the top floor of a building. If I carry half of them at a time, I must make two trips; if I carry one-fourth of them at a time, I must make four trips. The lighter the load, the more trips I must make. The work done is the same whatever way I carry them and is equal to the product of the total weight times the height to which the load must be carried.

89. The Wheel and Axle.—This is a name given in mechanics to the modification of levers that enables them to be rotated

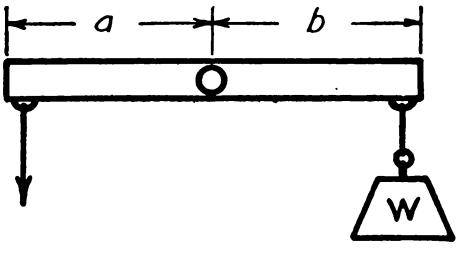


FIG. 49.—Simple lever pivoted at center.

continuously. That the wheel, pulley, or axle is a lever is not apparent at first glance, but a close study of Figs. 49 and 50 hould make it clear. The simple lever of Fig. 49 is a straight piece pivoted on a shaft which furnishes the fulcrum, and since

a and b are equal, its M.A. is 1. In Fig. 50 is a wheel or pulley with a rope running over it, and a weight W at one end of the rope is balanced by a force P at the other end. By dotting in on the wheel of Fig. 50 the lever of Fig. 49, we see at once that

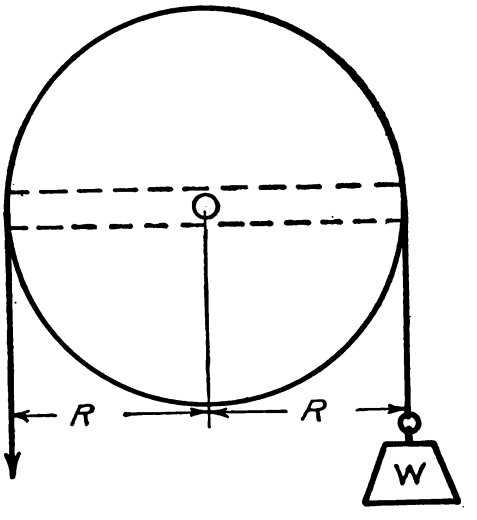


FIG. 50.—Wheel and axle.

the wheel is in reality a simple lever. The radius of the wheel is both the weight arm and the force arm and its M.A. is, therefore, $\frac{R}{R}$, or 1.

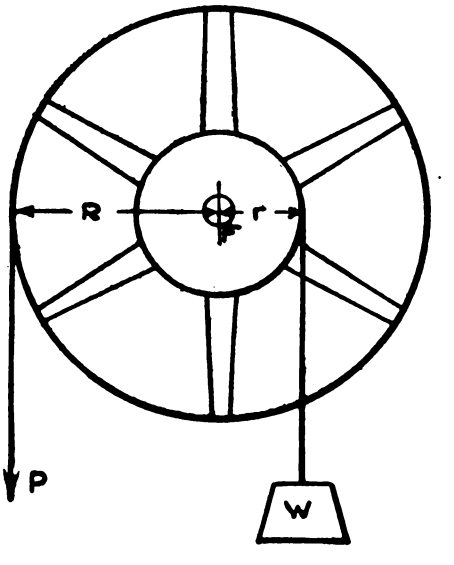


FIG. 51.—Wheel and axle.

By using two pulleys or wheels of different diameters attached to the same shaft, an M.A. greater than 1 can be obtained. This principle is illustrated by the wheel and axle of Fig. 51.

The force P exerted on the rope which is wound on the wheel, balances the weight W, hung from the rope which is wound on the axle. The force arm is thus equal to the radius, R, of the wheel, the weight arm equals the radius, r, of the axle, and the M.A. equals the ratio of force arm to weight arm, or M.A. $= \frac{R}{r}$.

Two pulleys on a countershaft may be likened to a lever in the same way. The belt which drives the countershaft furnishes the force P. The radius of this pulley is the force arm. The radius of the other countershaft pulley, which transmits the power to the machine, is the weight arm, and the pull in this belt

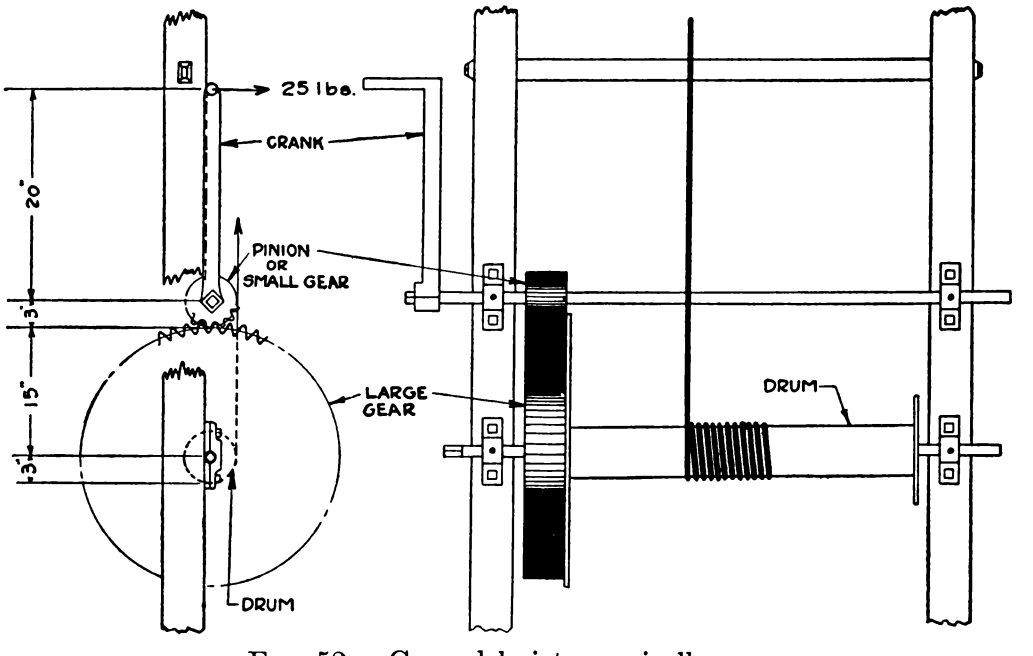


FIG. 52.—Geared hoist or windlass.

is the weight. Gears also are levers that can be rotated continuously. The simplest example of the use of the axle is probably the windlass, which we see used for hoisting, house-moving, etc.

A geared windlass, such as shown in Fig. 52, is a case of compound levers. The crank and pinion form the first lever, and the load on the gear teeth is transmitted to the teeth of the larger gear and becomes the force of the other lever, which consists of the large gear and the drum.

Example:

A geared windlass, such as shown in Fig. 52, has a crank 20 in. long; the small gear is 6 in. in diameter, the large gear is 30 in. in diameter, and the diameter of the drum is 6 in.

What load could be raised by a man exerting a force of 25 lb. on the crank?

$6 \div 2 = 3$ in., radius of pinion
$20 \div 3 = 6\frac{2}{3}$, M.A. of crank and pinion
$30 \div 2 = 15$ in., radius of gear
$6 \div 2 = 3$ in., radius of drum
$15 \div 3 = 5$, M.A. of gear and drum
$6\frac{2}{3} \times 5 = 33\frac{1}{3}$ in., total M.A.
$25 \times 33\frac{1}{3} = 833+$ lb., *Answer.*

If we do not know the sizes of the gears in inches, but know the number of teeth on each, we can calculate the total M.A. in the same manner by using the number of teeth instead of the radius of each gear. This does not give the correct value for the M.A. of the separate parts, such as the crank and pinion, but the product of all the M.A.'s or the total M.A. will be the correct value.

90. Efficiency.—In all classes of machines, the lever included, there is a certain amount of force, power, or energy that is lost within the machine itself due to friction. Previous to this, such losses have been neglected, and the forces were calculated on the assumption that the machines were frictionless.

It is easily seen that if the windlass of Fig. 52 were unoiled and rusty it would require a greater force at the crank to pull a given load on the rope than if all the bearing surfaces were oiled and in good running condition. What then becomes of the additional force applied at the crank in the first condition, since the force delivered to the rope is the same in both cases? As a matter of fact, this extra force is simply used up in overcoming the additional friction in the machine due to the rusty bearings, etc. For any useful purpose it is lost beyond recovery. In spite of the best methods of oiling bearings, smoothing the surfaces, etc., friction cannot be eliminated entirely, and its presence is a very important factor in industry.

To draw an analogy, a machine might be compared to a leaky hose; the faster the hose leaks, the faster must the water be supplied at one end in order to deliver a constant stream at the other end. Thus, in a machine, friction is a leak, and a certain amount of the force or energy supplied to the machine is lost through this leak. The greater the friction, the greater is the loss of power.

Again considering the windlass of Fig. 52, it will be noted from the example of the previous paragraph that the windlass has a M.A. equal to $33\frac{1}{3}$. From the principles of levers, a force of 25 lb. at the crank will exert a pull of $25 \times 33\frac{1}{3} = 833$ lb. on

the rope. This is theoretically true (neglecting friction), but actually, however, in spite of well-oiled bearings, a certain amount of force will be lost due to friction, and the pull on the rope will be less than 833 lb., possibly no more than 750 lb. Supposing this to be the case, we then have a machine *supplied* with sufficient force to exert a pull of 833 lb. but actually only *delivering* a pull of 750 lb. $\frac{750}{833} = .90$. In other words, the *force delivered* is only .9 or 90% of the *force supplied.* This fraction, $\frac{750}{833}$, or 90%, is caed the *efficiency* of the machine.

Efficiency may be defined as the ratio of the power or energy delivered up by a machine to that supplied to it.

In calculating efficiencies such as the above, the force supplied must be calculated at the delivery end of the machine in order to compare it with the force delivered. In other words, the machine is supplied with a 25-lb. force at the crank which is equivalent to a force of 833 lb. at the rope, or delivery end. If the efficiency of the windlass had been 75%, the actual pull on the rope would be $.75 \times 833 = 625$ lb. Expressed in words, 625 lb., the force delivered, is 75 % of 833 lb., the force supplied.

Example:

Assuming that the windlass of Fig. 52 has an efficiency of 85% and its M.A. is $33\frac{1}{3}$, what force will be required at the crank to deliver a pull of 1700 lb. at the rope?

Explanation: It is evident that 1700 lb. is the pull to be *delivered.* Since the efficiency is 85%, the pull delivered is 85% of the force supplied; therefore, 1700 is 85% of the force supplied.

$\therefore$ the force supplied $= \frac{1700}{85} \times 100 = 2{,}000$ lb.

Actual force at crank $= \frac{2000}{\text{M.A.}} = \frac{2000}{33\frac{1}{3}}$

Force at crank = 60 lb., *Answer.*

PROBLEMS

236. In the lever of Fig. 53, where should the fulcrum be placed so that the weight and the force will be balanced?

237. In the lever of Fig. 54 where should the weight of 1800 lb. be placed so that it may be lifted by a force of 200 lb.?

238. Figure 55 shows a safety valve V loaded with a 50-lb. weight at W. Find the total steam pressure on the bottom of V necessary to lift the valve.

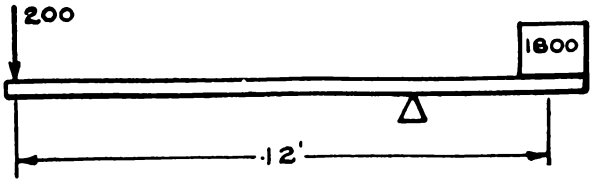


FIG. 53.—Illustration for Problem 236.

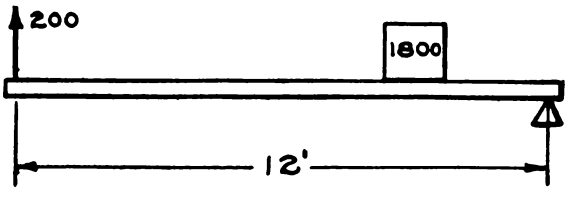


FIG. 54.—Illustration for Problem 237.

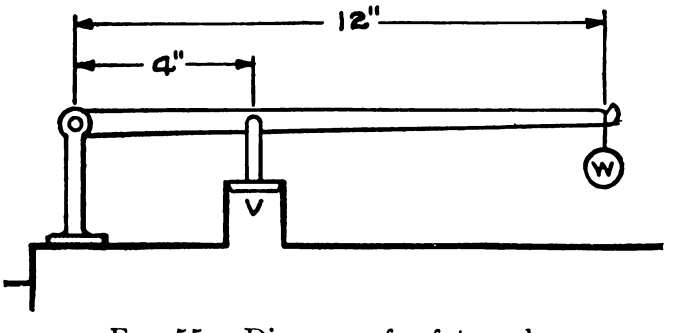


FIG. 55.—Diagram of safety valve.

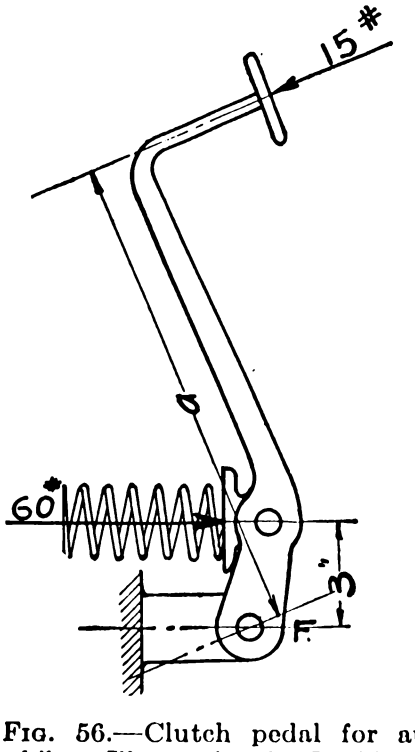


FIG. 56.—Clutch pedal for automobile. Illustration for Problem 240.

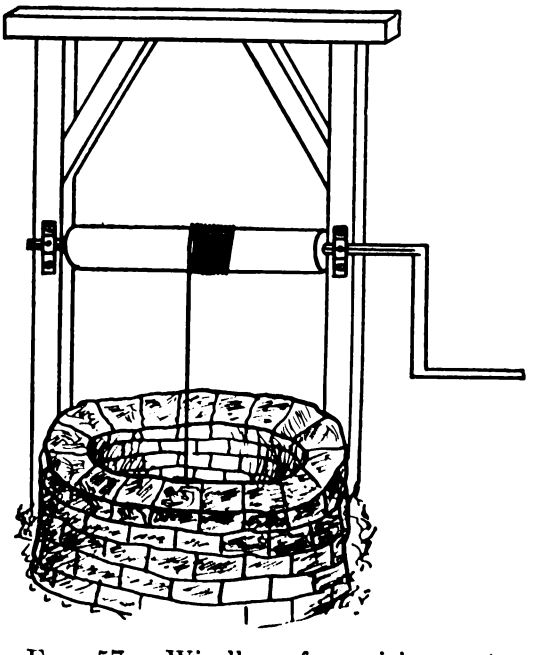


FIG. 57.—Windlass for raising water. Illustration for Problem 241.

239. From the result of Problem 238, find the steam pressure per square inch if V is $1\frac{1}{2}$ in. in diameter on the bottom where exposed to the steam.

240. Figure 56 shows the clutch pedal for an automobile. What must be the length of the power arm a in order that a foot pressure of 15 lb. can open the clutch against a spring pressure of 60 lb. having an arm of 3 in.?

241. Figure 57 shows an old-fashioned windlass for raising water. If the crank is 15 in. long and the drum is 5 in. in diameter, what pressure will be needed on the crank to raise a pail of water weighing 30 lb.?

242. Figure 58 represents in an elementary way the levers of a pair of

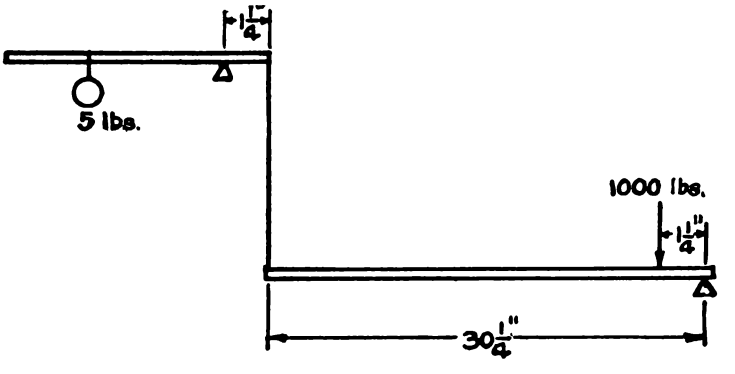


FIG. 58.—Elementary diagram of levers on platform scales.

platform scales. How far from the fulcrum must the 5-lb. weight be placed to balance the 1000-lb. weight located as shown?

243. The hoist of Fig. 52 has an 18-in. crank; the drum is 10 in. in diameter; the diameter of the large gear is 30 in. and of the small gear 6 in. What weight can be raised by a force of 25 lb. on the crank?

244. Calculate the pull required on the crank of the windlass of Problem 241, assuming that the efficiency of the apparatus is 75%.

CHAPTER XIII

TACKLE BLOCKS; THE INCLINED PLANE AND SCREW

91. Types of Blocks.—When a heavy weight is to be raised or moved through any considerable distance, either a windlass, such as described in Chap. XII, or tackle blocks can be used. As shown in the illustrations of the tackle blocks in this chapter, the revolving part is called the pulley or sheave; the framework surrounding the pulleys is called the block and, as generally used, includes both the frame and the sheaves contained in it.

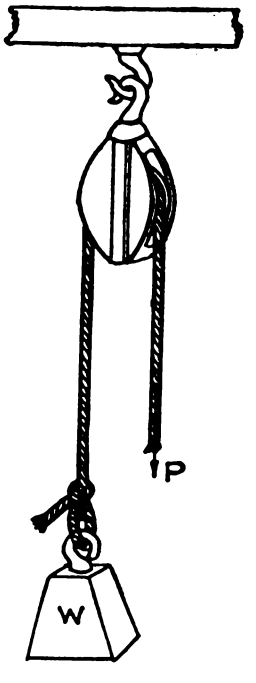


FIG. 59.—Single pulley arranged to give no mechanical advantage.

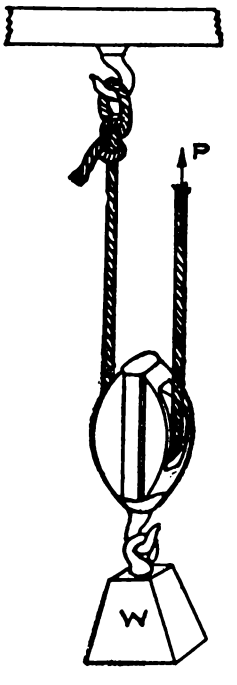


FIG. 60.—Single pulley arranged to give mechanical advantage of 2.

In Fig. 59 we have a single pulley, which serves merely to give a change of direction. There is no mechanical advantage in a single fixed pulley such as this. The pull on the rope at P is transmitted around the pulley and supports W on the other side. We can look at the pulley in this case as a lever with equal arms. P on one end must equal W on the other end. Such a block would be used solely for the convenience it affords, since it is usually easier to pull down than up.

In Fig. 60 the pull P is in the same direction that W is to be moved and is only one-half of W. As explained in Art. 88, the M.A. of a machine can be obtained by comparing the distances moved by the force and the weight. If a force must move five times as far as it lifts the weight, then the M.A. is 5, and the force is one-fifth of the weight. In Fig. 60 the M.A. is 2. This can be seen by raising W a certain distance. The rope on each side will be shortened this same distance and, therefore, P must be drawn up twice this distance. Since P moves twice as far as W, the force P will be one-half of W, and the M.A. will be 2.

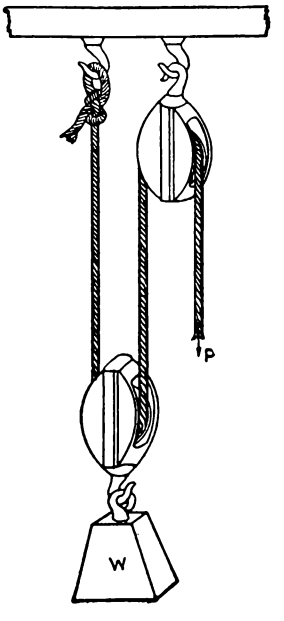


FIG. 61.—Single pulley attached to weight arranged to give mechanical advantage of 2. Upper pulley merely changes direction of pull on rope.

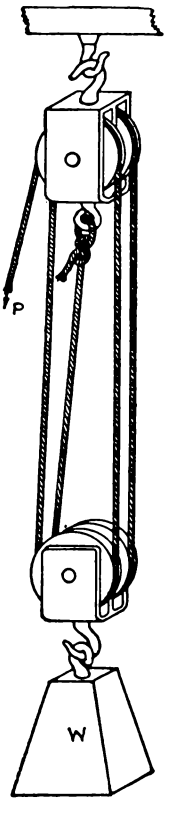


FIG. 62.—System of pulleys arranged to give mechanical advantage of 4.

In Fig. 61 we have merely added, to the device of Fig. 60, a fixed block above to change the direction of P. It makes no change in the relation of P and W, except as to direction.

In Fig. 62 we have two pulleys in the fixed block and two in the movable block. More pulleys might be used, but the principle is the same. A general rule for calculating their M.A.'s can be worked out for all cases. Proceeding as before, let us imagine that W and the movable block of Fig. 62 are lifted 1 ft. The

four ropes supporting W must each be shortened 1 ft., and it will be necessary to move P 4 ft. to accomplish this result. Hence, the M.A. of this system is 4, and P is $\frac{1}{4}$ of W, or W is 4 times P.

In general, we can say that the M.A. is equal to the number of ropes supporting the movable block and the load. The best way to find the M.A. is to draw a sketch of the blocks and to count the number of ropes *that are pulling on the movable block.* This number represents the M.A.

Whenever convenient, it is best to use as the movable block the one from which the free end of the rope runs. *This means that P will pull in the same direction that W is to be moved.* The M.A. is greater by 1 if P is pulling in the direction of motion. Notice in Fig. 62 that if we turned these blocks around and pulled the other way, fastening W to what is in the figure the fixed block, the mechanical advantage would be 5 instead of 4.

In all problems where there is any doubt, draw a rough sketch and count the number of ropes pulling on the movable block (Fig. 63).

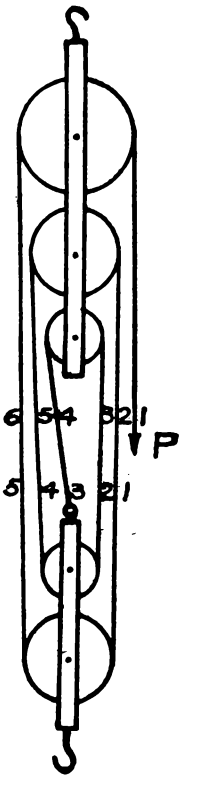


FIG 63.—Diagram to assist in determining mechanical advantage of system of pulleys.

Example:

How great a weight can be lifted by a pull of 150 lb. with a pair of pulley blocks, one being a three-sheave and the other a two-sheave block? Calculate, first, using the three-sheave as the movable block and, second, using the two-sheave block as the movable one.

Explanation: To avoid confusion, the sheaves are drawn one above the other, instead of parallel (Fig. 63). The free end of the rope must run from the three-sheave block. Starting from P, we wind the rope in and find that the inner end must be fastened to the two-sheave block. We count the ropes pulling on each block and find that, with the three-sheave block as the movable one, the M.A. is 6, and the weight lifted would be

$$150 \times 6 = 900 \text{ lb., } \textit{First Answer.}$$

With the two-sheave block as the movable one, the M.A. is 5 and the weight would be

$$150 \times 5 = 750 \text{ lb., } \textit{Second Answer.}$$

In practice, about 60% of these theoretical weights would be raised, the rest being lost in overcoming friction. Likewise, to lift a certain load, the actual pull required will be about $\frac{100}{60}$, or $1\frac{2}{3}$, of the theoretical pull.

92. Differential Pulleys.—In lifting heavy weights by hand, a very satisfactory apparatus to use is a differential hoist. This is a very simple and cheap apparatus, but it is not very efficient and is, therefore, not to be recommended for continuous use.

The pulleys are arranged as shown in Fig. 64. In the fixed block are two pulleys, A and B, A being somewhat larger than B. These pulleys, A and B, are fastened solidly together and rotate as one about a fixed axis; the pulley C is in the movable block. An endless chain passes over the pulleys as shown, the rims of the pulleys being grooved and fitted with lugs to prevent the chain from slipping. The loop np hangs free and is the pulling loop.

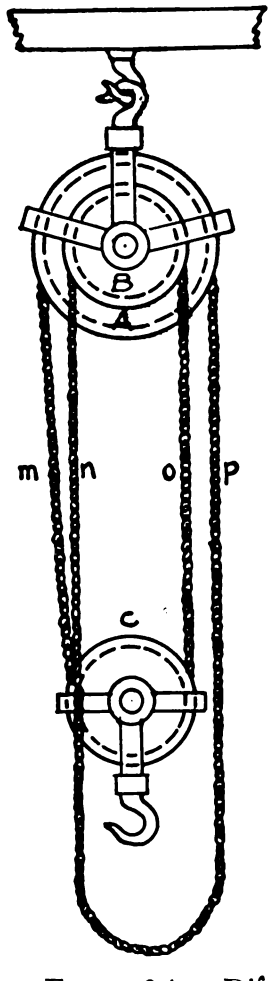


FIG. 64.—Differential hoist.

From the figure it is easily seen that if we pull down on p until pulley A is turned once around, the branch m will be *shortened* a length equal to the circumference of A. Since B is attached to A, it also will turn once around and the branch o will be *lengthened* a distance equal to the circumference of B.

Hence, the loop mo will be shortened by an amount equal to the difference of the circumferences of A and B; and the pulley C will rise one-half this amount.

We can express the difference in the distances moved by m and o as

$$(\pi \times D) - (\pi \times d) \text{ or } \pi \times (D - d)$$

where D and d represent the diameters of large and small pulleys, A and B. Hence, C will move up one-half of this, or $\frac{1}{2}$ of $\pi \times (D - d)$. To cause this motion of C upward, the chain p was moved a distance of $\pi \times D$.

The M.A. of the hoist is obtained by dividing the motion of P by the motion of W.

$$\text{M.A.} = \frac{\pi \times D}{\frac{1}{2}\pi \times (D - d)}$$

This can be simplified by canceling π out of both numerator and denominator of the fraction, leaving

$$\text{M.A.} = \frac{D}{\frac{1}{2} \text{ of } (D - d)}$$

This formula might be written as a rule in the following words: "The mechanical advantage of a differential hoist is obtained by dividing the diameter of the larger pulley in the upper block by half the difference between the diameters of the larger and smaller pulleys."

The efficiency of differential hoists is very low, an average figure being about 30%. Such a hoist will thus lift only about 30% of the theoretical load that could be lifted if it were without friction.

Example:

Calculate the pull a man must make on the chain of a differential hoist having 10 and 9 in. pulleys and an efficiency of 30% in order to lift a load of 600 lb.

$$D = 10 \text{ in.}$$
$$d = 9 \text{ in.}$$
$$D - d = 1 \text{ in.}$$
$$\frac{1}{2} \text{ of } (D - d) = \frac{1}{2} \text{ in.}$$
$$\text{M.A.} = \frac{D}{\frac{1}{2} \text{ of } (D - d)} = \frac{10}{\frac{1}{2}} = 20$$

The hoist must deliver a pull of 600 lb. Since the efficiency is 30%, 600 pounds is 30% of the theoretical pull supplied.

$$\text{Theoretical pull supplied} = \frac{600}{30} \times 100 = 2000 \text{ lb.}$$

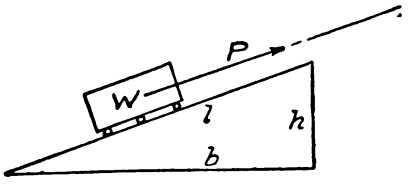
$$\text{Pull on chain} = \frac{2000}{\text{M.A.}} = \frac{2000}{20} = 100 \text{ lb., } \textit{Answer.}$$

93. The Use of Inclined Planes.—An inclined plane is a surface which slopes or is inclined from the horizontal. Anyone who has had experience in raising heavy bodies from one level to another knows that inclined planes are very useful for such work. The wedge is a form of an inclined plane, the powerful effect of which in splitting wood, quarrying stone, leveling machinery, and performing many other heavy duties is well known. The inclined plane, like the lever and the tackle block, enables us to lift a heavy weight with a small force.

94. Theory of the Inclined Plane.—The work done in moving a body up an inclined plane is merely the work of raising the body

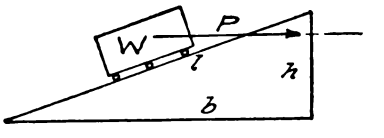
vertically. If we skid an engine base from the shop floor onto a flat car, the work accomplished is the raising of the base from the floor level to the car level, and is the same as if it were raised straight up by a crane, or by tackle blocks. The effect of the long incline is similar to that of a long force arm on a lever. It permits the force doing the work to use a greater distance, and hence the force will be smaller than the weight raised.

Neglecting friction or, in other words, supposing bodies to be perfectly smooth and hard, no work is done in moving the bodies



I. FORCE P PARALLEL TO THE INCLINED SURFACE.

FIG. 65.—Inclined plane—force acting parallel to incline.



II. FORCE P PARALLEL TO THE BASE.

FIG. 66.—Inclined plane—force acting parallel to base.

in a horizontal direction; and the work done upon a body when it is moved equals the weight of the body times the vertical height to which it is raised. In studying the theory of the inclined plane, we find that the force generally acts in one of two directions in raising the body: either parallel to the incline or parallel to the horizontal base.

In Case I (Fig. 65) the force P is exerted parallel to the incline, and, in raising the weight to the top, will act through a distance l. Meanwhile, it will raise W a distance h. Consequently, the *M.A. will be* $\frac{l}{h}$.

If we remember that the work put in equals the work got out of a machine (neglecting what is lost in friction) we see that we have the formula

$$P \times l = W \times h$$

To sum up, when the force is exerted parallel to the surface of the inclined plane, the force times the length of the inclined plane equals the weight times the vertical height through which the weight is raised by the plane; or the M.A. equals the length of the inclined surface divided by the height. If the weight to be raised is great as compared with the force available, a comparatively long incline must be used to give the necessary M.A.

In Case II (Fig. 66) the force acts parallel to the base of the inclined plane; that is, along the horizontal. This case is not often found in this elementary form, but is seen in jackscrews, in worm gearing, in wedges, and in cams, all of which are modifications of inclined planes. When the force acts parallel to the base, the work expended by it is the product of the force times the length of the base; the work accomplished is, as before, the product of the weight times the height.

$$P \times b = W \times h$$

$$\text{M.A.} = \frac{b}{h}$$

A comparison of these formulas with those for Case I shows that the M.A. is greater where the force P is exerted along the incline, as in Case I, because l, the length of the incline, or

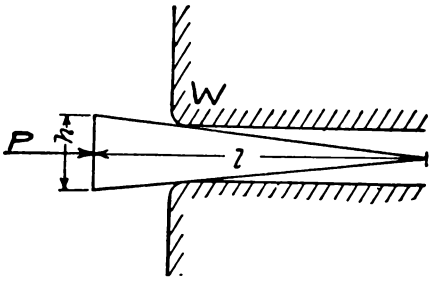


FIG. 67.—Diagram of wedge.

the hypotenuse of a right triangle, is greater than b, the length of the base.

There may be other cases where the force acts in some other direction, but they are seldom seen in practice.

95. The Wedge.—The wedge consists of two inclined planes placed base to base, the force acting parallel to the base, as shown in Fig. 67, where the horizontal center line of the wedge is the common base of the two inclined planes. Usually the wedge is moved instead of the object to be raised, but the effect is the same, and the force relations are the same as if the object itself were being moved up a stationary incline. In Fig. 67 it will be seen that the weight W will be raised a distance h when the wedge is driven a distance l. The work expended in driving the wedge is $P \times l$; the work accomplished in raising the weight is $W \times h$; and, neglecting friction, these are equal, or

$$P \times l = W \times h$$

From this we see that the M.A. of the wedge is:

$$\text{M.A.} = \frac{l}{h}$$

The relation of P and W might, if desired, be written as a proportion, as follows:

$$P : W = h : l$$

Example:

Figure 68 shows an adjustable pillow block for a Corliss engine, the bearing being raised or lowered by means of the wedge underneath. If the weight of the shaft and the flywheel upon this bearing is 6000 lb.,

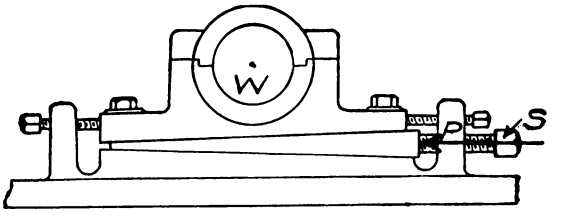


FIG. 68.—Application of wedge for adjustment of pillow block on Corliss engine.

and the wedge has a taper of 1 in. per foot of length, what pressure must be exerted on the wedge by the screw S in raising the shaft?

$$\text{M.A.} = 12$$
$$6000 \div 12 = 500 \text{ lb., } \textit{Answer.}$$

Explanation: If the taper of the wedge is 1 in. in 12 in., then a motion of 1 ft. would raise the bearing 1 in.; or a motion of 1 in. in the wedge would raise the bearing $\frac{1}{12}$ in. Hence, the M.A. of the wedge is 12, and $P = \frac{W}{12} =$ 500 lb.

Note.—There would also be required, in addition to this 500 lb., a force sufficient to overcome the friction on the top and bottom of the wedge, which is neglected in this solution.

96. The Jack Screw.—A screw is nothing but an inclined plane which, instead of being straight, is wrapped around or cut into a round rod or bar. Turning the screw gives the same effect as giving a straight push on an inclined plane or wedge.

When we raise an object with a jackscrew, such as shown in Fig. 69, the weight presses down on the screw and, consequently, is borne by the threads (which are the inclined planes). The threads are advanced and the weight is raised by a pull (which we will call P) on the end of the rod whose length is marked R. The distance the weight W is moved for one revolution of the screw equals the lead of the screw expressed as a fraction of an inch. The lead of a screw is the distance it advances lengthwise

in one turn or revolution. The force P moves through a distance equal to the circumference of a circle whose radius is the length of the handle; if we represent the length of the handle or lever by R, then the distance traversed by P in one revolution is $\pi \times 2 \times R$. If we let the letter L represent the lead of the screw, then we

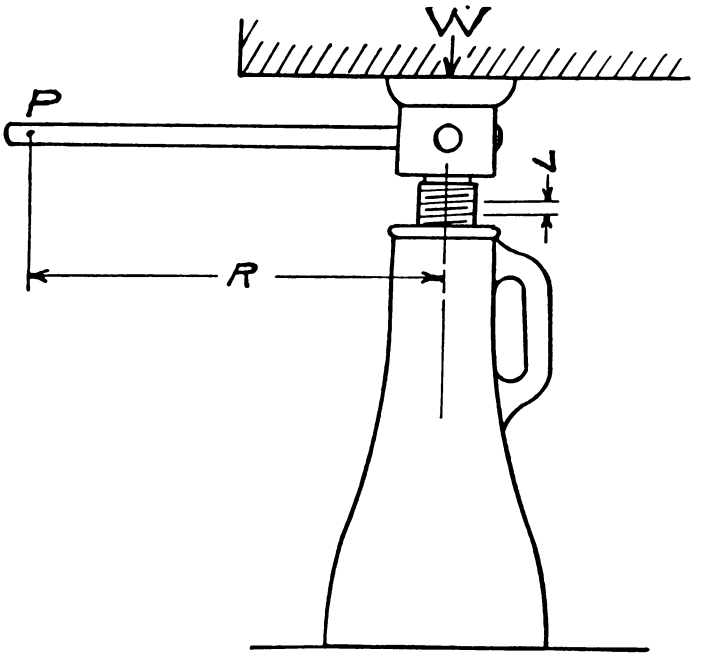


FIG. 69.—Jack screw.

will have the work accomplished in one revolution of the screw $= W \times L$. Meanwhile, the work expended in doing it

$$= P \times \pi \times 2 \times R$$

Assuming that there is no friction in the screw, we have

$$P \times \pi \times 2 \times R = W \times L$$

$$\text{or M.A.} = \frac{\text{Distance } P \text{ moves}}{\text{Distance } W \text{ is raised}} = \frac{\pi \times 2 \times R}{L}$$

If stated in words, these formulas would read: "The force multiplied by the circumference of the circle through which it moves equals the weight multiplied by the lead of the screw."

"The mechanical advantage of a jackscrew equals the circumference of the circle through which the force moves divided by the lead of the screw (the amount the screw advances in one turn)."

Example:

With a 1½-in. jackscrew having 3 threads per inch and a pull of 50 lb. at a radius of 18 in., calculate:

(*a*) The theoretical load that can be lifted by the screw.

(*b*) The actual load if the efficiency of the screw is 18 %.

(*a*) $\text{M.A.} = \dfrac{\pi \times 2 \times R}{L}$

$R = 18$ in. and $L = \frac{1}{3}$ in.

Hence,

$$\text{M.A.} = \frac{3.1416 \times 2 \times 18}{\frac{1}{3}}$$

$$= \frac{113.1}{\frac{1}{3}} = 339$$

$W = 339 \times 50 = 16{,}950$ lb., *Answer.*

(*b*) 18 % of 16,950 = 3051 lb., *Answer.*

Explanation: If there are 3 threads per inch, the lead is $\frac{1}{3}$ in., and if the radius is 18 in., the M.A. is 339.3. In theory then we should be able to lift 50 lb. × 339 = 16,950 lb. with this screw. But a screw has considerable friction and for this reason only 18 % of the energy expended in this case is effective, the remaining 82 % being all lost in friction. The actual weight lifted is, therefore, only 18 % of 16,950 lb., or 3051 lb.

The efficiency of the average jackscrew is between 10 and 20 %, and for a rough estimate an efficiency of 15 % might be assumed.

PROBLEMS

245. A weight of 2000 lb. is to be lifted with a four-sheave and a three-sheave pair of blocks as shown in Fig. 70. Neglecting friction, how great a pull is required on the rope to lift the load?

246. In the preceding problem, what pull would be required if the efficiency of the block was 80 %?

247. The small crane of Fig. 71 is shown lifting a 7-ton section of cast-iron pipe. The hoisting tackle has a single sheave in the lower or "hook" block and two sheaves in the upper block. What is the pull in the hoisting rope, neglecting friction?

248. A windlass and tackle block as shown in Fig. 72 are used for moving a house. If the team exerts a pull of 200 lb. at the end of the sweep, find the theoretical pull on the house (neglecting friction). Also find the actual pull if the efficiency of the apparatus is 65 %.

249. A load of 2 tons is to be lifted with a differential hoist. The pulleys are 12 in. and $11\frac{3}{8}$ in. in diameter.

(*a*) What is the theoretical pull required to lift the load?

(*b*) What is the actual pull required if the efficiency of the hoistis 30 %?

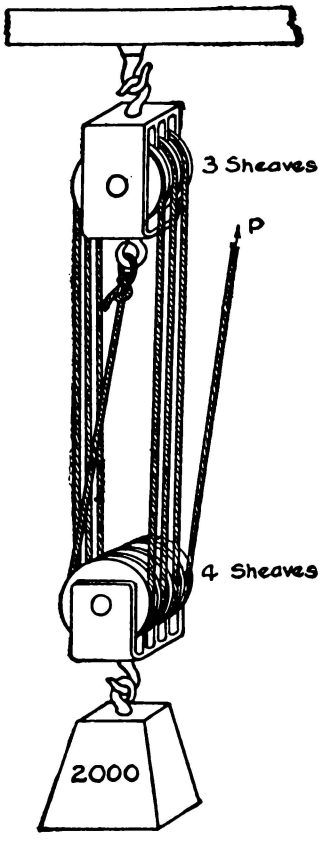


FIG. 70.—Illustration for Problem 245.

FIG. 71.—Crane lifting section of cast-iron pipe.

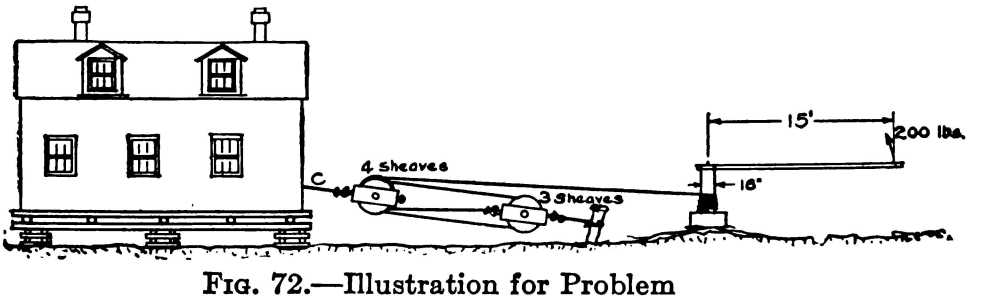


FIG. 72.—Illustration for Problem

250. An engine weighing 5 tons is to be loaded onto a car, the floor of which is 6 ft. from the ground. If 16-ft. timbers are used for the runway, find the pull necessary to draw the engine up the slope, neglecting friction.

251. A building is to be raised by means of four jackscrews. The screws are 2 in. in diameter, with four threads to the inch. The lever is 20 in. long, and a 30-lb. force is required on each handle. Calculate the weight of the building. assuming that the efficiency of the screws is 20%.

252. What would be the pull on the house in Fig. 72 if the house were going up a grade of 1 ft. rise in 72 ft. (Pull horizontal.)

253. The electric mine locomotive of Fig. 73 hauls a train of loaded cars up a grade which rises 1 ft. in every 100 ft. What pull must the locomotive exert at the drawbar for each ton of its load, neglecting friction?

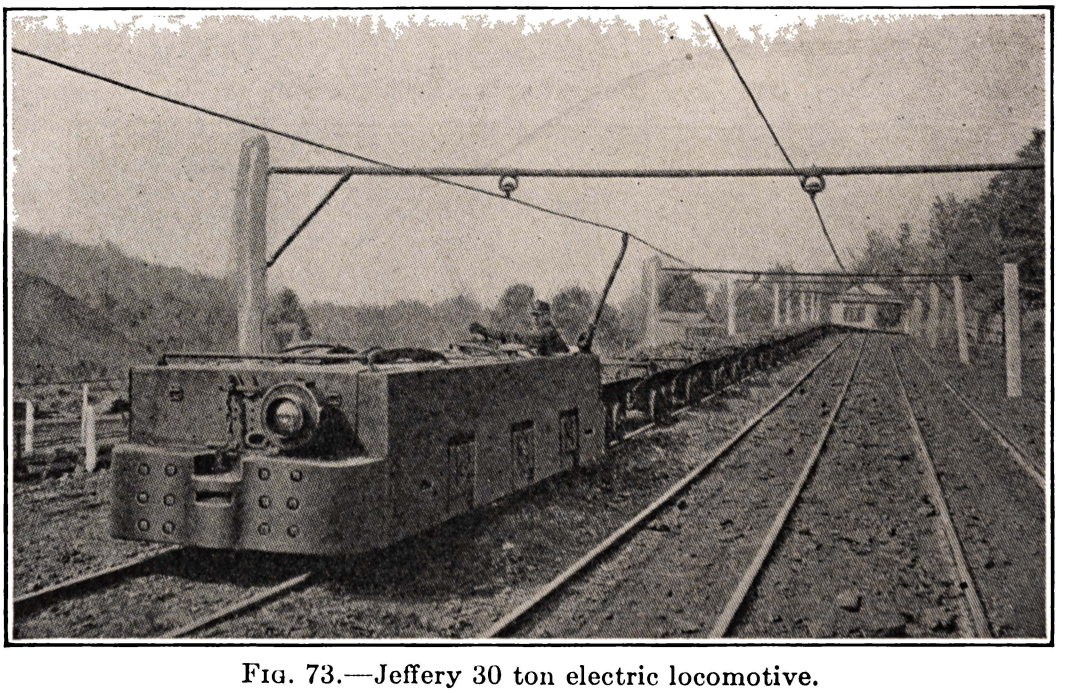


FIG. 73.—Jeffery 30 ton electric locomotive.

254. If, in the preceding problem, an additional pull of 30 lb. per ton of load is required to overcome friction, and the locomotive exerts a maximum drawbar pull of 14,400 lb., what is the "haulage capacity" of the locomotive in tons?

255. In Fig. 73*a* find P when $W = 350$ lb.

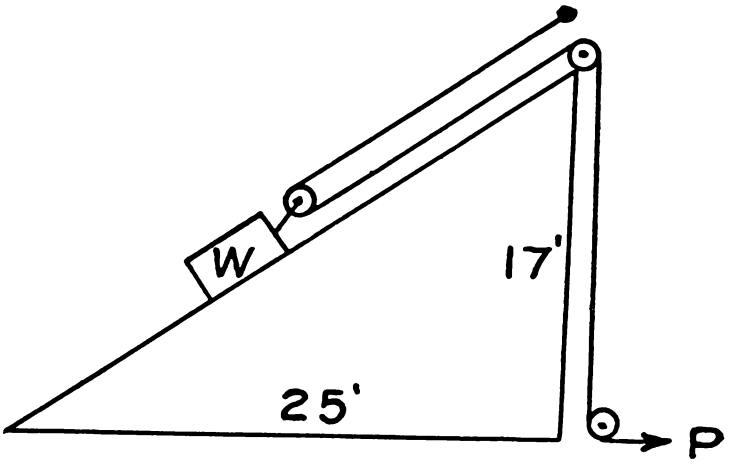


FIG. 73*a*.

CHAPTER XIV

WORK, POWER, AND ENERGY; HORSEPOWER OF BELTING

97. Work.—Whenever a force causes a body to move, work is done. Unless the body is moved, no work is accomplished. A man may push against a heavy casting for hours, but unless he moves it, he does no work, no matter how tired he may feel at the end of the time. It is evident that there are two factors to be considered in measuring work—*force* and *distance.* In the study of levers, tackle blocks, and inclined planes we dealt with the problem of work. In any of these machines the work delivered or accomplished in lifting a weight is measured by the product of the weight times the height it is raised. The work supplied, or put into the machine, is the product of the force exerted times the distance through which this force must move. It is a known fact that if we neglect the work lost in friction, the work supplied or put into a machine is equal to the work delivered or accomplished by it. The actual difference between the work supplied to the machine and the work delivered is the amount that is lost in friction. The following expressions may make these relations clearer.

Work lost in friction = Work put in − Work delivered

$$\text{Efficiency} = \frac{\text{Work delivered}}{\text{Work put in}}$$

98. Unit of Work.—The unit by which work is measured is called the *foot-pound.* This is the work done in overcoming a resistance of 1 lb. through a distance of 1 ft.; that is, if a weight of 1 lb. is lifted 1 ft., the work done is equal to 1 ft.-lb. All work may be measured by this standard. If a weight of 2 lb. is lifted through a distance of 1 ft., the work done is $2 \times 1 = 2$ ft.-lb.; or if a weight of 1 lb. is lifted a distance of 2 ft., the work done is $1 \times 2 = 2$ ft.-lb. *The work in foot-pounds is the product of the force in pounds times the distance in feet through which the force moves.*

In lifting a weight vertically, the resistance, and, hence, the force that must be exerted, are equal to the weight itself in pounds. The work done is the product of the weight times the vertical distance that it is raised. If a weight of 80 lb. is lifted a distance of 4 ft., the work done is 80 × 4, or 320 ft.-lb. It would require the same amount of work to lift 40 lb. 8 ft., or to lift 20 lb. 16 ft.

When a body is moved horizontally, the only resistance to be overcome is the friction. When a team of horses pulls a loaded wagon, the only resistances which it must overcome are the friction between the wheels and the axles and the resistance on the tires caused by the unevenness of the road. The actual work done by the horses is equal to the work required by friction. However, if the wagon is being drawn up a hill, in addition to the work done in overcoming friction, the horses will have to perform an additional amount of work equal to that of raising the wagon the height of the hill. Considering the wagon as a machine, the work supplied or "put in," is the total work done by the horses, that is, the work of friction plus the work of raising the wagon. The work done in overcoming friction is practically wasted; therefore, the work delivered or "got out" is only that required to raise the wagon the height of the hill.

The work necessary to pump a certain amount of water is the weight of the water times the height through which it is lifted or pumped, plus the work lost in friction between the water and the sides of the pipe. The work necessary to hoist a casting is the weight of the casting times the height it is raised, plus the work required to overcome friction in the hoist. The actual work accomplished is merely the weight of the casting times the height, since the work of friction is wasted. The work done by a belt is the effective pull of the belt times the distance in feet which the belt travels. The work done in hoisting an elevator is the weight of the cage and of the load it carries times the height of the lift. Numerous other illustrations of work will suggest themselves.

99. Power.—Power is the *rate* of doing work; that is, in calculating power the time required to do a certain number of foot-pounds of work is considered. If 10,000 lb. are lifted 7 ft., the *work* done is 70,000 ft.-lb., regardless of how long it takes. But, if one of two machines can do this in one-half the time that the other machine requires, then the first machine has twice the *power* of the second.

The engineer's unit of power is the *horsepower*, which may be defined as the ability to do 33,000 *ft.-lb. of work per minute.* Therefore, if a machine can perform 33,000 ft.-lb. of work in 1 min., its power is 1 hp.; if it can perform 66,000 ft.-lb. in 1 min., its power is 2 hp., etc. The horsepower required to perform a certain amount of work is found by dividing the foot-pounds done *per minute* by 33,000. If an engine can do 1,980,000 ft.-lb. in a minute, its horsepower will be 1,980,000 ÷ 33,000 = 60. An engine that can raise 66,000 lb. to a height of 10 ft. in 1 min. will do 66,000 lb. × 10 ft. = 660,000 ft.-lb. per minute, and this will equal $\frac{660,000}{33,000}$ = 20 hp. If another engine takes 4 min. to do this same amount of work, it is only one-fourth as powerful; the work done per minute will be $\frac{660,000}{4}$ = 165,000 ft.-lb. per minute; and its horsepower is $\frac{165,000}{33,000}$ = 5 hp.

Example:

An electric crane lifts a casting weighing 3 tons to a height of 20 ft. from the floor in 30 sec.; what is the horsepower used?

3 tons = 6000 lb.

6000 lb. × 20 ft. = 120,000 ft.-lb. done.

120,000 ft.-lb. done in 30 sec. = 240,000 ft.-lb. per 1 min.

$\frac{240000}{33000}$ = 7.27 hp. used.

In calculating horsepower, it should be remembered that the work done should be reduced to foot-pounds per minute before dividing by 33,000.

100. Horsepower of Belting.—A belt is used for the transmission of power from one shaft to another. The driving pulley exerts a certain pull in the belt, and this pull is transmitted by the belt and exerted on the rim of the driven pulley.

The power transmitted by any belt depends upon two things—the effective pull of the belt tending to turn the wheel, and the speed with which the belt travels. It is easily seen that these include the three items necessary to measure *power.* The pull of the belt is the *force.* The speed, given in feet per minute, includes both *distance* and *time.* Force, distance, and time, therefore, are the three items necessary for the measurement of power.

The total pull that a belt will stand depends upon its width and thickness. It should be wide enough and heavy enough to stand for a reasonable time the greatest tension put upon it. This is, of course, the tension on the driving side. This tension, however, does not represent the force tending to turn the pulley. The force tending to turn the pulley (or the Effective Pull, as it is called) is the difference in tension between the tight and the slack sides of the belt.

The effective pull that can be allowed in a belt depends primarily upon the width, thickness, and strength of the leather, or whatever material the belt is made of. In addition, every time a belt causes trouble from breaking or becoming loose, it means a considerable loss in time of the machine, of the men who are using it, and of the men required to make the repairs. Therefore, it should not be loaded so heavily as might otherwise be allowed. Leather belts are called "single," "double," "triple," or "quadruple," according to whether they are made of one, two, three, or four thicknesses of leather. Good practice allows an effective pull of 35 lb. in a single leather belt per inch of width. In a double belt a pull of 70 lb. per inch of width may be allowed. The allowable pull per inch of width times the width of the belt in inches gives the total effective pull or force which can be safely transmitted by the belt.

The total effective pull in the belt times the velocity in feet per minute will give the foot-pounds transmitted by it in 1 min. One horsepower is a rate of 33,000 ft.-lb. per minute; hence, the horsepower of a belt is obtained by dividing the foot-pounds transmitted by it per minute by 33,000. The velocity of the belt is calculated from the diameter and revolutions per minute of either one of the pulleys over which the belt travels. From these considerations, the formula for the horsepower that a belt will transmit may be written

$$H = \frac{P \times W \times V}{33{,}000}$$

where H = horsepower
P = effective pull allowed per inch of width
W = width of belt in inches
V = velocity of belt in feet per minute.

Stated in words, this formula would read as follows: "The horsepower that may be transmitted by a belt is found by multiplying

together the allowable pull per inch of width of the belt, the width of the belt in inches, and the velocity of the belt in feet per minute, and then dividing this product by 33,000."

Example:

Find the horsepower that should be carried by a 12-in. double leather belt if one of the pulleys is 14 in. in diameter and runs 1100 r.p.m.

$P = 70$ lb.

$W = 12$ in.

$V = \pi \times \frac{14}{12} \times 1100$

$= 4032$ ft. per minute

$H = \frac{P \times W \times V}{33000}$

$= \frac{70 \times 12 \times 4032}{33000}$

$= 102+$ hp., *Answer.*

Explanation: To get the horsepower, we must first find the values of P, W, and V. We will take P as 70 lb. since this is a double belt. W is given as 12 in. V, the velocity, is obtained by multiplying the circumference of the pulley by the r.p.m. which gives 4032. Multiplying these three together gives 3,387,000 ft.-lb. per minute, and dividing by 33,000 gives 102+ as the horsepower that this belt might be required to carry.

101. Widths of Belts.—It is possible, also, to develop a formula with which to calculate the width of belt required to transmit a certain horsepower at a given velocity.

One horsepower is 33,000 ft.-lb. per minute. Then the given number of horsepower multiplied by 33,000 gives the number of foot-pounds to be transmitted per minute.

$$\text{Foot-pounds per minute} = 33{,}000 \times H$$

If we know the velocity in feet per minute, we can divide the foot-pounds per minute by the velocity; the quotient will be the force or the effective pull in the belt.

$$\text{Force} = \frac{33{,}000 \times H}{V}$$

Now the force can be divided by the allowable pull per inch of width of belt. The result will be the necessary width.

$$W = \frac{33{,}000 \times H}{P \times V}$$

Stated in words, this formula would read: "To obtain the width of belt necessary for a certain horsepower multiply the horsepower by 33,000 and divide by the product of the allowable pull per inch of width of belt times the velocity of the belt in feet per minute."

Example:

Find the width of a single belt to transmit 10 hp. at a speed of 2000 ft. per minute.

$H = 10$

$V = 2000$

$P = 35$

$$W = \frac{33{,}000 \times H}{P \times V}$$

$$= \frac{33\cancel{000} \times \overset{2}{\cancel{1}0}}{\cancel{35} \times \cancel{2000}} = \frac{33}{7} = 4\tfrac{5}{7} \text{ in.}$$

Use 5-in. belt, *Answer.*

Explanation: We have given the horsepower and the velocity, and we know that for a single belt a pull of 35 lb. per inch is allowable. These data are all that are needed to calculate the width, which comes out $4\frac{5}{7}$ in. The next larger standard width is 5 in., so that is the size that would be used.

102. Rules for Belting. (1) *Belt Thickness.*—It is generally advisable to use single belting in all cases where one or both pulleys are under 12 in. in diameter, and double belting on pulleys 12 in. or larger. Triple and quadruple belts are used only for main drives where considerable power is to be transmitted and where a single or double belt would have to be of excessive width. A triple belt should not be run on a pulley less than 20 in. in diameter, nor a quadruple belt on a pulley less than 30 in. in diameter.

2. *Tension per Inch of Width.*—An effective pull of 35 lb. per inch of width of belt is allowable for single belts. For double belts an effective pull of 70 lb. per inch is allowable unless the belt is used over a pulley less than 12 in. in diameter, in which case only 50 lb. per inch should be allowed. A prominent manufacturer of rubber belting recommends 33 lb. per inch of width of belt for four-ply belts and 43 lb. for six-ply rubber belts.

3. *Belt Speeds.*—The most efficient speed for belts to run is from 4000 to 4500 ft. per minute. Belts will not hug the pulley and, therefore, will slip badly if run at a speed of over 1 mile per minute. These figures are seldom reached in machine shops. Belts for machine tool drives run from 1000 to 2000 ft. per minute, while main driving belts for line shafts are more often run about 3000 ft. per minute. On wood-working tools we find higher speeds, usually 4000 ft. per minute or over.

4. *Arrangement of Pulleys.*—It is desirable that the angle of the belt with the floor should not exceed 45 deg.; that is, the belt should be nearer the horizontal than the vertical. Figure 74 shows the effect of having a belt nearly vertical. Any sag in the belt causes it to drop away from the lower pulley and lose its grip

on it. Figure 75 shows the best arrangement. The belt should be nearly horizontal and should have the tight side of the belt underneath, if possible. This will increase the wrap of the belt

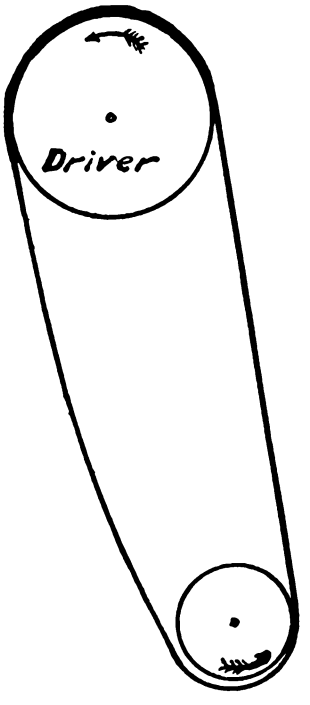


FIG. 74.—Arrangement of pulleys with belt nearly vertical.

FIG. 75.—Arrangement of pulleys with belt in horizontal position.

around the pulleys. If the lower side is the loose side, the wrap will be decreased by the sag.

It is also desirable, whenever possible, to arrange the shafting and machinery so that the belts will run in opposite directions

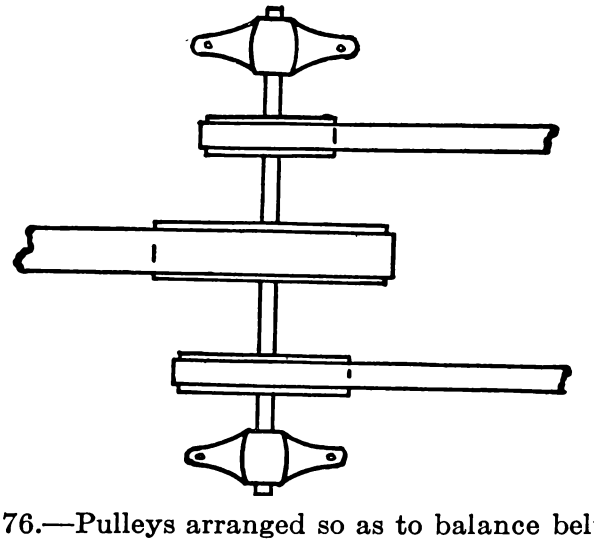


FIG. 76.—Pulleys arranged so as to balance belt pull.

from the shaft, as shown in Fig. 76. This arrangement balances somewhat the belt pulls, and reduces the friction and wear in the bearings.

For belts which are to be shifted, the pulley faces should be flat; all other pulleys should have the faces crowned (high in the center) about $\frac{3}{16}$ in. per foot of width.

5. *Grain and Flesh Sides.*—The grain side of the leather is the side from which the hair is removed. It is the smoother but weaker side of the leather, and should run next to the pulley surface. It wraps closer to the pulley surface and thus gets a better grip on the pulley. Furthermore, the flesh side, being stronger, is better able to stand the stretching which must occur in the outside of the belt in bending around a pulley.

6. *Belt Joints.*—Whenever possible, the ends of belts should be fastened together by splicing and cementing. A wide

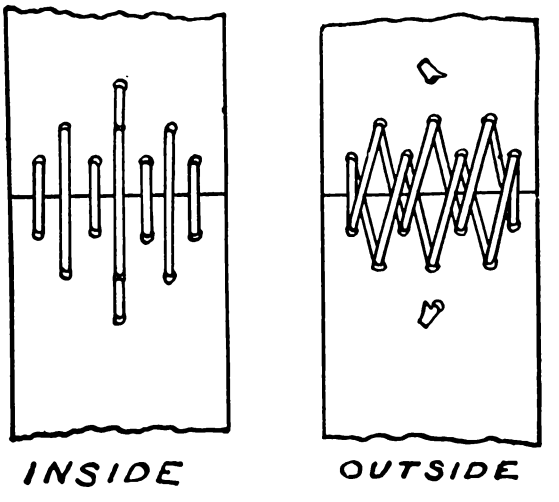


FIG. 77.—Excellent method of lacing belts.

cemented belt should never be run onto the pulleys, as one side is liable to be stretched out of true. Instead lift one shaft out of the bearings, place the belt on the pulleys, and force the shaft back into place. Of all other methods of fastening belts, the leather lacing is undoubtedly the best when properly done. In lacing a belt, begin at the center and lace both ways with equal tension. Figure 77 shows an excellent method of lacing belts. The lacing should be crossed on the *outside* of the belt. On the inside, the lacing should lie in line with the belt.

The following table from "Machinery's Handbook" gives the proper size of lacing and the number and arrangement of holes for belts of various widths. In each end of the belt there should be two rows of staggered holes, as shown in Fig. 77.

BELT LACES AND HOLES FOR LACED JOINTS

Width of belt, inches	Width of lace, inches	No. of holes	Distance of holes from end	
			First row, inches	Second row, inches
1 –1¾	¼	2 or 3	⅜	
2 –1½	5/16	3	⅜	¾
2¾–3¼	5/16	5	½	1
3½–4½	⅜	5	⅝	1⅛
5	⅜	7	⅝	1⅛
6	⅜	9	¾	1¼
8	½	11	¾	1⅜
10	½	13	1	1¾
12	½	15	1	1¾
14	½	17	1¼	2

PROBLEMS

256. A casting weighs 300 lb. How much work is required to lift it from the floor to a planer bed 3½ ft. above the floor?

257. How much work is required to pump 5000 gal. of water into a tank 150 ft. above the pump, neglecting friction?

258. What horsepower may be transmitted by a 3-in. single belt running at 2500 ft. per minute, allowing the effective pull given in Art. 102, Par. 2?

259. A 6-in. double belt is carried by a 48-in. pulley running 250 r.p.m. What is the allowable horsepower that may be transmitted?

260. A shop requires 50 hp. to run it. The main shaft runs 250 r.p.m. Select a main driving pulley (see "Belt Speed," Art. 102, Par. 3) and determine the width of double belt to run the shop.

261. A pumping station is to be designed to deliver 5,000 gal. per minute to a height of 150 ft. What theoretical horsepower is required to drive the pumps?

262. In Problem 261, what actual horsepower is required if the actual efficiency is 55%?

Note.—The actual horsepower is greater than the theoretical, in such a case, because extra power is required to overcome friction.

263. If a freight elevator and its load weigh 5000 lb., what theoretical horsepower is required to raise the elevator at a speed of 2 ft. per second?

264. In Problem 263, what actual horsepower is required, assuming an efficiency of 80%?

265. The electric mine locomotive of Fig. 73 exerts a maximum drawbar pull of 14,400 lb. on a 1% grade, at a speed of 6 miles per hour. What horsepower is developed at the drawbar?

266. Find the work done in the following problem: an engine weighing 5 tons is to be loaded onto a car, the floor of which is 6 ft. from the ground. The timbers for the runway are 16 ft. long. Friction is to be neglected.

267. If the work of Problem 266 is done in 1.3 min., what is the average horsepower developed?

$F = AP$

$P = \frac{F}{A}$

CHAPTER XV

HORSEPOWER OF ENGINES

103. Steam Engines.—In the last chapter the meaning of the term horsepower was explained and its application to belting was discussed. We will now take up the calculations of the horsepowers of steam and gas engines.

One horsepower was given as the ability to do 33,000 ft.-lb. of work in 1 min. From this we see that the best way to get the horsepower of any engine is to find out how many foot-pounds of work it does in 1 min. and then to divide the number of foot-pounds delivered in a minute by 33,000.

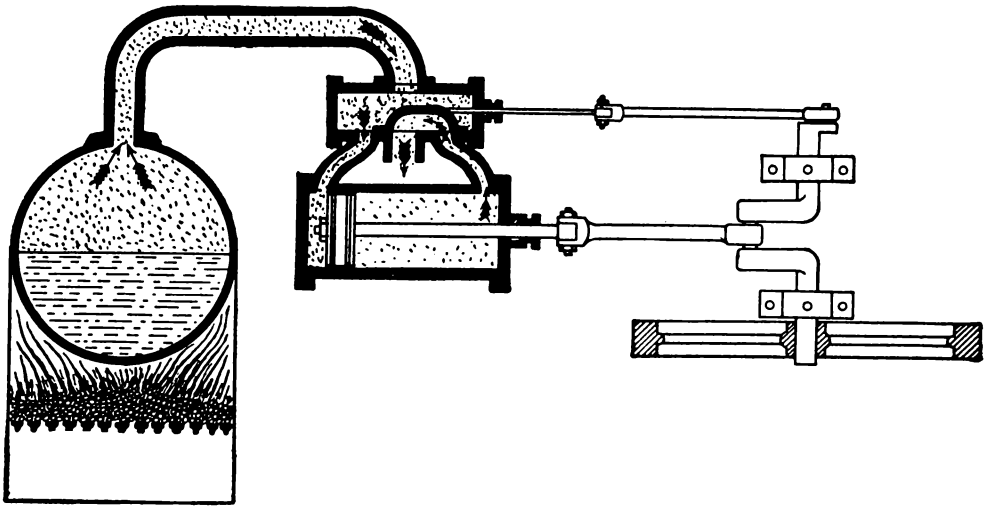


FIG. 78.—Simple steam boiler and engine.

Let us study the action of the steam in the cylinder of the ordinary double-acting steam engine. In Fig. 78 is shown a section of a very simple boiler and engine. We find that steam enters one end of the cylinder behind the piston and pushes the piston toward the other end of the cylinder. Meanwhile, the valve is moved to the other end of the valve chest. The operation is then reversed, and the piston is pushed back to the starting point. It has thus made two strokes while the flywheel has made one complete revolution (the motion of the piston from one end of the cylinder to the other is called a "stroke"). During both strokes of the piston the steam pressure was acting on the piston,

forcing it to move; therefore, both strokes were what is called "power strokes," or "working strokes." This type of engine is known as a double-acting steam engine, since the steam acts on both sides of the piston and there are two power strokes to each revolution. In the single-acting steam engine, the steam is admitted to one end of the cylinder only, and, therefore, there is but one power stroke to each revolution.

As the piston moves from one end of the stroke to the other, the pressure of the steam in the cylinder does not remain the same, but varies according to the action of the valve in cutting off the admission of the steam and the expansion of the steam. However, it is possible to obtain the average pressure per square inch throughout the stroke, or what is called the "mean effective pressure." This average pressure, or m.e.p., multiplied by the piston area in square inches gives the total average pressure or force acting against the piston. The total pressure or force exerted on the piston multiplied by the length of the stroke in feet gives the number of foot-pounds of work performed *during one stroke.* This result, when multiplied by the number of working strokes per minute, gives the foot-pounds of work per minute, and this divided by 33,000 gives the horsepower.

The following are the symbols generally used:

$hp.$ = horsepower.

P = mean pressure in pounds per square inch.

A = area of piston in *square inches.*

L = length of stroke in *feet.*

N = number of working *strokes* per minute.

$P \times A$ = total pressure on piston.

$P \times A \times L$ = foot-pounds of work done per stroke.

$P \times A \times L \times N$ = foot-pounds of work done per minute, and hence

$$hp. = \frac{P \times A \times L \times N}{33{,}000} \text{ or, as usually written,}$$

$$hp. = \frac{P \times L \times A \times N}{33{,}000}.$$

In the latter form, the letters in the numerator spell the word *Plan,* and the formula is thus easily remembered.

In the common steam engine, there are two *working* strokes for every revolution of the engine, and N is twice the revolutions per

minute. A few steam engines, like the vertical Westinghouse engine, are single-acting and, hence, have only one working stroke of each piston per revolution. Unless otherwise stated, it will be assumed in working problems that a steam engine is double-acting.

Example:

Find the horsepower of a 32- by 54-in. steam engine running at 94 r.p.m. with an m.e.p. (mean effective pressure) of 60 lb. per sq. in.

Note.—In giving the dimensions of an engine cylinder, the first number represents the diameter and the second number the stroke.

P = 60 lb. per sq. in.

L = 54 in. = $4\frac{1}{2}$ ft.

A = Area of 32-in. piston = 804.25 sq. in.

N = Number of strokes per minute = $94 \times 2 = 188$

$$hp. = \frac{P \times L \times A \times N}{33{,}000}$$

$$= \frac{60 \times 4.5 \times 804.25 \times 188}{33{,}000} = 1237+ \text{ hp., } \textit{Answer.}$$

Notice particularly that the area of the piston is expressed in square inches, because the pressure is given in pounds per square inch; but that the stroke is reduced to feet, because we measure work in foot-pounds and, consequently, must express in feet the distance which the piston moves.

If an engine has more than one cylinder, the horsepower of each can be calculated and the results added; or if the cylinders are arranged to do equal amounts of work, we can find the horsepower of one cylinder and multiply this by the number of cylinders.

The m.e.p. can be obtained for any engine by the use of a device called an "indicator," which draws a diagram showing just what the pressure is in the cylinder at each point in the stroke. From this diagram, we can calculate the average or mean effective pressure for the stroke. This pressure must not be confused with the boiler pressure or the pressure in the steam pipe. For instance, when the steam comes from the boiler to the engine at 100-lb. pressure, the mean pressure in the cylinder will not be 100 lb., for it would be very wasteful to use steam from the boiler for the full stroke as the expansive power of the steam would not be utilized. Instead, the m.e.p. will be from 20 to 85% of the boiler pressure, depending upon the type of the engine and the load it is carrying. Horsepower calculated as

explained here is called indicated horsepower because an indicator is used to determine it. The indicated horsepower represents the power delivered to the piston by the steam.

104. Gas Engines.—The most common type of gas or gasoline engine works on what is called the *four-stroke cycle.* Such an engine is called a *four-cycle* engine. Figure 79 shows in four views the operation of such an engine. Four strokes, or two revolutions, are required for each explosion that occurs in the

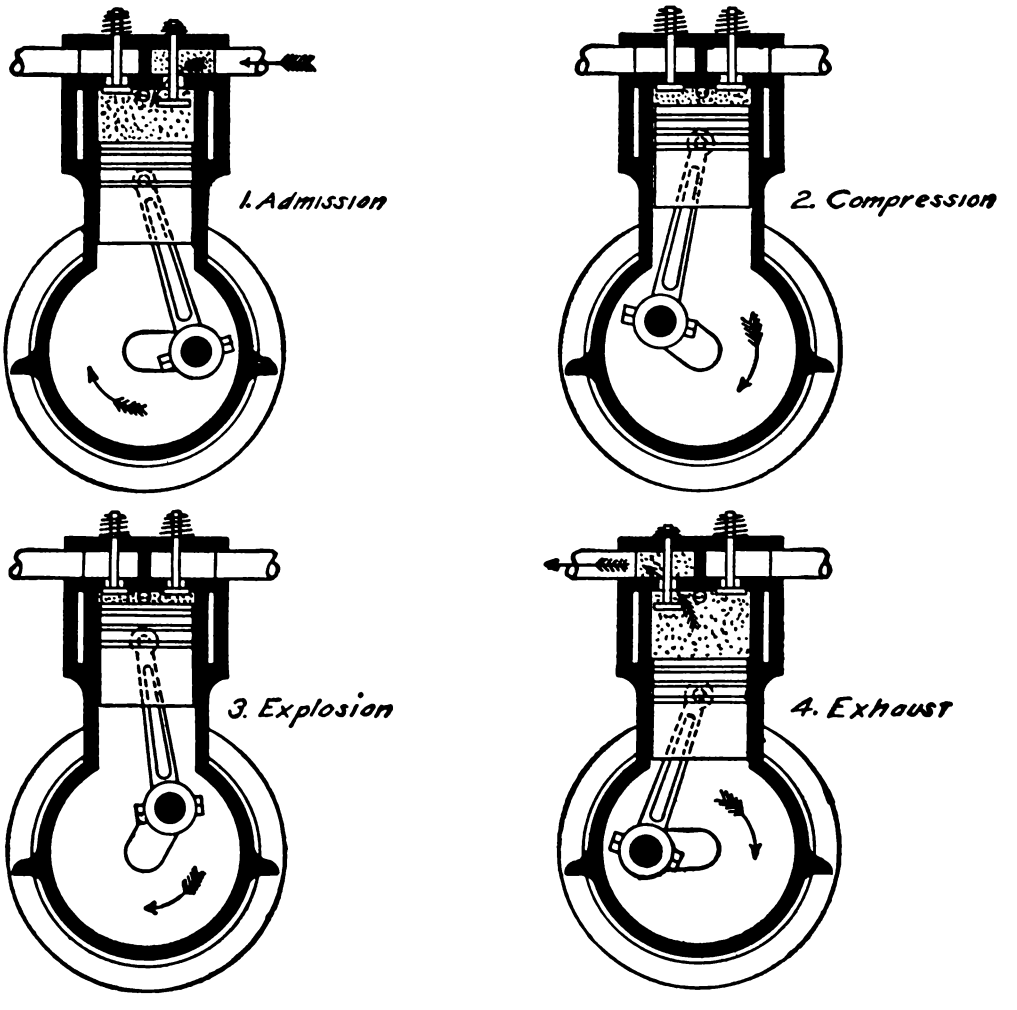


FIG. 79.—The four strokes of a four-cycle gas engine.

cylinder. Consequently, in calculating the horsepower of a single-cylinder gas engine, the number of working strokes (or N in the horsepower formula) is one-half of the r.p.m. There is another type of gasoline engine called the *two-cycle* engine. A single-cylinder, two-cycle engine has one working stroke for each revolution of the crankshaft and N is, therefore, the same as the number of r.p.m.

The m.e.p. of a gas engine is from 40 to 100 lb. per square inch, depending principally upon the fuel used. For gasoline or

natural gas or illuminating gas it is usually between 80 and 90 lb. per square inch.

Example:

Calculate the indicated horsepower of a single-cylinder, 5- by 8-in., four-cycle gasoline engine running at 450 r.p.m. with a m.e.p. of 80 lb. per square inch.

$$P = 80 \text{ lb. per sq. in.}$$

$$L = \frac{8}{12} = \frac{2}{3} \text{ ft.}$$

$$A = .7854 \times 5^2 = 19.6 \text{ sq. in.}$$

$$N = \frac{450}{2} = 225 \text{ working strokes per min.}$$

$$\text{Then, } hp. = \frac{P \times L \times A \times N}{33{,}000}$$

$$= \frac{80 \times \frac{2}{3} \times 19.6 \times 225}{33{,}000} = 7.13 \text{ hp., } \textit{Answer.}$$

105. Air Compressors.—An air compressor is like a double-acting steam engine in appearance; but, instead of delivering power, it requires power from some other source to run it. This power is stored in the air and later is recovered when the air is used. An air compressor takes air into the cylinder, raises its pressure by compressing it, and then forces it into the air line or the storage tank. When the horsepower required to drive a compressor is being calculated, the same formula can be used as for a steam engine. The value of P to use is not the pressure to which the air is raised, but the average or mean pressure during the stroke. It is usually somewhat less than half the final air pressure; for example, when an air compressor is delivering air at 80-lb. pressure, the mean pressure on the piston is about 33 lb.

Most air compressors are double-acting, though there are many small single-acting ones.

Example:

A double-acting 12- by 14-in. air compressor is running 150 r.p.m. It is supplying air at 100 lb., and the mean pressure in the cylinder is 37 lb. per square inch. Calculate the horsepower necessary to run it.

$$P = 37 \text{ lb. per sq. in.}$$

$$L = 14 \text{ in.} = \frac{14}{12} \text{ ft.}$$

A = area of 12-in. piston = 113.1 sq. in.
$N = 150 \times 2 = 300$ strokes per minute.

$$\text{Then, } hp. = \frac{P \times L \times A \times N}{33{,}000}$$

$$= \frac{37 \times 14 \times 113.1 \times 300}{12 \times 33000} = 44.4, \textit{ Answer.}$$

In this case 12 appears in the denominator in order to reduce the 14 *inches* to *feet*.

106. Brake Horsepower.—The brake horsepower of an engine is the power actually available for outside use. It, therefore, is equal to the indicated horsepower minus the power lost in friction in the engine. Brake horsepower can be readily determined by putting a brake on the rim of the flywheel and thus absorbing and measuring the power actually delivered.

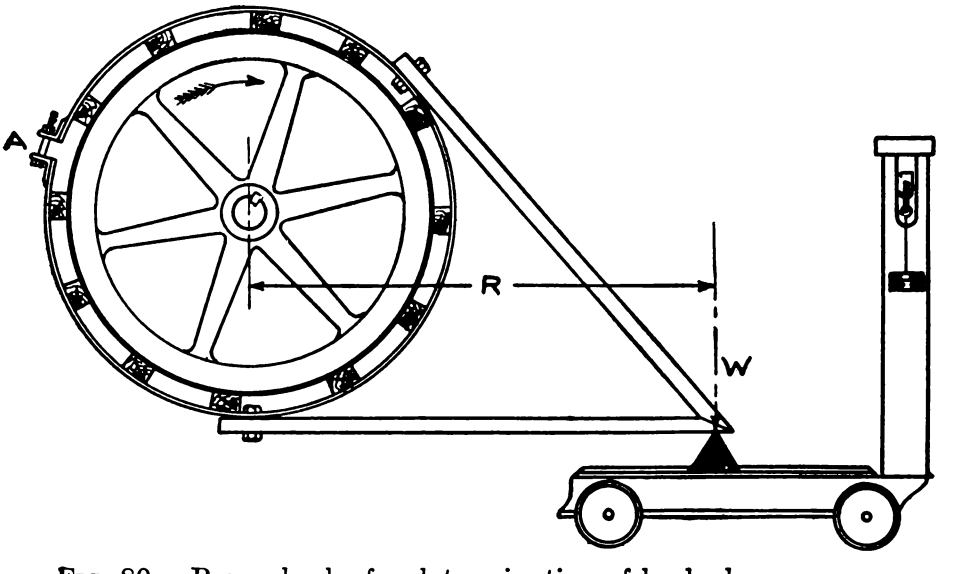


FIG. 80.—Prony brake for determination of brake horsepower.

Figure 80 shows such a brake arranged for use. This form is known as the "Prony Brake." It consists of a steel or leather band carrying a number of wooden blocks. By tightening the bolt at A, the friction between the blocks and the rim of the wheel can be varied at will. The corresponding pull which this friction gives at a distance R ft. from the shaft is weighed by a platform scale or spring balance. From the scale reading must be deducted the weight due to the unbalanced weight of the brake arms, which can be determined by reading the scales when the brake is loose and the engine is not running. If an engine is capable of maintaining a certain net pressure W on the scale, and meanwhile maintains a speed of N revolutions per minute, we can readily see that this is equivalent to an effective belt pull

of W pounds on a pulley of radius R running at N revolutions; or it can be considered as being equivalent to raising a weight equal to W by means of a rope wound around a pulley of radius R turning at N revolutions per minute. This weight would be lifted at the rate of

$$3.1416 \times 2 \times R \times N \text{ ft. per minute}$$

The work done in foot-pounds per minute will be

$$W \times \text{ft. per min.}$$

or $W \times 3.1416 \times 2 \times R \times N$ = foot-pounds per minute. The brake horsepower is this amount divided by 33,000.

$$\text{B.hp.} = \frac{W \times 3.1416 \times 2 \times R \times N}{33{,}000}$$

The brake and wheel rim will naturally get hot during a test, as all of the work done by the engine is transformed back into heat at the rubbing surfaces of the pulley rim and the brake. It is necessary to keep a stream of water playing on the rim to remove this heat. It is also best to have special brake wheels for testing. These have thin rims and inwardly extending flanges on the rims so that a film of water can be maintained on the inner surface of the rim.

Example:

Suppose that at the time of testing the 5- × 8-in. gas engine in Art. 104, we also determined the brake horsepower by means of a Prony brake having a radius of 3 ft., and that a net pressure of 22 lb. was exerted on the scales (the speed of the engine was 450 r.p.m.). Let us calculate the brake horsepower.

$3.1416 \times 2 \times 3 \times 450 = 8482$
$22 \times 8482 = 186{,}604$ ft.-lb.
$186{,}604 \div 33{,}000 = 5.65$, *Answer.*

Explanation: Our data are equivalent to those of hoisting a weight of 22 lb. by a rope winding upon a pulley of 3-ft. radius turning at 450 r.p.m. The 22-lb. weight is raised 8482 ft. per minute, and the work done per minute is 22 lb. × 8482 ft. = 186,604 ft.-lb. per minute. Hence, the brake horsepower of the engine is 5.65.

107. Frictional Horsepower.—If this engine gave 7.13 indicated horsepower (i.hp.), but the power available at the flywheel was only 5.65, it stands to reason that the difference, or 1.48 hp., was lost between the cylinder and the flywheel. The explanation is that this power is expended simply in overcoming the friction of the engine; and this horsepower is, therefore, called

the frictional horsepower. At zero brake horsepower, the entire indicated horsepower is used in overcoming friction.

108. Mechanical Efficiency.—The ratio of the brake horsepower to the indicated horsepower gives the mechanical efficiency, meaning the efficiency of the mechanism in transmitting the power through it from piston to flywheel. This is usually expressed in per cent. In the case of the engine of which we figured the indicated horsepower and brake horsepower, the mechanical efficiency is:

$$E = \frac{5.65}{7.13} = .792 = 79.2\%$$

The mechanical efficiency of a gas engine is lower than that of a steam engine on account of the idle strokes which use up work in friction while no power is being generated, but at full load a well-built gas engine should show over 80% mechanical efficiency. The mechanical efficiency of a steam engine should be above 90% at full load. In every case, however, the efficiency is less than 100%.

PROBLEMS

268. The cage in a mine weighs 2200 lb., and the load hoisted is 3 tons. The hoisting speed is 20 ft. per second. Calculate the horsepower necessary, allowing 25% additional for friction and rope losses.

269. A 10-in. by 12-in. air compressor runs at a speed of 150 r.p.m. The m.e.p. is 30 lb. per square inch. Calculate the horsepower required to run it, neglecting friction losses.

270. If the mechanical efficiency of the air compressor in the preceding problem is 60%, what actual horsepower is required to run it?

271. What is the friction loss in Problem 269 if the mechanical efficiency is 64%?

272. Find the indicated horsepower of a 10-in. by 12-in. steam engine which runs at a speed of 250 r.p.m. if the m.e.p. is 60 lb. per square inch?

273. What will be the indicated horsepower of a single-cylinder, four-cycle gas engine, using the following data:
Size of cylinder = 12 in. by 16 in.
R.p.m. = 225
M.e.p. = 78 lb. per square inch
Number of working strokes = $\frac{1}{2}$ of the number of revolutions.

274. In Problem 272 what will the mean effective pressure have to be if the engine is to develop 28 hp. at the brake? The mechanical efficiency is 70%.

275. A body can do as much work in descending as is required to raise it. Knowing this fact, calculate the horsepower that could be developed by a waterpower which discharges 800 cu. ft. of water per second from a height of 13.6 ft., assuming that 25% of the theoretical power is lost in the wheel and in friction.

276. What would be the brake horsepower of a steam engine which exerted a net pressure of 100 lb. on the scales, at a radius of 4 ft., when running at 250 r.p.m.?

277. A hydraulic turbine is direct-connected to a direct-current generator. During a normal season the waterpower available to the turbine is theoretically equivalent to 500 hp. If the full load efficiency of the turbine is 80% and that of the generator 92%, what horsepower is developed by the generator at full load?

278. A centrifugal pump running at 1500 r.p.m. requires 90 hp. to drive it and is to be belt-driven from a 48-in. pulley on a high-speed automatic engine running at 275 r.p.m. What should be the diameter and width of face of the pulley on the pump if the pulley is to be 1 in. wider than the belt? A single ply belt is to be used.

$P = F/A$

CHAPTER XVI

MECHANICS OF FLUIDS

109. Fluids.—Nearly every shop of any size contains some devices which are operated by water or air pressure. Consequently, a knowledge of how these machines work is very valuable.

A *Fluid* is any substance which has no particular form, but always shapes itself to the vessel which contains it. Water, oil, air, steam, gas are fluids. In some ways water, oil, and similar substances are different from the lighter substances—air, steam, etc. To distinguish between these, we give the name of *Liquids* to such substances as water and oil; while air, steam, etc., are given the general name of *Gases*. In some respects liquids and gases are alike, and in others they are different. The chief difference is that liquids have definite volumes; they cannot be compressed or expanded any visible amount, while gases can be readily compressed or expanded to almost any extent. For all practical purposes we can say that liquids cannot be compressed. The third form of matter—*Solids*—needs no explanation. The differences in these three forms can be stated as follows:

A Solid has a definite shape and volume.

A Liquid has no definite shape, but has a definite volume.

A Gas has neither a definite shape nor volume.

There are some substances that exist in states in between the solid and the liquid form. Among these are tar, glue, putty, gelatine, etc.

110. Specific Gravity.—By specific gravity of a substance we mean its *relative* weight as compared with the same volume of water. Thus we say that the specific gravity of cast iron is 7.21, meaning that cast iron is 7.21 times as heavy as water. A cubic foot of water weighs 62.4 lb., and a cubic foot of cast iron weighs about 450 lb. The quotient $\frac{450}{62.4}(= 7.21)$ is the specific gravity of the iron.

In many handbooks we find tables of specific gravities, and when we wish to get the actual weight per cubic inch or per cubic foot of some substance, we must multiply the weight of the unit of water by the specific gravity of the substance.

Example:

The specific gravity of alcohol is .8. How much does a gallon of alcohol weigh?

1 gal. of water = $8\frac{1}{3}$ lb.
1 gal. of alcohol = $.8 \times 8\frac{1}{3} = 6\frac{2}{3}$ lb., *Answer.*

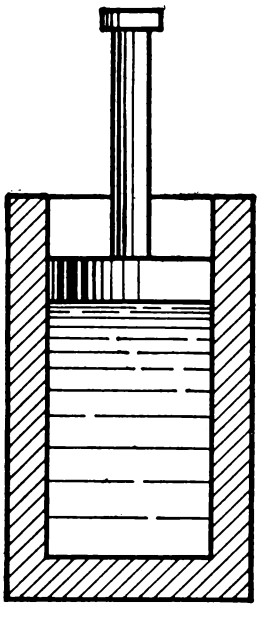


FIG. 81.—Cylindrical vessel fitted with piston.

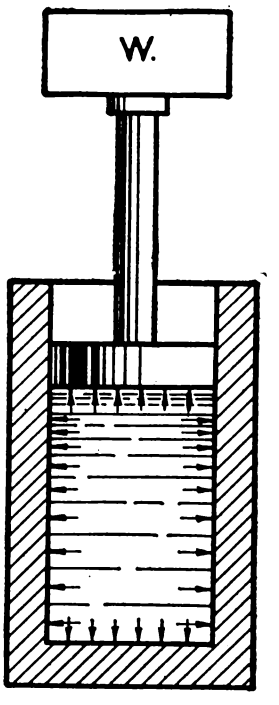


FIG. 82.—Cylindrical vessel containing water under piston sustaining weight *W*.

If a substance has a specific gravity less than 1, it will float in water, because it is lighter than the same volume of water. If the specific gravity is greater than 1, the substance is heavier than water and will sink. A substance that will float in water may sink in some other liquid if it has a greater specific gravity than the liquid in question. For example, a piece of apple wood will float in water but will sink when placed in gasoline, the specific gravity of the wood being about .76 and that of gasoline about .71. A piece of iron will sink in water but will float when placed in mercury (quicksilver), the specific gravity of mercury being 13.6 and that of iron about 7.21.

111. Transmission of Pressure through Fluids.—One of the most useful properties of all fluids is that of transmitting pressure in all directions. In Fig. 81 is shown a cylindrical vessel partly filled with water and fitted with a piston and piston rod. Assuming that the piston and rod are practically weightless, and that the piston fits snugly in the cylinder, preventing the water from leaking past the piston, it is then apparent that the piston will rest lightly on top of the water. If now a weight W be placed upon the piston rod, as in Fig. 82, the piston will then press down against the water with a total pressure of W pounds. Assuming the area of the bottom of the piston to be 10 sq. in., then each square inch of piston surface exerts a pressure of $\frac{W}{10}$ lb. against the water surface. In order to support the load W, the water must also press up against the bottom of the piston with a total pressure of W lb. It is clear that the pressure exerted against the piston by each square inch of water surface must also be $\frac{W}{10}$ lb. This pressure of $\frac{W}{10}$ lb. per square inch is not only exerted at the upper surface of the water but is transmitted equally to all parts of the water, and every square inch of water coming in contact with the vessel exerts a pressure of $\frac{W}{10}$ lb. against it (neglecting the weight of the water itself). The small arrows in Fig. 82 indicate how the water pressure is exerted on all sides of the vessel.

Example:

If the piston of Fig. 82 were 8 in. in diameter, and the weight W equaled 5026 lb., what would be the water pressure per square inch?

Area of piston $= 8 \times 8 \times .7854$

Area of piston $= 50.26$ sq. in.

$$\text{Pressure} = \frac{W}{\text{Area}} = \frac{5026}{50.26}$$

Pressure = 100 lb. per sq. in., *Answer.*

Explanation: The area of the piston is 50.26 sq. in., which is also the area of water surface in contact with the piston. The total load supported is, therefore, 5026 lb. Each square inch of water surface must press up against the piston with a pressure of $5026 \div 50.26 =$ 100 lb. per square inch, which pressure is transmitted to all parts of the water.

112. The Hydraulic Jack.—The property which water has of transmitting pressure in any direction is made use of in many ways. The same property is, of course, common to other fluids, such as oil, air, etc. Wherever we find a powerful, slow-moving force required in a shop, we usually find some hydraulic machine. (The word "hydraulic" refers to the use of water, but it is often applied to machines using any liquid—water, oil, or alcohol.)

Figure 83 illustrates the principle used in these hydraulic machines. The small cylinder is connected to a large cylinder having a piston supporting a load W. A small force P is exerted on the small piston producing a pressure per square inch

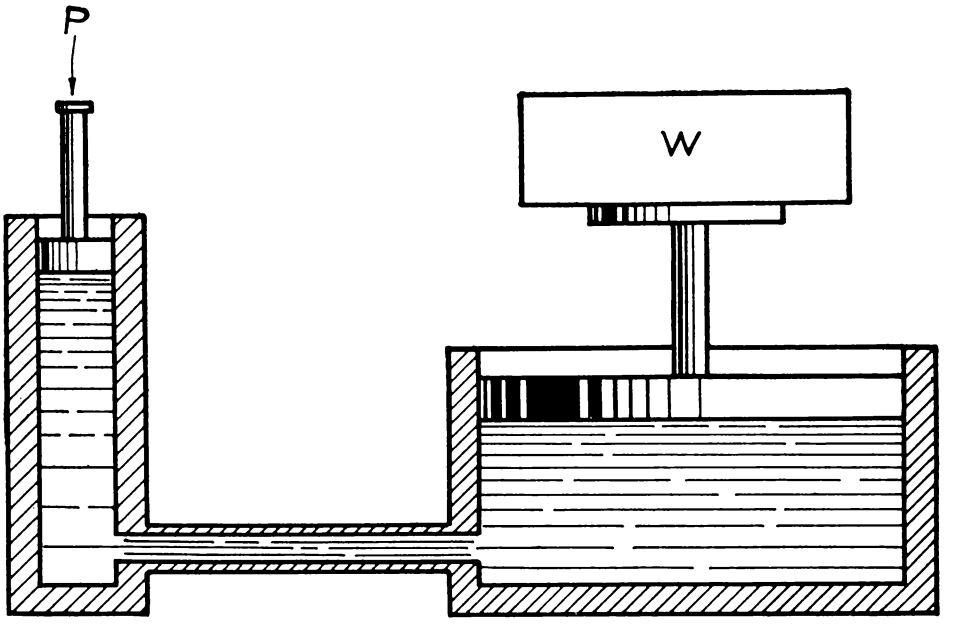


FIG. 83.—Diagram showing principle of hydraulic jack.

in the water equal to P divided by the area of the small piston. This pressure is transmitted to the large cylinder, where the same pressure per square inch is exerted against the underside of the large piston. The total upward pressure of the water on the large piston equals the pressure per square inch times the area of the piston. The load W supported by the piston will equal the total upward pressure of the water.

Let A = area of large piston
and a = area of small piston

Then, pressure per square inch $= \frac{P}{a}$

and the load $W = A \times \frac{P}{a}$ or $\frac{A}{a} \times P$

From this it is plain that the ratio, $\frac{A}{a}$, of the areas times the force P equals the load W. *The ratio of the areas is thus seen to be the mechanical advantage of the machine.* If the area of the large piston is 10 times the area of the small piston, the load W will be 10 times the force P, etc.

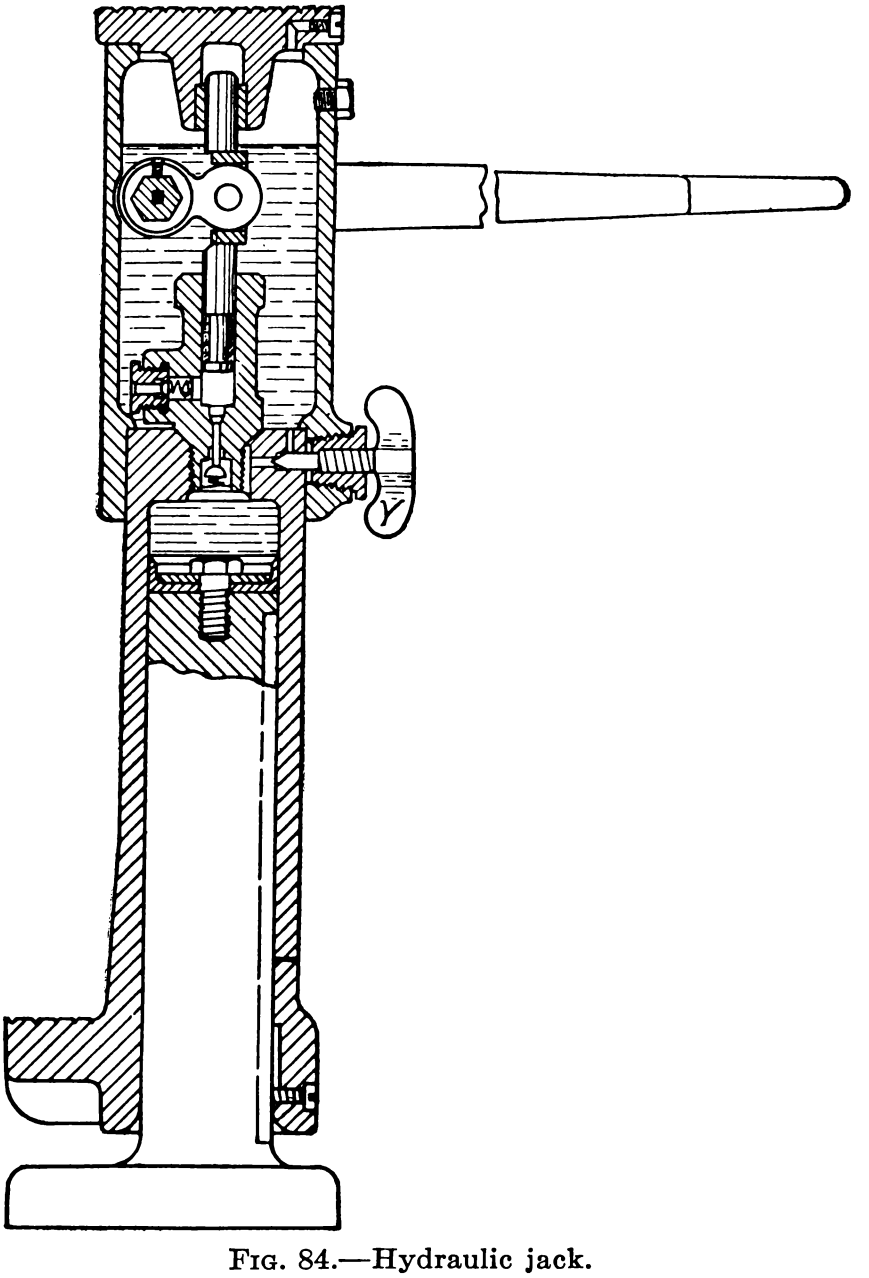


FIG. 84.—Hydraulic jack.

Like the lever, the jackscrew, and the pulley, this increase in force is obtained only by a decrease in the distance the weight is moved. The *work* done on the small piston is theoretically the same as the work obtained from the large piston. For example, suppose that the large piston has 100 times the area of the small one and we push the small piston down 1 in.; the water that is

pushed out of the small cylinder will have to spread out over the entire area of the large piston; the large piston will, therefore, be raised only $\frac{1}{100}$th of the distance that the small piston was moved. We have here an application of the law that the work put into a machine is equal, neglecting friction, to the work done by it.

Force $\times$ Distance moved = Weight $\times$ Distance raised

In Fig. 83 the motion that can be given to W is very limited, but by using a pump with valves, instead of the simple plunger P, we can continue to force water into the large cylinder and thus give a considerable movement to W.

Figure 84 shows a common form of hydraulic jack which operates on this principle. The top part contains a reservoir for the liquid, and also has a small pump operated by a hand lever on the outside of the jack. By working the lever, the liquid is pumped into the lower part of the jack between the plunger and the casing, thus raising the load. The load may be lowered by loosening the lowering screw Y. This opens a passage to the reservoir, and the load on the jack forces the liquid back through this passage to the reservoir.

In calculating the M.A. of a hydraulic jack, we must consider the M.A. of the lever which operates the pump, as well as the advantage due to the relative sizes of the two pistons, known as the pump and the ram.

Example:

If the ram of Fig. 84 is 3 in. in diameter and the pump is 1 in. in diameter, while the lever is 15 in. long and is connected to the pump at a distance of $1\frac{1}{2}$ in. from the fulcrum, what is the M.A. of the entire jack?

$\frac{.7854 \times 3^2}{.7854 \times 1^2} = \frac{9}{1} = 9$

$\frac{15}{1.5} = 10$

$9 \times 10 = 90$, *Answer.*

Explanation: The areas of the ram and pumps are as 9:1, hence their M A. is 9. The lever has a M.A. of 10. Hence, that of the whole jack is $9 \times 10 = 90$, and a force applied at the end of the lever would be multiplied 90 times. This force would, however, move through a distance 90 times as great as the distance the load would be raised.

The hydraulic jack has usually an efficiency of over 70% and is, therefore, a much more efficient lifting device than the jack-screw. A mixture containing one-third alcohol and two-thirds water should be used in jacks. The alcohol is added to prevent freezing.

113. Hydraulic Machinery.—In the shop, we often find water pressure used to operate presses, punches, shears, riveters, hoists, and sometimes elevators. These machines are seldom operated by handpower but have water supplied under pressure from a central pumping plant. The admission of the water and the consequent motion of the machine are controlled by hand-operated valves. Most of these hydraulic machines are used where tremendous forces are required. Therefore, very high water pressures are used, occasionally as high as 3000 lb. per square inch, 1500 lb. per square inch being a common working pressure for hydraulic machines.

Figure 85 shows a press operated by hydraulic pressure. It will be noticed that the movable head is connected to two pistons—a large one for doing the work on the down stroke, and a smaller one above, used only for the idle or return stroke of the press.

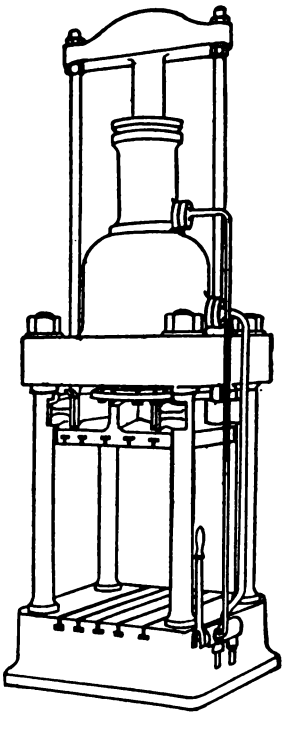


FIG. 85.—Hydraulic press.

114. Hydraulic Heads.—Water pressure for use in hydraulic machines is secured in two ways: by the use of pumps, and by the use of hydraulic heads. Beneath the surface of the ocean, or any body of water, a certain pressure exists in the water, due to its own weight, and this pressure becomes greater and greater as the distance from the surface increases.

The same effect can be produced by elevating water to a high tower or tank or by making use of some natural source of water which is at an elevation. The pressure existing at any point in the water due to its own weight is entirely independent of the amount of water or the shape of the container, but is directly proportional to the vertical height of the water surface above that point. This vertical height is known as the "head." Figures 86 and 87 represent two different water tanks, each having a pipe leading from the bottom. At the lower end of the pipe in both systems the "head" or vertical height is 50 ft. The pressure produced by a 1-ft. head of water, that is, at a point 1 ft. below the surface, equals .434 lb. per square inch; therefore, the pressure under a 50-ft. head, will be 50 times as great or $50 \times .434 = 21.7$ lb. per square inch.

Water towers and tanks are used quite frequently in towns and cities to furnish the pressure in the water supply systems.

The shape and size of the tank have no influence on the pressure of the water. The water in the tank, except a column of it directly over the outlet, is supported by the bottom of the tank and has no effect on the pressure in the pipe. The pressure at the bottom of the pipe would be the same if the pipe alone extended up to the same height as the tank. Also the size of the

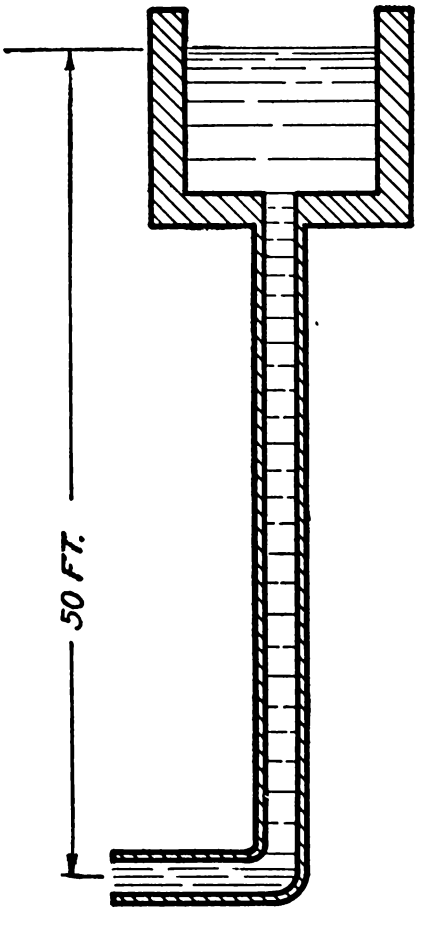


FIG. 86.—Water tank with vertical outlet.

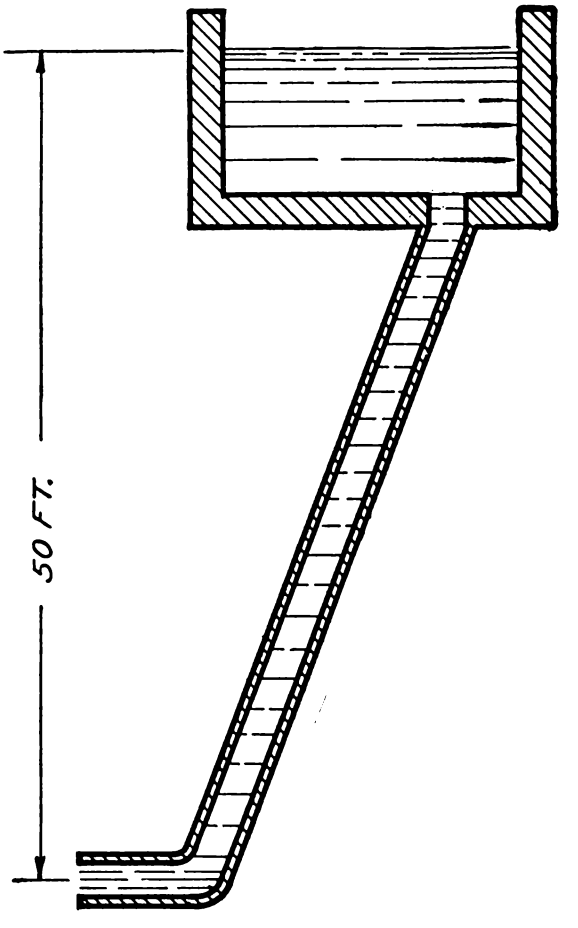


FIG. 87.—Water tank with inclined outlet.

pipe has no effect on the pressure *per square inch.* The water in a large pipe will weigh more than in a small pipe; but the pressure will be spread over a larger area, and if the heights are the same, the pressure per square inch will be the same.

In pumping water to an elevated tank or reservoir, the pressure required per square inch is also determined in the same manner and is .434 times the height in feet to which the water is raised, plus an allowance for friction in the pipes. Thus, to pump water to a height of 100 ft. requires a pressure somewhat greater than $.434 \times 100 = 43.4$ lb. per square inch.

115. Steam and Air.—Steam and air under pressure are also used to operate various kinds of shop machinery. Being more

elastic than water, they are preferred where the machines are to be operated quickly. Devices using air are called "pneumatic appliances," among the most common of which are pneumatic drills, hammers, and hoists. The air for operating these is supplied by air compressors located in the power house. These take the air from out-of-doors and compress it into a smaller volume. The resistance of the air to this compression causes it, in its effort to escape, to exert a pressure on the walls of the tank or pipe containing it. The more the air is compressed, the greater is the pressure exerted by it. The air pressure used in shop work is usually about 80 lb. per square inch. The air is conducted through pipes and hose to the point where it is to be used where its pressure is exerted on the piston of the appliance which is to be driven.

PROBLEMS

279. The specific gravity of lignum-vitae (a hard wood) is 1.328. Will this wood float or will it sink in water?

280. What weight on the small piston of Fig. 83 would support a weight of 30,000 lb. on the large piston if the small piston is 1 in. in diameter and the large one 12 in. in diameter?

281. If a hydraulic press works with a water pressure of 1500 lb. per square inch, what must be the diameter of the ram if a total pressure of 75,000 lb. is to be produced?

282. If the air hoist of Fig. 88 has a cylinder 10 in. in diameter inside, and the piston rod is $1\frac{1}{4}$ in. in diameter, and an air pressure of 80 lb. per square inch is exerted on the bottom of the piston, what weight can be lifted by the hoist?

283. If a city wishes to maintain a water pressure of 80 lb. per square inch at the hydrants, how high above the streets must be the water level in the standpipe?

284. What water pressure must a pump be capable of producing in order to force the water to a reservoir at an elevation of 3000 ft. above the pump?

285. Figure 89 shows the principle of one form of hydraulic elevator, the car being fastened directly to a long ram which is raised by water pressure. The ram and car are partially balanced by a counterweight, so that only 500 lb. of the dead load are supported by the water pressure. If this elevator is operated by water from the city mains at 80 lb. pressure per square inch, and the ram has a diameter of 10 in., what external or "live" load can be lifted, allowing 30% for losses in friction, etc.?

286. A centrifugal pump is to deliver 2000 gal. of water per minute against a pressure of 75 lb. per square inch. What horsepower

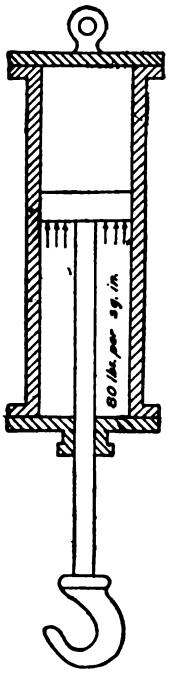


FIG. 88.—Illustration for Problem 282.

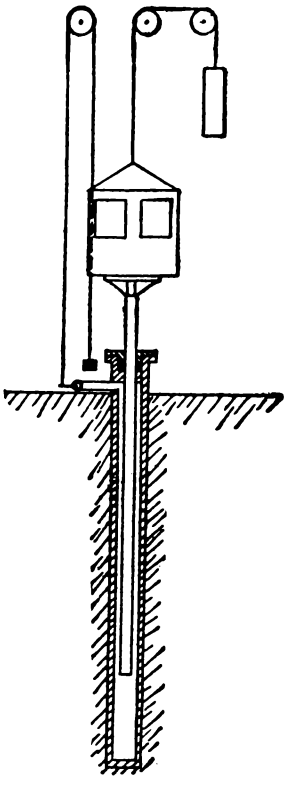


FIG. 89.—Illustration for Problem 285.

engine is required to run the pump, allowing 30% extra for friction losses?

287. A gravity oiling system has an oil tank placed 10 ft. above the bearings to be lubricated. The tank is connected by small tubes to the various bearings. If the specific gravity of the oil is .88, what pressure will the oil have at the bearings?

288. Four hydraulic jacks similar to Fig. 84 are used to raise a machine weighing 4 tons. Each jack has a lever 20 in. long, connected to the pump at 2 in. from the fulcrum. The diameters of the ram and pump piston are 4 in. and 1 in., respectively. What force is required on each lever if the efficiency of the jacks is 75%?

289. The radius of the bottom of a jug, which is filled with oil, is .3 of a foot. The diameter of the cork in the jug is 1.2 in. If the cork exerts a pressure of 7 lb. on its bottom end, what is the total pressure exerted on the bottom of the jug?

290. If copper has a specific gravity of 9, how much will 147 cu. in. weigh?

CHAPTER XVII

HEAT

116. Nature of Heat.—Heat is a form of energy; that is, it is capable of doing work. This we see amply illustrated in the steam engine and the gas engine, where heat is used in producing work. The steam engine uses heat which has been imparted to the steam in the boiler. Part of the heat of the steam is changed to work in the engine, and the rest is rejected in the exhaust. Heat is not a substance as was formerly supposed—it cannot be weighed and cannot exist by itself.

Heat is usually obtained by the burning of some variety of fuel, such as coal, wood, gas, or oil. In burning, the fuel unites with oxygen, one of the constituents of air, and this process, called "combustion," generates the heat. We cannot get heat by this process, therefore, without air. No fuel will burn without a supply of air. As soon as we shut off the air from a fire, combustion stops and no more heat is generated. A fire may continue to give off heat for some time after the air is cut off, but this heat comes from the cooling of the hot fuel in the fire. Of the heat generated during combustion, some of it goes through the furnace walls to the surrounding air; some goes to heat up the bed of coals and any object that may be placed in the fire to be heated; but the greater part of the heat goes off in the gases that are formed by the union of the fuel with the air. It is to save this heat that we sometimes see steam boilers set up in connection with the furnaces of large forge shops.

There are other ways of generating heat besides that of combustion. One method that is coming into considerable use and which is especially interesting to shop men because of the ease with which it can be controlled is by the use of electricity. We now have electric annealing and hardening furnaces for use in toolrooms, where a close regulation of the heat is very desirable. Electric furnaces are used for the production of aluminum and carborundum. We also have electric welding as an example of the production of heat from electricity.

Another method of heat generation that is frequently encountered in shops, often where it is not desired, is the production of heat from work. We have seen how heat is turned into work, but here we have work turned into heat. One common case of this is in bearings, where heat is produced from the work that is spent in overcoming the friction. Another example is seen in the heating of a lathe tool when it is taking a heavy cut, or in the heating of the tool when it is being ground. In either event, the work spent in removing the metal goes into heat.

117. Temperatures.—Temperature is the indication of the intensity of the heat in a body, or, in simple words, the "hotness" of a body. To lower the temperature of any body it is necessary to remove heat from the body, and to raise the temperature, heat must be added to it. It must be emphasized here that temperature is not a measure of quantity of heat. For example, it takes much more heat to heat a barrel of water from the freezing to the boiling temperature than it does to heat a quart of water from the freezing to the boiling temperature. Although the quantities of heat added to each are entirely different, the temperatures are the same. The common method of measuring temperature is by means of an instrument known as a thermometer, which usually consists of a glass tube which is partly filled with mercury and which has the air exhausted from the other part of it. The mercury expands or contracts as the temperature rises or falls and, therefore, the height of the column of mercury is a measure of the temperature. Alcohol is often used instead of mercury for outdoor thermometers where the mercury might freeze.

The two kinds of thermometers most commonly used are the Fahrenheit (abbreviated Fahr. or F.), which is mostly used in English-speaking countries, and the Centigrade (abbreviated C.), which is used more extensively in Europe, but also in the United States for scientific work.

On the Fahrenheit thermometer the temperature at which water freezes is 32°, and the temperature at which it boils is 212° (at atmospheric pressure). The zero temperature on this scale is, therefore, 32° below the freezing point, and the boiling temperature is $212 - 32 = 180°$ above the freezing point.

On the Centigrade thermometer, the temperature at which water freezes is the zero point of the scale, and the temperature

at which it boils is 100°. The boiling temperature is, therefore, 100° above the freezing temperature.

If a Fahrenheit and a Centigrade thermometer are both immersed in a pan of water and ice (therefore, at the freezing temperature) the Fahrenheit thermometer will register a temperature of 32° and the Centigrade a temperature of 0°. If the water is then heated until it boils (assuming standard atmospheric conditions), the Fahrenheit thermometer will read 212° and the Centigrade 100.° In heating the water from the freezing point to the boiling point, the temperature is increased or raised through 180° on the Fahrenheit scale and 100° on the Centigrade scale. It is at once apparent that a temperature change of 100° on the Centigrade scale is the same as a temperature change of 180° on the Fahrenheit scale. Figure 90 shows the two types of thermometer scales placed side by side. It can easily be seen that 1° on the Centigrade scale is larger than 1° on the Fahrenheit scale. As a matter of fact, each Centigrade degree equals $\frac{180}{100}$ or $\frac{9}{5}$ Fahrenheit degrees. Similarly, 1 Fahrenheit degree equals $\frac{100}{180}$ or $\frac{5}{9}$ of a Centigrade degree.

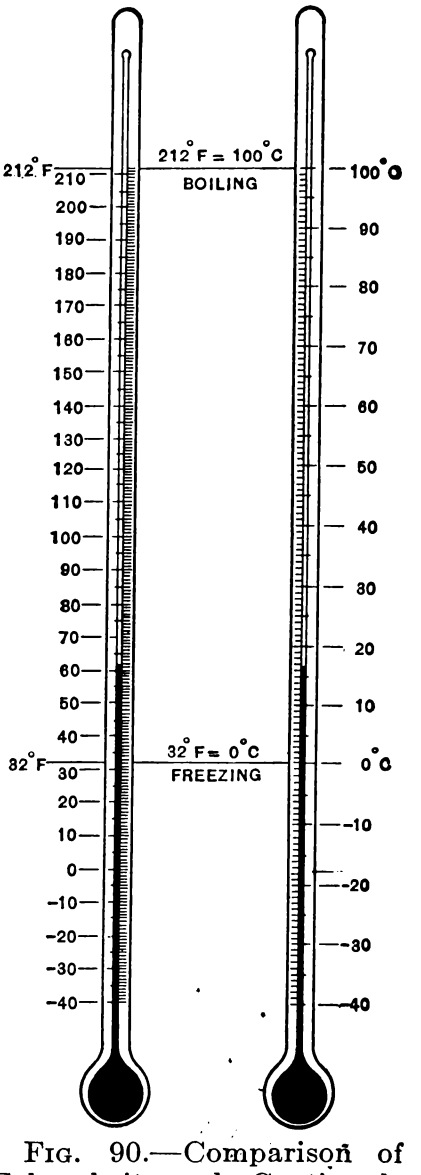


FIG. 90.—Comparison of Fahrenheit and Centigrade temperature scales.

If a weather report from some town in Europe states that the temperature during the day was about 0° C., we would know immediately that it was just about at the freezing point, or a temperature of 32° F. If the following day the temperature was given as 20° C. we would know that the temperature that day was 20 Centigrade degrees above the freezing point. One Centigrade degree equals $\frac{9}{5}$ Fahrenheit degrees; therefore, 20 Centigrade degrees equals $20 \times \frac{9}{5} = 36$ Fahrenheit degrees. In other words, then, the temperature was 36 Fahrenheit degrees above the freezing point. Since the freezing point on the Fahrenheit scale is at a temperature of 32° F., then the actual tempera-

ture that day on the Fahrenheit scale was 32 + 36 = 68° F. Therefore, a temperature of 20° C. is the same as a temperature of 68° F.

To change a reading on the Centigrade scale to the corresponding Fahrenheit reading: First multiply the degrees Centigrade by $\frac{9}{5}$. This gives an equivalent number of degrees on the Fahrenheit scale. To this add 32, in order to have the reading from the Fahrenheit zero.

To change a reading on the Fahrenheit scale to the corresponding Centigrade reading: First subtract 32. This gives the number of Fahrenheit degrees above freezing (which is the Centigrade zero). Multiply the result by $\frac{5}{9}$, thus obtaining the desired Centigrade reading.

Examples:

1. Change 30° C. to the corresponding Fahrenheit reading.

$30 \times \frac{9}{5} = 54$, the equivalent number of Fahrenheit degrees above the freezing point (32°).

54 + 32 = 86° F., the reading on a Fahrenheit thermometer.

2. What would a Centigrade thermometer read when a Fahrenheit thermometer stood at 72°?

72 − 32 = 40, the number of Fahrenheit degrees above freezing.

$40 \times \frac{5}{9} = 22\frac{2}{9}$° C., *Answer.*

When it is necessary to measure the temperatures in furnaces, as in the heat treatment of steel, a thermometer cannot be used on account of the very high temperatures. As the mercury thermometer is ordinarily made, it should not be used for temperatures above 500°, but, by filling the glass tube above the mercury with nitrogen gas under pressure, a thermometer can be made that may be read up to 1000°. For higher temperatures, devices called pyrometers are used. There are numerous kinds of pyrometers, but the one most used in the shops for furnace temperatures is what is called the Le Chatelier pyrometer. In this pyrometer, one end of a porcelain tube about $\frac{5}{8}$ in. in diameter and from 12 to 40 in. long is thrust into the furnace and held there or, if frequent readings are to be taken, it may be placed there permanently. Inside this tube are some wires of special composition that generate an electric current when they get hot. From the other end of the tube a pair of wires runs to a small box containing a "galvanometer," that is, a device for indicating the strength of the electric current generated. This has a needle

swinging over a dial, and the dial is usually laid off in degrees so the temperature is read direct. Many of these pyrometers have Centigrade graduations, but one should be sure which scale a pyrometer has before he uses it.

For example, suppose we wanted to get 1000° F. and by mistake had 1000° C. instead.

$$\left(\frac{9}{5} \times 1000\right) + 32 = 1832° \text{ F.}$$

$$1000° \text{ C.} = 1832° \text{ F.}$$

This shows that it would be pretty serious to use the wrong scale.

It has for years been the practice of the older shop men to tell the temperature of steel or iron by its color. This method has its disadvantages, however, as so much depends on the sensitiveness of the man's eye and on whether the work is being done in bright sunlight or in a dark corner of the shop. A bar will show red in the dark when it would still be black in the sunlight.

For the lower range of temperatures (those used in tempering tools) we can judge the temperature by the color which will appear on a polished steel surface when heated in the air. These tempering colors and their uses for carbon tool steels are about as follows:

430° F.	Very pale yellow	Scrapers Hammer faces Lathe, shaper, and planer tools
460°	Straw-yellow	Milling cutters Taps and dies
480°	Dark straw color	Punches and dies Knives Reamers
500°	Brownish-yellow	Stone cutting tools Twist drills
520°	Yellow tinged with purple	Drift pins
530°	Light purple	Augers Cold chisels for steel
550°	Dark purple	Hatchets Cold chisels for iron Screwdrivers Springs
570°	Dark blue	Saws for wood
610°	Pale blue	
630°	Blue tinged with green	

More uniform results can be obtained if the steel is heated in a bath of sand or of oil, the bath being maintained at the desired

temperature and a pyrometer being used to observe the temperature. For higher temperatures, molten lead or mineral salts, such as common salt, barium chloride, potassium chloride, and potassium cyanide, are used.

When steel and iron are heated to higher temperatures, they successively become red, orange, and white. These colors and the corresponding temperatures are approximately as follows:

First signs of red	957° F.
Dull red	1290°
Dark cherry	1470°
Cherry-red	1655°
Bright cherry	1830°
Dull orange	2010°
Bright orange	2190°
White heat	2370°
Bright white—welding heat	2550°
Dazzling white	2730° to 2910°

118. Expansion and Contraction.—Nearly all substances expand when they are heated and contract when heat is removed from them, or they are cooled. This phenomenon is greatest in gases and least in solids, but even in solids it is of enough importance to be extremely useful at times, or to cause considerable trouble if no allowance has been made for it.

There are a few metal alloys which, within certain limits, do not change their volumes with changes of temperature, and there are also some which between certain temperatures will even expand when cooled and contract when heated. A nickel steel containing 36% nickel has practically no expansion or contraction with changes in temperature and is, therefore, used in some cases for accurate measurements where expansion of the measuring instruments would introduce serious errors.

When a solid body is heated, it expands in all directions if free to do so. As a rule, however, we are concerned only with the change of one dimension and not with the change in volume. Thus, in the case of a steam pipe we do not care about the change in thickness or in diameter, but we are concerned with the change in length. On the other hand, when a bearing gets hot and seizes, it is the change in diameter that causes the trouble. There are few machinists who have not had the experience, in boring a sleeve to fit a certain shaft, of having a free fit when tested just after taking a cut through the sleeve, and then later of finding

that the sleeve fitted so tight that it had to be driven off the shaft. Of course, the explanation is that the sleeve becomes warm when being bored in the lathe, while the shaft is much cooler. When the sleeve cools to the temperature of the shaft, it contracts and seizes or "freezes" to the shaft. In accurate tool work the effect of differences in temperature between the measuring instruments and the work may become serious. For this reason many gages are provided with rubber or wooden handles which do not conduct heat readily. They thus prevent the heat of the hand from getting into the gages and expanding them.

The foregoing discussion gives some idea of the troubles caused by these properties of materials; let us now see of what benefit they are. We have already seen the use that is made of the expansion of mercury in thermometers. There are numerous heat-regulating devices (called *thermostats*) which depend on the expansion or contraction of a bar to perform the desired operations. We find these used for regulating house-heating boilers and furnaces, incubators, and other devices where uniform temperatures are required. Probably the greatest shop use of expansion and contraction is in making shrink fits. When we want to fasten securely and permanently one piece of metal around another, we generally shrink the first onto the second. This process is used for attaching all sorts of bands and collars to shafts, cylinders, and the like, for putting tires on locomotive wheels, and for similar work. The erecting engineer uses it to put in the links in a sectional flywheel rim or to draw up bolts in the hub or in any other place where he wants to make a rigid permanent joint.

The amount that any substance will expand when heated, or contract when cooled, depends upon three things: the kind of material of which the body is made, the length of the body, and the amount which its temperature is changed. By "length" is meant the dimension along which the expansion is to be found. This might be the diameter, or the width, height, or any other dimension. That the expansion also depends on the temperature change is easily apparent. Thus a body heated from 60 to 80° F. has its temperature changed 20°, and its expansion will be twice as great as when its temperature change is only 10°, such as from 60 to 70° F. or from 30 to 40° F. The actual temperature of the body is ordinarily of no consequence. It will expand just as much when heated from 10 to 11° F. as when heated from 80 to 81° F., since the temperature change in both cases is 1°.

Under ordinary conditions of temperature most substances expand uniformly; that is, for each degree of temperature change the expansion (or contraction) is a definite part of the original length. This fraction of its own length which a body expands or contracts for 1° change in temperature is called its "coefficient of expansion." The coefficients of expansion for different metals have been determined by careful experiments, and complete tables of such coefficients may be found in various handbooks. In many cases the values given in different books do not agree, but this is undoubtedly due to variations in the compositions of the metals used in the tests. The following values are taken from the most reliable authorities and are sufficiently accurate for most purposes.

COEFFICIENTS OF EXPANSION

Metal	Coefficient
Aluminum	.00001234
Brass	.00001
Cast iron	.0000055 to .000006
Wrought iron and machine steel	.0000065
36 % nickel steel	.0000003

The above values are based on a temperature rise of 1° F. For 1° C. change in temperature the coefficients would be $\frac{9}{5}$ of those just given. The student is not expected to memorize these values. It must be remembered that if the length is given in feet, the expansion calculated will be in feet; and if the length is in inches, the expansion calculated will be in inches. To get the actual expansion (or contraction) of any body for 1° change in temperature, multiply the length of the body by its coefficient of expansion. The expansion for a change of temperature of 10° will be 10 times that for 1°; for a change of temperature of 50°, the expansion will be 50 times as great, etc.

Example:

What will be the expansion in a steam pipe (steel) 200 ft. ong when subjected to a temperature of 300° F. if erected when the temperature was 60° F.?

Coefficient of expansion = .0000065 (from table)
Temperature change = 300° − 60° = 240°
Expansion per degree = 200 × .0000065
= 00130. ft.
Total expansion..... = 240 × .0013 = .312 ft., *Answer.*

The law of expansion and contraction may be expressed by a formula as follows:

$$E = T \times C \times L$$

where E is the *change* in length
T is the *change* in temperature
C is the coefficient of linear expansion
L is the original length of the body

2. The piston of a gas engine when running has a temperature about 400° higher than the cylinder in which it is running. What allowance must be made for expansion when fitting the piston if the cylinder is 12 in. in diameter? (The piston is made of cast iron.)

The piston must be made smaller than the cylinder by the amount that a 12-in. piston would expand with a temperature rise of 400°.

Temperature change	= 400°
Coefficient of expansion	= .000006
Expansion per degree	= .000006 × 12 = .000072 in.
Total expansion	= .000072 × 400 = .0288 in., *Answer.*

119. Allowances for Shrink Fits—In making a shrink fit, the collar or band, or whatever is to be shrunk on, is bored slightly smaller than the outside diameter of the part on which it is to be shrunk. It is then heated and thus expanded until it can be slipped into place. When it cools, it cannot return to its original size but is in a stretched condition. It, therefore, exerts a powerful grip on the article over which it has been shrunk.

Practice differs considerably in the allowances that are made for shrink fits. A rule which has been widely and successfully used is to allow $\frac{1}{1000}$ in. for each inch of diameter. According to this rule, if we were shrinking a crank on a 6-in. shaft, the crank should be bored .006 in. small or else the shaft turned .006 in. oversize and the crank bored exactly 6 in. For a 10-in. shaft we would allow .010 in, and so on for other sizes.

This could be expressed by the following formulas:

$$A = .001 \times D, \text{ or, since } .001 = \frac{1}{1000}, \text{ this could be written}$$

$$A = \frac{D}{1000}$$

where A stands for "allowance"
and D for the diameter.

Assuming that an allowance of .001 inch per inch of diameter is used in shrinking a steel tire onto a locomotive wheel, let us see what temperature change is required to perform this operation. Since the tire is of steel, each inch of diameter will expand .0000065 in. for every degree of temperature rise. The total expansion for each inch of diameter must, of course, be .001 inch. Therefore, the number of degrees temperature rise required will be

$$.001 \div .0000065 = 154°, \textit{Answer.}$$

If the wheel and tire were at a temperature of 70° F. to begin with, the final temperature of the tire would be 70 + 154 = 224° F.

In practice, the tire would be heated to a still higher temperature in order to expand it enough so that it could be slipped on easily and quickly before it had time to cool off or to warm the wheel.

120. Unit of Heat.—Although heat is not a substance nor can it be weighed, still it is possible to measure heat through its effect upon the temperature of a body. In English-speaking countries the unit of heat in general use is known as the British thermal unit, or "B.t.u." The B.t.u. is that quantity of heat required to raise the temperature of 1 lb. of pure water 1° F. In other words, when 1 lb. of pure water is heated so that its temperature rises 1°, say, for example, from 60 to 61° F., the amount of heat absorbed by the water equals 1 B.t.u. It follows, then, that 2 lb. of water heated so that its temperature rises 1° will absorb 2 B.t.u.'s, and 3 lb. of water so heated will absorb 3 B.t.u.'s, etc. Similarly for each degree of temperature rise, a pound of water will absorb 1 B.t.u. Thus if the above pound of water is heated from 60 to 70° F. it will absorb 10 B.t.u.'s, etc.

121. Efficiencies of Boilers and Engines.—We have already mentioned the manner in which heat is transformed into work in the steam engine and the gas engine. In these two cases, through the effect of heat, we obtain a working fluid (steam or gas) under high pressure, and this working fluid by virtue of its pressure exerts a force upon the piston of an engine and performs work.

In the case of the steam engine, for example, heat is obtained by burning coal, oil, or gas in a suitable furnace or "firebox."

This heat of combustion is then utilized as much as possible to heat the water in the boiler and thus generate steam. Finally, the steam is conducted to an engine and there performs its work.

In such a process but very little of the heat generated in the firebox is turned into work at the engine, because of the large amount that is lost. Some of it goes up the chimney, some escapes into the boiler room, a considerable amount of it is lost at the engine by radiation and by condensation in the cylinder, and a very large amount is carried away by the exhaust steam.

The average coal burned under boilers, when burned as completely as possible, will generate about 12,500 B.t.u.'s per pound of coal. However, in the average power plant, for each pound of coal fired, only about 7500 B.t.u.'s are absorbed by the boiler, and only about 600 B.t.u.'s are transformed into work at the engine flywheel. We see from this that for every 12,500 B.t.u.'s supplied to the power plant, only about 600 B.t.u.'s are delivered as work. The average over-all efficiency of a power plant is, therefore, about

$$\frac{600}{12{,}500} = .048, \text{ or } 4.8\,\%.$$

Considering the boiler (including furnace) and engine separately, it is plain that the average boiler delivers to the engine about 7500 B.t.u.'s (in the steam) for each 12,500 B.t.u.'s supplied to the boiler, and the average engine delivers up as work about 600 B.t.u.'s of the 7500 B.t.u.'s supplied to it. Their respective efficiencies then are as follows:

$$\text{Av. eff. of boiler} = \frac{7{,}500}{12{,}500} = .6, \text{ or } 60\,\%.$$

$$\text{Av. eff. of engine} = \frac{600}{7500} = 0.8, \text{ or } 8\,\%.$$

Boiler efficiencies over 80 have been obtained, but 50% is more common, and it is probable that there are many cases where boiler efficiencies are still lower. Steam engine efficiencies greater than 20% have also been obtained.

PROBLEMS

291. Convert the following Centigrade temperatures to Fahrenheit:

(*a*) 22°
(*b*) 76°
(*c*) 124°
(*d*) 7°
(*e*) −2°

F C
212 − 40 · 68 · 32 · 0

292. In testing direct-current generators, it is customary to specify that under full load the temperature of the armature shall not rise more than 40° C. above a room temperature of 25° C.; that is, the temperature of the armature under these conditions should not exceed 65° C. In making a test a Fahrenheit thermometer was used. The room was at a temperature of 77° F., and the temperature of the armature at the end of the run was 180° F. Did the generator meet the specifications? What was the temperature change in Centigrade degrees?

293. A number of steel rails are welded together until the length is 3000 ft. at a temperature of 115° F. What will be the change in length if the temperature is changed to 2° C.?

294. The steel rails to be laid in a railroad track are each 30 ft. in length. If the maximum temperature variation from summer to winter is 140° F., what allowance in inches must be made for the expansion in each rail?

295. If we wished to maintain a tempering bath at a temperature of 500° F., what should be the reading on a Centigrade pyrometer?

296. In erecting a long steam line that will have a variation in temperature of 320° F., how far apart should the expansion joints be placed if each joint can take care of a movement of 3 in.?

297. If a brass bushing measures 2 in. just after boring, when its temperature is 95° F., what will it measure when it has cooled to 65° F.?

298. A steel link 2 ft. long is made $\frac{1}{16}$ in. too short for the slot in the flywheel rim into which it is to be shrunk. How hot must the link be before it will go into the slot if its temperature before heating is 65° F.?

299. A horizontal steam turbine and dynamo are to be direct-connected, their shaft centers being $3\frac{1}{2}$ ft. above the bedplate. If the bearings are lined up at a temperature of 70° F., how much will they be out of alignment under running conditions when the temperature of the dynamo frame is 80° F. and that of the turbine is 215° F., both frames being of cast iron?

300. A steam power plant during a period of 1 mo. burned an average of $2\frac{1}{8}$ lb. of coal per developed horsepower-hour. If 1 hp.-hr. is equivalent to 2545 B.t.u., what was the overall efficiency of the plant, assuming the coal had an average heating value of 12,000 B.t.u. per pound.

CHAPTER XVIII

STRENGTH OF MATERIALS

122. Stress.—When a load or force is applied to any piece of material it tends to alter the shape of the piece. This can easily be demonstrated with an ordinary piece of rubber. A pull will stretch the rubber, making it longer and thinner, and a pressure will squeeze it into a shorter and thicker piece. Other materials, such as wood, iron, etc., change in shape under a load, the same as rubber, but the change is so slight, in most cases, as to be invisible to the naked eye.

If a $\frac{1}{2}$-in. manila rope is used to suspend a load of 800 lb., the stretch in the rope produced by the load will be an appreciable amount, and in all probability the rope will be very near the breaking point. At any point in the rope each fiber will be stretched taut, indicating that the tendency of the load is to tear the fibers apart. The fibers themselves resist being stretched and torn apart, and this resisting force in the rope itself, counteracting the pull of the load, is called "stress." Thus we see that the load on the rope sets up a stress in every fiber.

In the same manner a load or force applied to any piece of material sets up an internal resistance, or stress, in the piece itself which opposes the tendency of the load to alter its shape or rupture the piece entirely. In the above case, if we could cut the $\frac{1}{2}$-in. rope at any point and attach the cut ends to a spring scale, the scale would read 800 lb., showing that the load tends to pull the rope apart at any section with a force of 800 lb. It follows, then, that the total internal resistance or stress exerted by the fibers to keep them from pulling apart must equal the load, or 800 lb.

There are three different kinds of stresses that can be produced, depending upon how the load is applied.

1. Tensile stress (pulling stress).
2. Compressive stress (crushing stress).
3. Shearing stress (cutting stress).

Figure 91 shows how these different stresses are produced.

There are two other kinds of stresses, but they are really special cases of the three just given. They are:

(*a*) Bending stress (really a combination of tension on one side and compression on the other).

(*b*) Torsional or twisting stress (a form of shearing stress)

123. Unit Stress.—"Unit stress" is the stress on a unit area; for example, if the cross-sectional area of a bar is 10 sq. in.

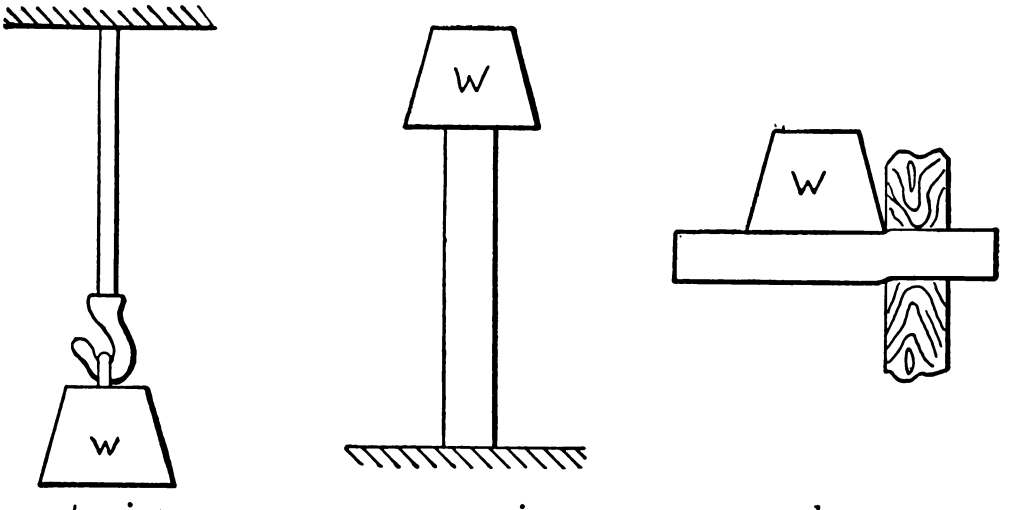


FIG. 91.—Simple illustrations of tension, compression, and shear.

and the total stress on this section is 700 lb., the unit stress will be 700 divided by 10, or 70 lb. per square inch. It is apparent, then, that the total stress equals the unit stress times the area of the section, and, therefore, that the load equals the unit stress times the area. Letting S stand for unit stress, W for load, and A for area, we get

$$W = AS$$

and

$$S = \frac{W}{A}.$$

The foregoing statements apply only to the simple stresses (1, 2, and 3 of Art. 122), that is, to such cases of simple, direct stress as are illustrated in Fig. 91. In Fig. 91 suppose that the post under compression is a square timber 10 in. on a side, and the load W is 16,000 lb. The cross-sectional area is then $10 \times 10 = 100$ sq. in. and the unit stress will be,

$$S = \frac{W}{A} = \frac{16{,}000}{100}$$

$S = 160$ lb. per square inch

124. Ultimate Strength.—By taking specimens of different materials and loading them until they break it has been possible to find out just what stress each kind of material will stand. The unit stress to which a material must be subjected in order to break it is called the *ultimate strength* of the material. The strength of most materials differs for the different methods of loading as shown in Fig. 91.

The unit stress at which a material will break under tension is called the "tensile strength," or the ultimate strength in tension.

The unit stress at which a material will break in compression is called the "compressive strength." or the ultimate strength in compression.

The unit stress at which a material will break in shear is called the "shearing strength," or the ultimate strength in shear.

The following table gives the average values for the most used materials.

ULTIMATE STRENGTHS (POUNDS PER SQUARE INCH)

Material	Tension	Compression	Shear
Timber.....	8,000	5,000	3,000 (across grain)
Cast iron.....	20,000	90,000	20,000
Wrought iron.....	50,000	50,000	40,000
Machine steel.....	65,000	65,000	50,000

125. Safe Working Stresses.—The unit stress which exists in a member while under load is usually referred to as the "working stress." When parts for machines, structures, bridges, etc. are being designed, the customary procedure is to determine first what unit stress the material for any particular part can stand with safety. This is called the "safe working stress" or "safe stress," and its determination is largely a matter of judgment and experience. The "safe working stress" is generally found by dividing the ultimate strength by some number called a "factor of safety." A factor of safety of 5 is one that is used quite commonly for parts subjected to steady loads.

The following table gives the average range of safe working stresses for some of the most used materials. From the table we note that for cast iron the safe stress in tension is 3000–4000, which means between 3000 and 4000 lb. per square inch.

SAFE WORKING STRESSES (POUNDS PER SQUARE INCH)

Material	S_t Tension	S_c Compression	S_s Shear
Timber	700	700	500
Cast iron	3– 4,000	15–18,000	3– 4,000
Wrought iron	8–10,000	8–10,000	7– 9,000
Machine steel	10–16,000	10–16,000	8–12,000

Instead of writing "safe stress in pounds per square inch" for tension, compression, or shear, the symbols S_t, S_c, and S_s are used. So if A = area in square inches, then the load W which can be carried safely = Area $\times$ safe stress per square inch or

$$A \times S_t = W \text{ (tension)}$$
$$A \times S_c = W \text{ (compression)}$$
$$A \times S_s = W \text{ (shear)}$$

Or, in general, for all stresses

$$A \times S = W$$

In designing a part of a machine, however, the load that the part is to carry is known. The question is to determine its size. Having decided on the safe stress that the material will stand, we see at once that the load divided by the safe stress per square inch will give the area in square inches required to carry the load, or,

$$A = \frac{W}{S}$$

Knowing the area, it is a simple problem to calculate the diameter, or the dimensions of the piece.

126. Strength of Bolts.—There is a well-known saying that "a chain is only as strong as its weakest link." This means, in general, that any mechanism must be so designed that its weakest part will be strong enough to stand the greatest load that may come on it.

In the design of a bolt it is quite clear that the weakest section of the bolt is at the root of the threads where the cross-sectional area is the least. The area at the root of the threads, therefore,

must not be less than the area required to support the load safely, as found by the formula, $A = \frac{W}{S}$. The bolt used should have an area, at the root of the threads, nearest to but not less than the required area.

Example:

What size of steel eyebolt will support a weight of 5000 lb.? Take 12,000 lb. as the safe load in tension.

$$\text{Then, } A = \frac{W}{S} = \frac{5{,}000}{12{,}000}$$

$$A = \frac{5}{12} \text{ sq. in.} = .416 \text{ sq. in}$$

The necessary area to support the weight, therefore, is .416 sq. in. Of course, the example could be completed by saying $.7854\ D^2 = .416$ sq. in., where D = diameter at the root of the thread. By then solving for D we would get the diameter at the root of the threads. But the Bolt Tables afford an easier method than this. In the following table .4193 is given as the area of a $\frac{7}{8}$-in. bolt at the root of the thread. Therefore, a $\frac{7}{8}$-in. eyebolt would probably be used.

BOLT TABLE—U. S. S. THREADS

Diameter, inches	Threads to inch	Diameter at bottom of thread	Area of body of bolt	Area at bottom of thread
$\frac{1}{4}$	20	.1850	.0491	.0269
$\frac{5}{16}$	18	.2403	.0767	.0454
$\frac{3}{8}$	16	.2938	.1104	.0678
$\frac{7}{16}$	14	.3447	.1503	.0933
$\frac{1}{2}$	13	.4001	.1963	.1257
$\frac{9}{16}$	12	.4542	.2485	.1621
$\frac{5}{8}$	11	.5069	.3068	.2018
$\frac{3}{4}$	10	.6201	.4418	.3020
$\frac{7}{8}$	9	.7307	.6013	.4193
1	8	.8376	.7854	.5510
$1\frac{1}{8}$	7	.9394	.9940	.6931
$1\frac{1}{4}$	7	1.0644	1.2272	.8899
$1\frac{3}{8}$	6	1.1585	1.4849	1.0541
$1\frac{1}{2}$	6	1.2835	1.7671	1.2938
$1\frac{5}{8}$	$5\frac{1}{2}$	1.3888	2.0739	1.5149
$1\frac{3}{4}$	5	1.4902	2.4053	1.7441
2	$4\frac{1}{2}$	1.7113	3.1416	2.3001
$2\frac{1}{4}$	$4\frac{1}{2}$	1.9613	3.9761	3.0213
$2\frac{1}{2}$	4	2.1752	4.9087	3.7163
$2\frac{3}{4}$	4	2.4252	5.9396	4.6196
3	$3\frac{1}{2}$	2.6288	7.0686	5.4277

In figuring the allowable loads for steel bolts, it is best not to allow over 12,000 lb. stress per square inch, and 10,000 lb. is perhaps even more usual on account of the sharp root of the threads, which makes a bolt liable to develop cracks at this point.

127. Strength of Hemp Ropes.—It is quite common in calculating the strength of ropes and cables to assume that the section of the rope is a solid circle. Of course, the strands of the rope do not completely fill the circle; but if we find by test the allowable safe strength *per square inch* on this basis, it will be perfectly safe to make calculations for other sizes of ropes on the same basis. The safe working stress based on the full area of the circle is 1420 lb. per square inch. The Nominal Area (as the area of the full circle by which the rope is designated is called) is $A = .7854 \times D^2$. The safe stress is 1420 lb. per square inch, and, consequently, the weight that can be supported by a rope of diameter D is

$$W = S \times A$$
$$= 1420 \times .7854 \times D^2$$

Here we have two constant numbers (1420 and .7854) that would be used every time we were to calculate the safe strength of a rope. If this were to be done often we would not want to multiply these together every time, so we can combine them.

$$1420 \times .7854 = 1120, \text{ approximately}$$

Hence,
$$W = 1120 \times D^2$$

Example:

Find the safe load on a hemp rope of $\frac{1}{2}$ in. diameter.

$$D = \frac{1}{2} \text{ and } D^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$
$$W = 1120 \times D^2$$
$$= 1120 \times \frac{1}{4} = 280 \text{ lb., } \textit{Answer.}$$

128. Wire Ropes and Cables.—For wire ropes made of crucible steel, a safe working load of 15,000 lb. per square inch of nominal area is allowable. For cables of Swedish iron but half this value should be used.

129. Strength of Chains.—It has been demonstrated by repeated tests that a welded joint cannot be safely loaded so heavily as a solid piece of material. Of course, there are welds that are practically as strong as the stock, but it is not safe to depend upon them. For this reason, the safe working load per square

inch for chain links is often given as 9000 lb., which is just $\frac{3}{4}$ of 12,000 lb.

If D = the diameter of the rod of which the links are made

$$A = 2 \times .7854 \times D^2$$

$$W = S_t \times A$$

$$W = 9000 \times 2 \times .7854 \times D^2$$

Combining the constant numbers, this can be simplified into

$$W = 14{,}000 \times D^2$$

130. Columns.—The previous examples were cases of tension. The size of a rod or timber subjected to compression is computed in the same way unless it is long in comparison with its thickness. When a bar under compression has a length greater than 10 times its least thickness, it is called a *Column* and must be considered by the use of complicated formulas which take account of its length. It can be seen by taking a yardstick, or similar piece, that it is much easier to break than a piece of shorter length but otherwise of the same dimensions. A long piece, when compressed, will buckle in the center and break under a light thrust or compression.

PROBLEMS

301. A small hoist is so designed that the maximum load will cause a pull of about 1750 lb. in the hoisting rope. What diameter hemp rope should be used?

302. What size of hemp rope would be necessary to withstand safely a pull of 3750 lb.?

303. What would be the safe load for a $\frac{3}{4}$-in. chain?

304. The boiler of Fig. 92 is suspended from the steel beams by four rods, each rod being threaded at the top and secured by a nut. When the boiler is completely set up and operating, the total weight supported by the rods will be about 15,000 lb. Determine the diameter of the rods, allowing a safe tensile stress of 12,000 lb. per square inch.

305. A manufacturer of jackscrews states that a $2\frac{1}{2}$-in. screw is capable of raising 28 tons. If the diameter of the screw at the base of the threads is 1.82 in., what is the compressive stress per square inch in the screw when carrying 28 tons?

306. A soft steel test bar having a diameter of .8 in. is pulled in two by a load of 31,500 lb. What was the breaking tensile stress per square inch?

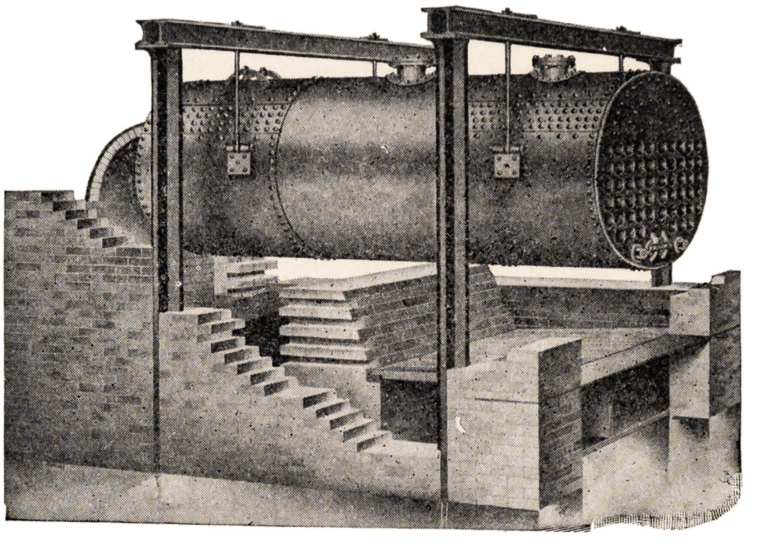


FIG. 92.—Steam boiler setting.

307. The cylinder head of a small steam engine, Fig. 93, having a cylinder diameter of 7 in. is held on by six studs of $\frac{3}{4}$-in. diameter. When there is a steam pressure of 125 lb. per square inch in the cylinder, what will be the pull on each stud? And what will be the stress per square inch in each stud, due to the steam pressure?

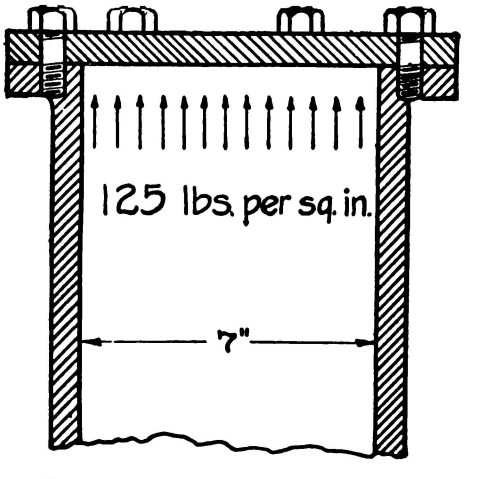


FIG. 93.—Cylinder head of small steam engine.

308. With a cylinder diameter of 10 in. and an air pressure of 100 lb. per square inch, find the greatest weight that can be lifted by the air hoist shown in Fig. 94. Also find the size of piston rod necessary, assuming that it is screwed into the piston. Neglect space occupied by rod.

309. Work out a formula for the strength of crucible steel cables on the same plan as that given for hemp rope.

310. What is the greatest load that should be lifted with a pair of tackle blocks having three pulleys in the movable block and two in the fixed block, and using a $\frac{3}{4}$-in. rope?

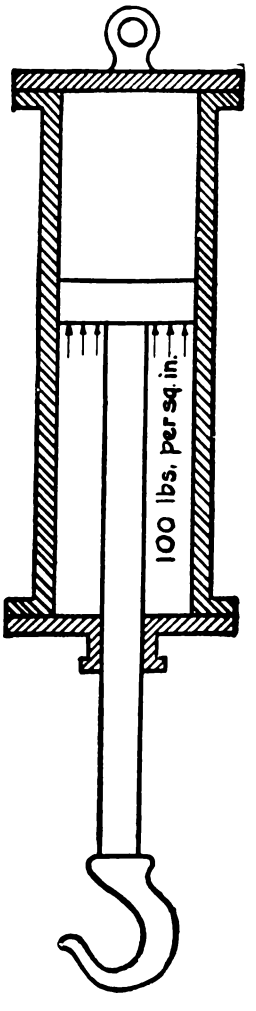


FIG. 94.—Illustration for Problem 308.

CHAPTER XIX

LOGARITHMS

131. Common Logarithms.—Logarithm is the name of a group of numbers the properties of which enable us to divide or multiply large numbers, or obtain powers and roots of numbers, more quickly and easily than by ordinary arithmetical processes. The Common System of logarithms, having the number 10 as its base, is the system in general use, and the one which we shall consider in this chapter.

Any number has a corresponding logarithm, and this logarithm is fixed for that particular number. For example, the logarithm of 3 is 0.4771 and always has that value in this system. Some other logarithms are:

Logarithm of 17 is 1.2304
Logarithm of 200 is 2.3010
Logarithm of 983 is 2.9926

132. Mantissas and Characteristics.—*Every logarithm is made up of two distinct parts, the part to the right of the decimal point called the mantissa, and the part to the left of the decimal point called the characteristic.* In the logarithm of 3, mentioned above, the mantissa is .4771, and the characteristic is 0. The mantissa is determined by the significant figures in the number, without regard to the decimal point, and the characteristic is determined by the location of the decimal point in the number.

To obtain the characteristic, the following simple method is used. If there is one figure to the left of the decimal point in the number, the characteristic of its logarithm is 0; if there are two figures, the characteristic is 1; three figures, the characteristic is 2; etc. *In other words, the characteristic is always one less than the number of figures to the left of the decimal point in the number.*

To illustrate this rule, let us find the logarithms of the numbers, 3, 30, 300, and 3000. The mantissa for all four logarithms is .4771, since the significant figure without reference to the decimal point is 3 in all the numbers. Applying the above rule for the

characteristic, it is evident that the characteristic for the log of 3 is 0; of 30 is 1; of 300 is 2; and of 3000 is 3. Now combining the characteristics and mantissas, we get

$$\begin{aligned} \text{Log } 3 &= 0.4771 \\ \text{Log } 30 &= 1.4771 \\ \text{Log } 300 &= 2.4771 \\ \text{Log } 3000 &= 3.4771 \end{aligned}$$

133. Use of Logarithmic Tables.—A table of four-place logarithms for numbers from 1 to 1000 is given on pages 241 and 242. Larger tables are available in various publications and textbooks. The tables give only the mantissas of the logarithms. The characteristics must be determined by inspection. As stated before, the mantissas depend solely upon the significant figures in the number without any regard for the decimal point. Figures 95 and 96, showing sections of the logarithmic tables, demonstrate clearly the method of finding the mantissas.

	0	1	2	3	4	5	6	7	8	9	Proportional parts		
											1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	**0792**	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24

FIG. 95.—Location of mantissa for 120.

Example:

Find the log of 120.

The significant figures are "12." Figure 95 illustrates how to find the mantissa from the tables. At the left-hand edge of the table are the numbers. The number 12 is indicated in heavy type. Its mantissa is in the first column to the right and is .0792 as indicated. The number 120 has three figures to the left of the decimal point, so the characteristic is 2.

Therefore, Log 120 = 2.0792

Referring to the tables, it will be noted that the number column at the left-hand edge contains only numbers of two sig-

nificant figures. The mantissas for numbers of three significant figures are found as illustrated in the following example.

Example:

Find the logs of the following numbers: 6, 56.3, and 5870.

In Fig. 96 the number 6 (or 60) is indicated in heavy type, and its mantissa, .7782, is in the first column to the right of it. The next number, 56.3, has three significant figures. The first two figures, 56, we locate in the left-hand column and the third figure, 3, in the row of figures from 0 to 9 at the top. The mantissa is found in the column beneath the 3 and opposite the 56, and its value .7505, as indicated. Similarly the mantissa, .7686, for the number 5870 is opposite the 58 and in the column beneath the 7 Knowing the mantissas we can write the logarithms.

$$\begin{aligned} \text{Log } 6 &= 0.7782 \\ \text{Log } 56.3 &= 1.7505 \\ \text{Log } 5870 &= 3.7686 \end{aligned}$$

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	**7505**	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	**7686**	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	**7782**	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6

FIG. 96.—Mantissas for 6, 56.3, and 5870.

Note: At this point, the student should solve Problems 311 to 315 inc. before proceeding with the balance of the text.

134. Interpolation.—Since the tables shown do not contain numbers of more than three significant figures, it is necessary to interpolate to find the logs of numbers containing four or more significant figures. For example, we will find the log of 2364. The nearest numbers to this value for which we can find the mantissas are 2370 and 2360, and their mantissas are .3747 and .3729. The mantissa for 2364 is somewhere between these two values. The difference between the numbers 2370 and 2360 is 10. The number 2364 is 4 greater than 2360; thus 2364 is $\frac{4}{10}$ of the difference, or $\frac{4}{10}$ of 10 greater than 2360. Similarly, the mantissa for the log of 2364 is greater than .3729 (the mantissa of 2360) by $\frac{4}{10}$ of the difference between their mantissas, .3747 and .3729. The difference between .3747 and .3729 is .0018, and $\frac{4}{10}$ of .0018

= .00072. Adding this correction to .3729 gives us the mantissa for the log of 2364.

$$\text{Mantissa for log } .2364 = .3729 + .0007 = .3736$$
$$\text{Log } .2364 = 3.3736$$

Example:

Find the log of 152,600.

The significant figures are 1526.

$$\text{Mantissa for log } 1530 = .1847$$
$$\text{Mantissa for log } 1520 = .1818$$
$$\text{Difference} = \frac{}{10} = .0029$$
$$\text{Correction for } 1526 = \frac{6}{10} \text{ of } .0029 = .00174$$
$$\text{Mantissa for } 1526 = .1818 + .0017 = .1835$$
$$\text{Log } 152{,}600 = 5.1835, \textit{Answer.}$$

To facilitate interpolation, a small table of corrections, or proportional parts, already worked out, is given at the right of the mantissas. By the use of these tables we can find the corrections very quickly. Let us take the example just worked—to find the log of 152,600. The first three figures are 152. The mantissa for 152 is opposite the "15" and below the "2." As found before, its value is .1818. The last significant figure in the number is 6, which we locate in the top row of figures in the correction table. Directly beneath it and opposite the mantissa, .1818, is the correction, 17, to be added to the mantissa. The new mantissa will then be .1818 + 17 = .1835. By inspection, the characteristic is 5, so the log of 152,600 is 5.1835.

In the case of a number having more than four significant figures, it is sufficiently accurate in this work to drop the additional figures, giving the fourth figure the nearest integral values. Thus, if we were to find the log of 964.83, we would drop the 3, leaving the four significant figures 964.8+, and find the log as before. Or if the number were 964.862, we would drop the 62 and increase the fourth figure by one, making it 964.9.

The student should check the following logarithms by the method just explained to be sure he understands it correctly.

$$\text{Log } 3\,764 = 0.5757$$
$$\text{Log } 67316.3 = 4.8281$$
$$\text{Log } 97 = 1.9868$$

135. Antilogarithms.—When the logarithm of a number is known and we wish to find the number, we call it finding the antilogarithm.

Suppose we have given the logarithm 3.4829. To find the antilogarithm is simply to find the number whose logarithm is 3.4829. By searching in the mantissa columns of the table, we find the mantissa .4829. It is found opposite the number 30 and in the column beneath the "4," so that the significant figures of the number are 304. The characteristic of 3 indicates four figures to the left of the decimal point. Pointing off four places we get the number, 3040, whose logarithm is 3.4829.

If the exact mantissa does not appear in the tables, find the mantissa which has the nearest value less than the given mantissa. This determines the first three figures of the number. The fourth figure is found by taking the difference between the two mantissas and locating this difference (or the nearest value thereto) in the correction table on a horizontal line with the mantissa. The small number at the top, vertically above the correction, is the desired fourth figure of the number.

Example:

Find the antilog of 1.4836.

The mantissa .4836 is not given in the table, and the nearest mantissa less than .4836 is .4829.

The number corresponding to the mantissa .4829 is 304. 4836 − 4829 = 7 (difference).

Looking in the correction table, opposite 4829 is the figure 7. The corresponding number above the 7 is 5. Therefore, antilog 1.4836 = 30.45.

136. Multiplication by Logarithms.—*To multiply by means of logarithms, add together the logarithms of the numbers to be multiplied, and the resulting sum is the logarithm of the product. The product is then obtained by finding the antilog of the sum.*

Examples:

1. $4 \times 5 \times 3$ = what?

Log 4 + Log 5 + Log 3 = Log of Answer.

Log 4 = 0.6021
Log 5 = 0.6990
Log 3 = 0.4771
Log of answer = 1.7782
Antilog 1.7782 = 60, *Answer.*

Check.—$4 \times 5 \times 3 = 60$

2. Multiply 675.9 by 73.94.

Log 675.9 = 2.8299
Log 73.94 = 1.8688
Log of product = 4.6987
Antilog 4.6987 = 49,970, *Answer.*

137. Division by Logarithms.—Division is the reverse of multiplication, and so in division by logarithms the log of the divisor is subtracted from the log of the dividend to obtain the log of the quotient.

Examples:

1. Divide 42 by 3.

$$\begin{aligned} \text{Log } 42 - \log 3 &= \text{Log of Answer.} \\ \text{Log } 42 &= 1.6232 \\ \text{Log } 3 &= 0.4771 \\ \text{Log of quotient} &= 1.1461 \\ \text{Antilog } 1.1461 &= 14, \textit{Answer.} \end{aligned}$$

2. Solve the following using logarithms:

$$\frac{594 \times 6.2 \times 2000}{42.1 \times 27{,}432} = ?$$

(Multiply the numbers in the denominator and numerator separately and then divide.)

$$\begin{aligned} \text{Log } 594 &= 2.7738 & \text{Log } 42.1 &= 1.6243 \\ \text{Log } 6.2 &= 0.7924 & \text{Log } 27{,}432 &= 4.4383 \\ \text{Log } 2000 &= 3.3010 & \text{Log of denominator} &= 6.0626 \\ \text{Log of numerator} &= 6.8672 \end{aligned}$$

$$\begin{aligned} \text{Log of numerator} &= 6.8672 \\ \text{Log of denominator} &= 6.0626 \\ \text{Difference} &= 0.8046, \text{ log of answer} \\ \text{Antilog } 0.8046 &= 6.377, \textit{Answer.} \end{aligned}$$

138. Roots and Powers.—The calculation of roots and powers of numbers is simplified considerably by the use of logarithms. To obtain any root of a number, simply divide the log of the number by the figure which indicates the root. The quotient thus obtained is the log of the root desired, and its antilog is the root.

Example:

Find $\sqrt[4]{281}$.

$$\begin{aligned} \text{Log } 281 &= 2.4487 \\ 2.4487 \div 4 &= .6122, \text{ log of answer} \\ \text{Antilog } .6122 &= 4.095, \textit{Answer.} \\ \text{Therefore, } \sqrt[4]{281} &= 4.095 \end{aligned}$$

To obtain any desired power of a number, multiply the log of the number by the figure which indicates the power. The product is the log of the desired power, and its antilog is the power.

Example:

Find the value of 62^2.

$$\begin{aligned} \text{Log } 62 &= 1.7924 \\ 2 \times 1.7924 &= 3.5848, \text{ log of answer} \\ \text{Antilog } 3.5848 &= 3844 \\ \text{Therefore, } 62^2 &= 3844 \end{aligned}$$

139. Logarithms of Decimals.—Previous to this we have not considered the logarithms of numbers whose values are less than 1; in other words, decimal fractions. The logarithms of decimals, such as .03, .0024, etc., are somewhat more complicated since the characteristic becomes less than zero or what is called a negative number.

A negative number is that obtained by subtracting any number from another smaller than itself, as, for instance, $4 - 6 = -2$. The negative result is read as "minus 2."

The mantissas for the logs of decimals are obtained exactly as they would be for any other number, since the decimal point does not affect the mantissa. The mantissa always has a positive value.

As stated before, the characteristic is negative, and its value is one more than the number of ciphers, or zeros, immediately following the decimal point, that is, between the decimal point and the first significant figure. Thus, the mantissa for the log of .03 is .4771 and the characteristic is -2 (minus 2) since there is one cipher immediately following the decimal point. The log of .03 would thus be $-2 + .4771$. This method of writing the characteristic makes it rather difficult to use, so it is customary to substitute for the negative characteristic an equivalent subtraction.

Instead of -1, we can write $9 - 10$.
Instead of -2, we can write $8 - 10$.
Instead of -3, we can write $7 - 10$, etc.

The mantissa and the characteristic are then combined as follows

$$\begin{aligned} \text{Characteristic for log } .03 &= -2, \text{ or } 8 - 10 \\ \text{Mantissa for log } .03 &= .4771 \\ \text{Log } .03 &= 8.4771 - 10 \end{aligned}$$

The following logs will illustrate the point:

$$\begin{aligned} \text{Log } .007 &= 7.8451 - 10 \quad (\text{characteristic is } -3) \\ \text{Log } .635 &= 9.8028 - 10 \quad (\text{characteristic is } -1) \\ \text{Log } .00041 &= 6.6128 - 10 \quad (\text{characteristic is } -4) \end{aligned}$$

To make clear how these are used in multiplication, division, etc. we will work the following examples.

Examples:

1. Multiply $327.6 \times .0729 \times .0028$.

$$\begin{aligned} \text{Log } 327.6 &= 2.5153 \\ \text{Log } .0729 &= 8.8627 - 10 \\ \text{Log } .0028 &= 7.4472 - 10 \\ \text{Log of product} &= 18.8252 - 20 \end{aligned}$$

The characteristic $18 - 20 = -2$, which is the same as $8 - 10$, and indicates that there should be one cipher after the decimal point. The mantissa is .8252, and we find the number as explained in Art. 132.

Antilog $18.8252 - 20 = .06687$, *Answer.*

2. Divide 825 by .00872.

$$\begin{aligned} \text{Log } 825. &= 2.9165 \\ \text{Log } .00872 &= 7.9405 - 10 \end{aligned}$$

To divide we must subtract the log of .00872 from the log of 825, which it is evident we cannot do in the form they are now written. However, by adding $10 - 10$ to the log of 825, we do not change the value of the log and make it possible to perform the subtraction.

$$\begin{aligned} \text{Log } 825 &= 2.9165, \text{ or } 12.9165 - 10 \\ \text{Log } .00872 &= \qquad 7.9405 - 10 \\ \text{Log of quotient} &= 4.9760 \end{aligned}$$

Antilog $4.9760 = 94{,}620$, *Answer.*

3. Divide .000276 by 6930.

$$\begin{aligned} \text{Log } .000276 &= 6.4409 - 10 \\ \text{Log } .6930 &= 3.8407 \\ \text{Log of quotient} &= 2.6002 - 10 \end{aligned}$$

The characteristic is $2 - 10$, or -8, which indicates 7 ciphers (one less than 8) after the decimal point.

Antilog $2.6002 - 10 = .00000003983$, *Answer.*

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Proportional parts								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	9	13	17	21	26	30	34	38
											4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	12	15	19	23	27	31	35
											4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	11	14	18	21	25	28	32
											3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	7	10	13	16	20	23	26	30
											3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	19	22	25	28
											3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	9	11	14	17	20	23	26
											3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	14	16	19	22	24
											3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	3	5	8	10	13	15	18	20	23
											2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
											2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
											2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Proportional parts 1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 4 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8167	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 4	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 4	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9509	9614	9519	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

PROBLEMS

Work the following problems with logarithms:

311. Find the logarithm of 13.

312. Find the logarithms of 6 and 9.

313. Find the logarithm of 82.

314. Find the logarithm of 825.

315. Find the logarithm of 126.

316. Find the antilog of 3.0569.

317. Find the product of 343×25.

318. Find the antilog of 2.9119.

319. Multiply $63 \times 180 \times 4.3$.

320. Divide 725 by 72.

321.

(*a*) $42 \times 186 = ?$
(*b*) $73 \times 792 = ?$
(*c*) $41.5 \times 62.3 = ?$
(*d*) $29.7 \div 289.1 = ?$
(*e*) $42.8 \div 21.7 = ?$
(*f*) $794 \div 28.75 = ?$

322. (*a*) $\dfrac{29 \times 43 \times 7}{46 \times 49 \times .5} = ?$

(*b*) $\dfrac{\frac{1}{2} + .721 + 74}{72 \div 8.46} = ?$

(*c*) $\sqrt{\dfrac{29 \times 41}{73 \times 8}} = ?$

(*d*) $\sqrt[3]{741} = ?$

(*e*) $6^4 \times 48.2 = ?$

(*f*) $3^5 \times 4.1^2 \times \sqrt[3]{927} = ?$

323. Calculate the following:

(*a*) $93{,}760 \times 51 = ?$
(*b*) $10.243 \div 241 = ?$

Note.—It is necessary to add $10 - 10$ to the log of 10.243 in (*b*) before subtracting the logs.

324. The indicated horsepower of a steam engine is given by the formula $\text{hp.} = \dfrac{Plan}{33{,}000}$ (see Chap. XV). Calculate the horsepower of an engine having a 24-in. stroke, 12-in. cylinder diameter, m.e.p. of 60 lb. per square inch, and running at 90 r.p.m.

325. A cubical tank is to contain 422 cu. ft. of water when full. Calculate the dimensions of the tank.

326. Determine the speed of a belt running on a pulley 1.25 ft. in diameter, the pulley making 545 r.p.m.

CHAPTER XX

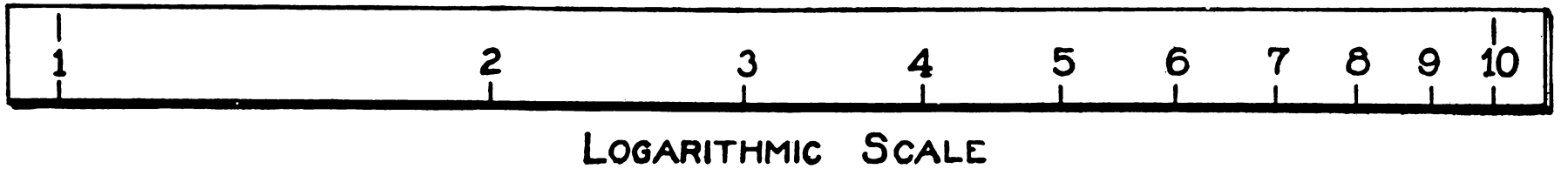
THE SLIDE RULE

140. Use of the Slide Rule.—The slide rule is an instrument for making rapid calculations in operations involving multiplication, division, powers, and roots. It is based upon the principle of logarithms, and, in fact, merely provides a shorter method of calculating by the use of logarithms. Addition and subtraction cannot be performed on the slide rule.

141. Logarithmic Scale.—The cumbersome table of logarithms is replaced in the slide rule by the logarithmic scale. Figure 97 shows such a scale in its simplest form, made to a length of 10 in., and below it an ordinary 10-in. rule. It will be noted that the numbered divisions on the logarithmic scale do not come at equal intervals; also that the beginning of the scale is numbered 1 instead of 0, and that each succeeding division is smaller than the one before it. The 10-in. rule measures inches, and the length from the zero point to any of the numbered divisions is that number of inches. Thus from 0 to 4 is 4 in. The logarithmic scale, however, measures logarithms, and the length from the end division, at 1, to any other division is proportional to the logarithm of that division. Thus the length from 1 to 2 is proportional to the logarithm of 2; the length from 1 to 3 is proportional to the logarithm of 3, and so on. The first point on the scale is numbered 1 since the logarithm of 1 is 0, and division No. 1 has zero length.

If we placed the 10-in. rule alongside the logarithmic scale and measured accurately the lengths from 1 to 2, 1 to 3, and 1 to 4, we would find that they measure 3.020 in., 4.771 in., and 6.040 in. respectively, as indicated in Fig. 98.

The logarithms of 2, 3, and 4, from the tables, are .3020, .4771, and .6040. By comparing the logarithms with the measured lengths we see that the lengths from 1 to 2, 1 to 3, and 1 to 4 are directly proportional to the logarithms of 2, 3, and 4, since each measured length, in inches, is exactly 10 times the value of the logarithm.



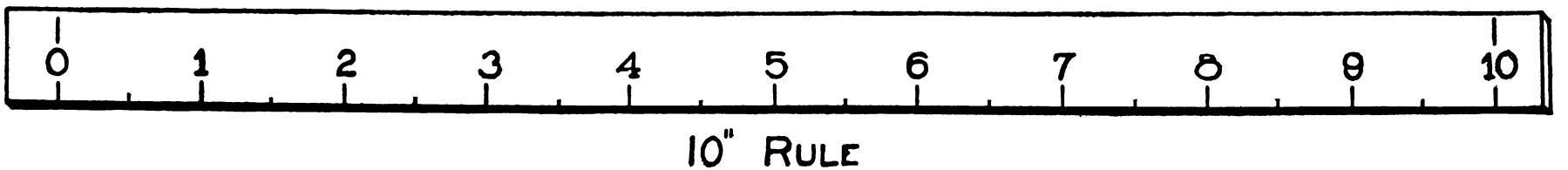


FIG. 97.—Simple logarithmic scale 10" long.

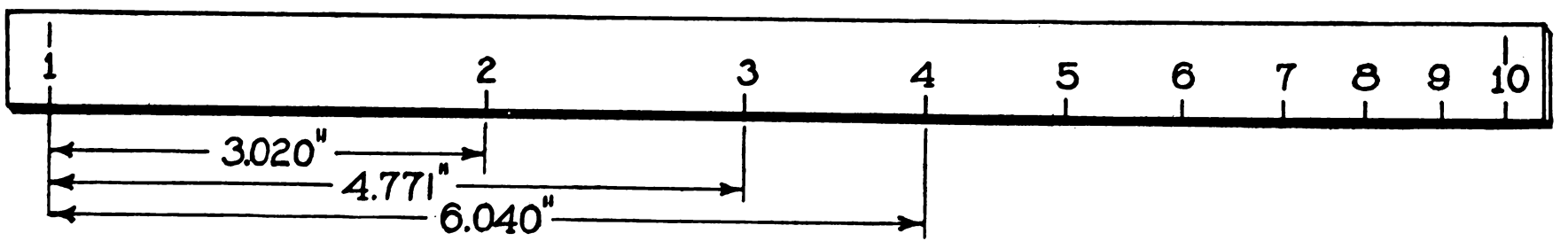


FIG. 98.—Measurement of 2, 3, and 4 on logarithmic scale.

142. Calculation by the Logarithmic Scale.—The same principles apply to the use of the logarithmic scale as to the actual logarithms; namely, to multiply, add the logarithms, and to divide, subtract the logarithms. The only difference is that logarithms on the scale are represented by lengths instead of numbers, and we add and subtract lengths.

To multiply two numbers, such as 2×3, simply lay off from the scale the length from 1 to 2 and add to it the length from 1 to 3, exactly as though you had a foot rule and laid off 2 in. and then added 3 in. to it. In Fig. 99 we have multiplied 2×3, step by step. In the first operation the scale is laid upon the paper and the length from 1 to 2 marked off, which distance is the logarithm of 2. In the second operation the scale is moved to the right and the length from 1 to 3, or the logarithm 3, is added to the length already measured, thus giving the sum of the two lengths, or log 2 + log 3.

From our knowledge of logarithms we know that log 2 + log 3 = log of product. We, therefore, take our logarithmic scale, in the third operation, and measuring the total length (log 2 + log 3), we find that it equals the length from 1 to 6, which is the log of 6.

$$\text{Log } 2 + \text{Log } 3 = \text{Log } 6$$

Therefore,

$$2 \times 3 = 6, \textit{ Answer.}$$

Since in division we subtract logarithms, the process of dividing one number by another with the logarithmic scale is just the reverse of multiplying. To illustrate, let us divide 6 by 3. In this case we must subtract the log of 3 from the log of 6 to obtain the log of the quotient, which will require three operations, as shown in Fig. 100.

First operation: Mark off the length from 1 to 6 which is the log of 6.

Second operation: Move the scale to the right and subtract the length 1 to 3, or the log 3, from the log 6.

Third operation: The difference found in the second operation is the log of the quotient, so we move the scale to the left and measure the log quotient. As the scale indicates, the log quotient equals the log 2; therefore,

$$\text{Log } 6 - \text{Log } 3 = \text{Log } 2$$

$$6 \div 3 = 2, \textit{ Answer.}$$

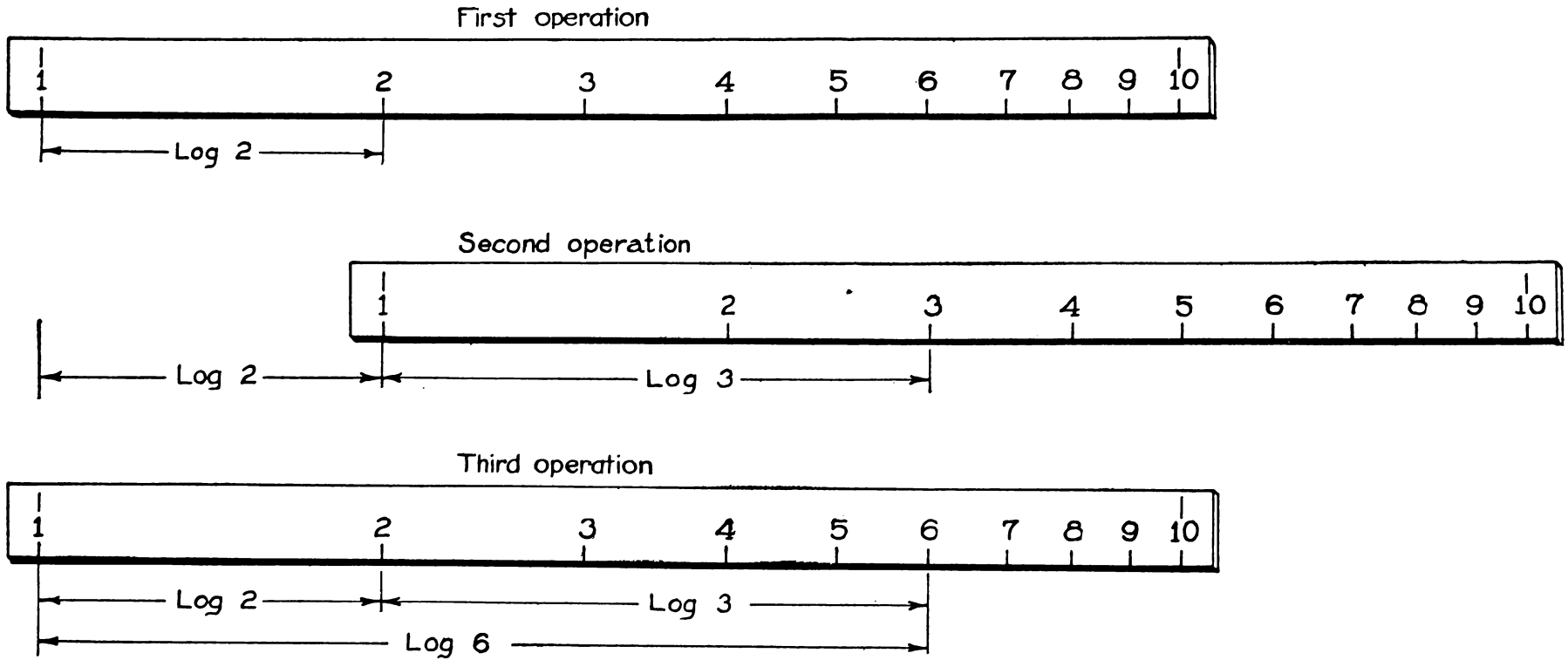


FIG. 99.—Operations in multiplying 2 by 3 by use of logarithmic scale

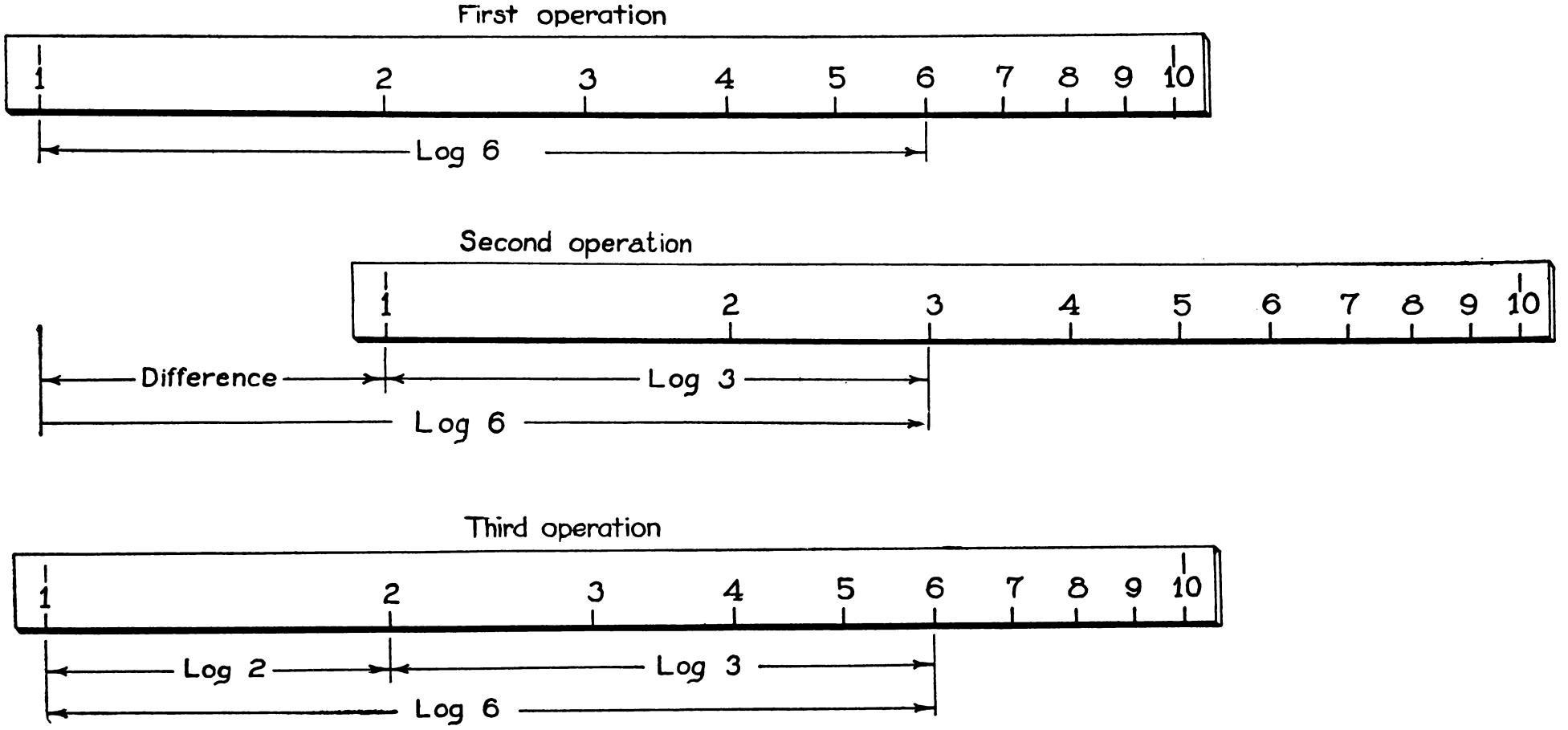


FIG. 100.—Operations in dividing 6 by 3, using logarithmic scale.

Thus by the aid of the logarithmic scale we have performed the multiplication and division of two numbers without having to consider the actual numerical value of the logarithms themselves.

The single logarithmic scale which we have just described and used is the most elementary form of slide rule. A similar scale was used for many years before the modern type of instrument was developed.

143. The Simple Slide Rule.—If we possessed two logarithmic scales exactly alike, the three operations required for multiplication or division, Figs. 99 and 100, could be combined into a single operation, thereby effecting a considerable saving in time.

Figure 101 illustrates the method of performing the multiplication, 2×3, with two logarithmic scales. The log of 2 and the log of 3 are added by placing the two scales together with the 1 of scale C opposite the 2 of scale D. The total length (log 2 + log 3) can then be read from the scale D at the point which is opposite the 3 of scale C. As the figure shows, the sixth division of scale D is opposite the 3 of scale C, and it follows that

$$\text{Log } 2 + \text{Log } 3 = \text{Log } 6$$
$$2 \times 3 = 6, \textit{Answer.}$$

To perform a division, such as $8 \div 4$, the log 4 is subtracted from the log 8 by placing the 4 of scale C above the 8 of scale D, as in Fig. 102. The difference, which is the log of the quotient. is measured on the scale D.

$$\text{Log } 8 - \text{Log } 4 = \text{Log } 2$$
$$8 \div 4 = 2, \textit{Answer.}$$

The question now arises, how shall we multiply numbers other than those from 1 to 10, for example, 2×400? Apparently there is no such value as 400 on the scale.

In Fig. 103 we have extended the two logarithmic scales, C and D, so as to contain numbers from 1 to 1000, and with these scales have multiplied 2×400 by the same method that we used with the smaller scales.

$$\text{Log } 2 + \text{Log } 400 = \text{Log } 800$$
$$2 \times 400 = 800, \textit{Answer.}$$

It is evident from the figure that each of these scales could be divided into three sections, the lengths from 1 to 10, from 10 to 100, and from 100 to 1000, and also that these three sections

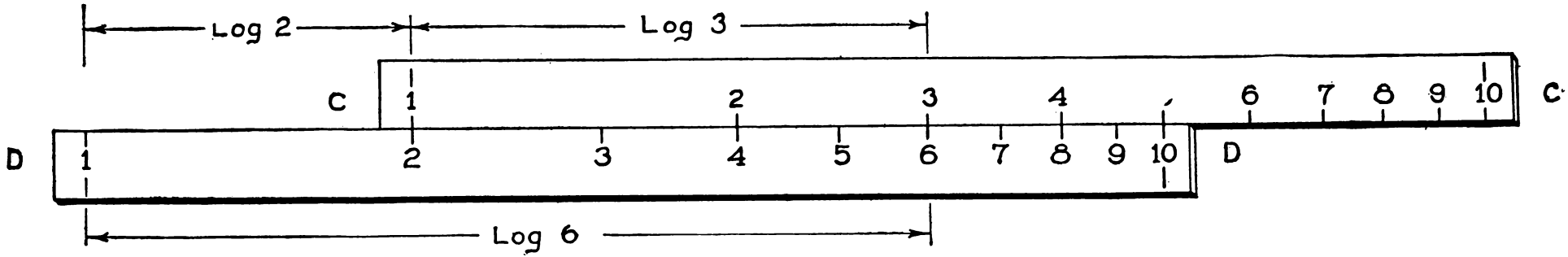


FIG. 101.—Method of multiplying 2 by 3, using two logarithmic scales.

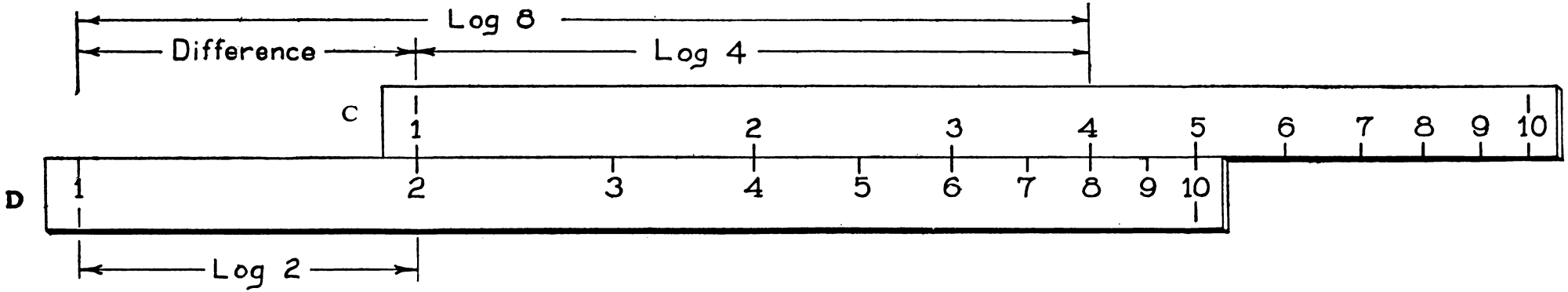


FIG. 102.—Method of dividing 8 by 4, using two logarithmic scales.

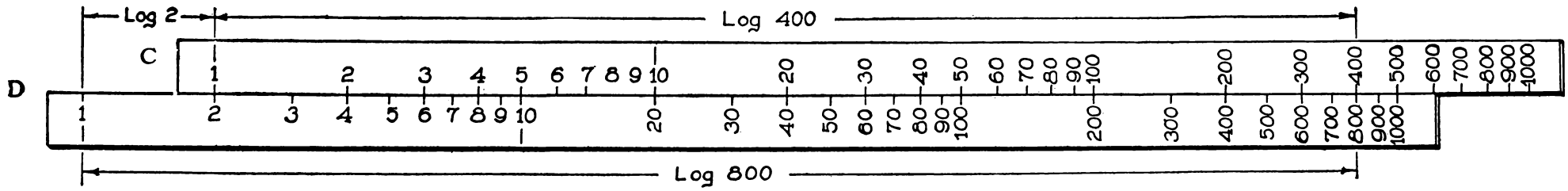


FIG. 103.—Logarithmic scales extended so as to contain numbers from 1 to 1000.

are exact repetitions of each other, except for the number of ciphers following the figures on each. As a matter of fact, no matter how far we extend the scale either to the right or to the left, there will be a continuation of repeated sections, each exactly the same as the first.

Let us consider the multiplication illustrated in Fig. 103. The answer, 800, is found on the *D* scale beneath the 400 on *C*. At the same time, it will be noted that 80 on *D* comes under 40 on *C*, and 8 on *D* comes under 4 on *C*. In other words, the actual digit, 8, in the answer can be found beneath the 4 and the complete answer obtained by adding 2 ciphers to the 8, making it 800. The reason for this is at once apparent. Since all sections of the scale are exactly alike, it follows that the section from 1 to 10 may also serve for any other section, and any point on the section from 1 to 10, such as 4, 7, etc., may also be read as .4 or .7, 40 or 70, 400 or 700, 4000 or 7000, etc. *The readings on the scale simply supply the actual digits in the number, and we may assume any position of the decimal point that we desire.* Then to multiply 2×400 with the simple scale, we set the 1 of the scale *C* above the 2, as in Fig. 104, and opposite the 4 of scale *C*, which we read as 400, is found the answer, 8, on the scale *D*, which we read as 800.

The two logarithmic scales used in this manner form the basis of the modern slide rule. However, to be of practical use, the logarithmic scale must be graduated between divisions so that intermediate points on the scale can be read directly or estimated. Similarly a foot rule would be of no use to the machinist if it had only the inch marks upon it, without the $\frac{1}{8}$-, $\frac{1}{16}$-, and $\frac{1}{32}$-in. divisions which enable him to read intermediate points, such as $5\frac{7}{16}$ in., $7\frac{1}{8}$ in., etc. Figure 105 illustrates the logarithmic scale graduated between divisions. The space between each numbered division is divided into 10, or multiples of 10, small divisions. Intermediate points are read as illustrated in the figure.

To multiply two numbers, such as 2.25×3.2, add the logarithms of these numbers as in Fig. 106 by placing the 1 of scale *C* above the point 2.25 of scale *D*. The product is then read from scale *D* opposite the point 3.2 of scale *C*.

$$\text{Log } 2.25 + \text{Log } 3.2 = \text{Log } 7.2$$

$$2.25 \times 3.2 = 7.2, \textit{Answer.}$$

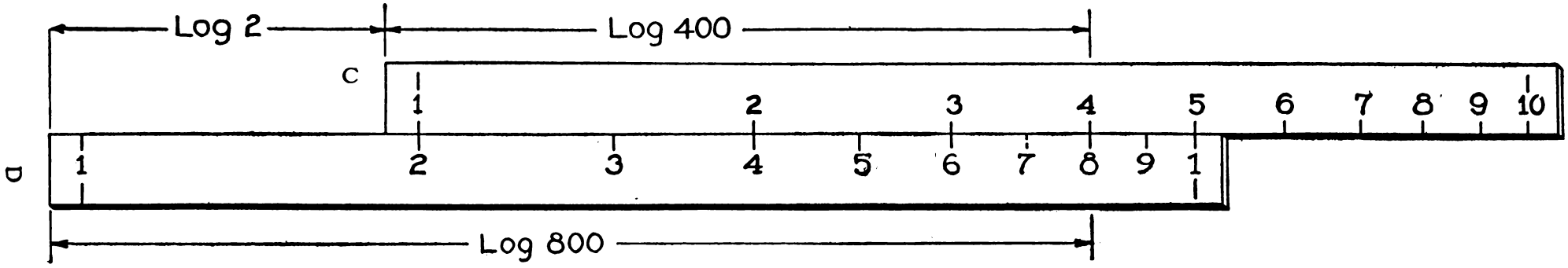


FIG. 104.—Multiplication of 400 by 2.

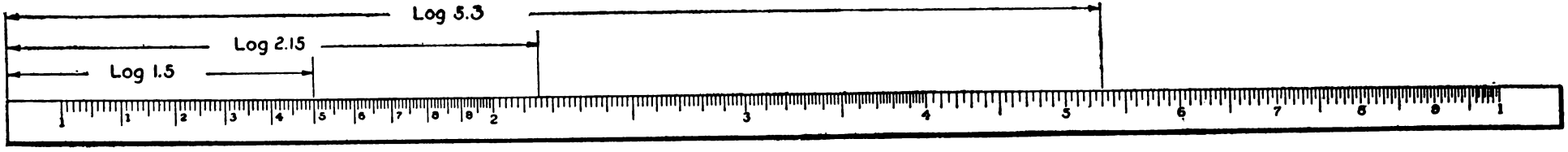


FIG. 105.—Logarithmic scale graduated between main divisions.

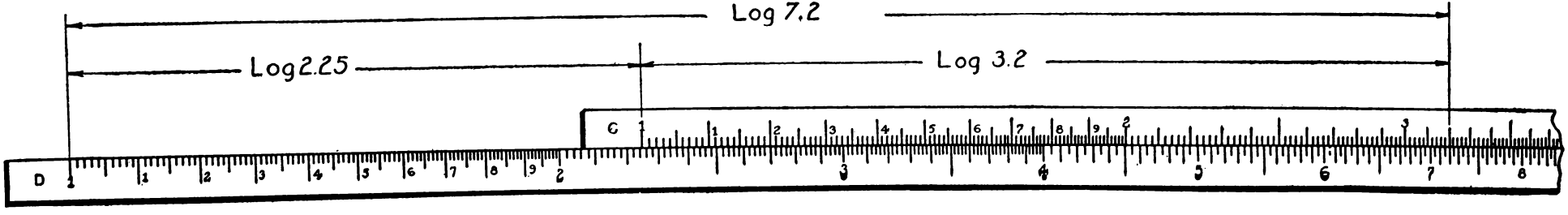


FIG. 106.—Multiplication of 2.25 by 3.2, using graduated logarithmic scales.

The same setting of the scales would also be used to multiply 2250×32, $.0225 \times 320$, etc. The respective answers in these two cases would be read 72,000 and 7.2.

144. Description of a Slide Rule.—In the preceding paragraphs we have described the logarithmic scale and the principles underlying its use as a calculator. We will now take up its practical application to the slide rule.

One of the many useful types of slide rules for ordinary purposes is shown in Fig. 107. There are four scales on the rule, *A*, *B*, *C*, and *D*, all of which are logarithmic scales. The two upper scales, *A* and *B*, both consist of two logarithmic scales

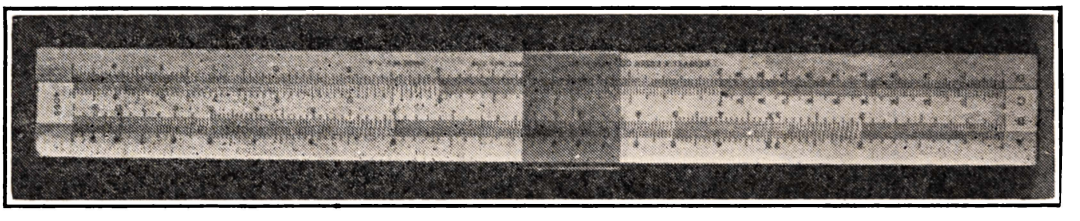


FIG. 107.—Student slide rule.

each of which is one-half the length of the scales *C* and *D*. Scales *C* and *D* are exactly the same as the two scales of Fig. 106 and in general these two scales are used for all the ordinary operations of multiplication and division. The two center scales, *B* and *C*, are on the slide, which permits of their being moved back and forth. In this manner scale *C* can be placed in any desired position above scale *D*. The glass runner is used to locate points on the scales and to hold readings while the slide is being moved.

145. Slide Rule Settings.—The fundamental principles underlying the use of the logarithmic scale are all that one really needs to know in order to learn the operation of the slide rule. There are certain features about its use, however, which require some explanation to a beginner.

One of the first difficulties many students encounter is that of reading the points on the scales correctly. The first significant figure of the number corresponds to the large numbered divisions on the scale; the second digit of the number corresponds to tenths of a division; and the third and fourth digits indicate the fraction of a tenth division, which it is usually necessary to estimate. The tenths of a division are generally subdivided to assist in locating points, but the tenth marks between the first and second divisions, only, are numbered. Figure 108 indicates

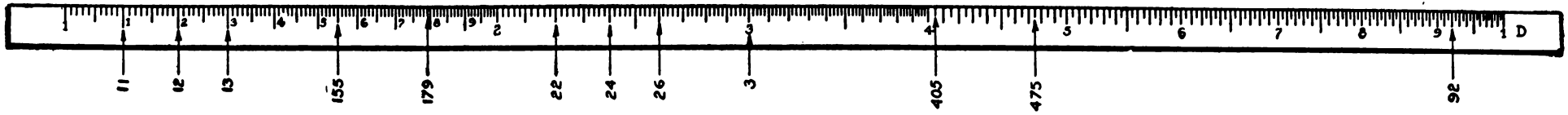


FIG. 108.—Method of locating numbers on slide rule scale.

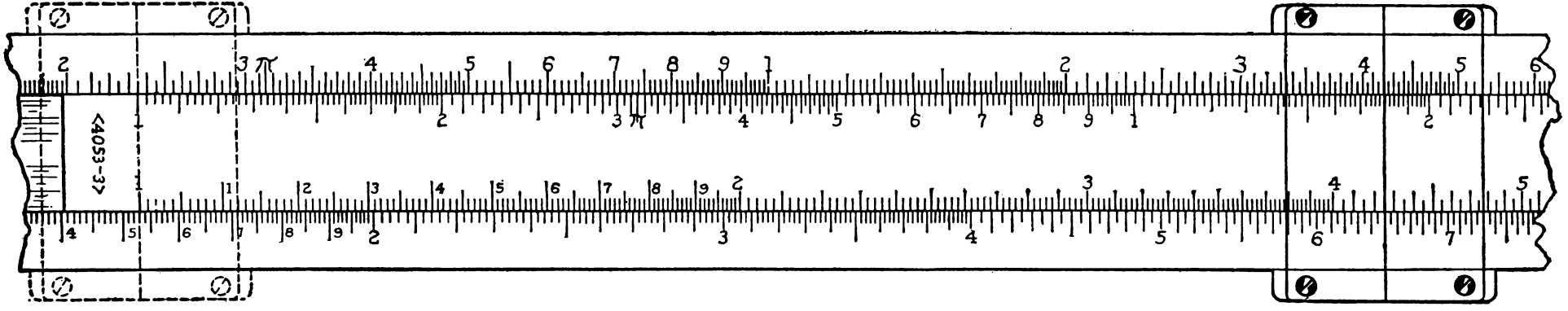


FIG. 109.—Slide rule setting for multiplication of 153 by 0.0425.

the location of several points on scale *D*. The student should study these readings until he is perfectly certain that he understands them. The ordinary 10-in. logarithmic scale can be read to only three places accurately with the fourth place estimated.

To multiply, locate the point on scale *D* corresponding to one of the numbers to be multiplied; set the unit figure, or 1, of scale *C* opposite that point; then locate the other number on scale *C*, and directly opposite on scale *D* will be found the answer. Lastly, determine the proper place for the decimal point in the answer.

Examples:

1. Multiply 153 × .0425.

First, locate the point 153 on scale *D* and place the 1 of scale *C* directly opposite. (Placing the hair line of the glass indicator over the point 153 assists materially in locating the 1 of scale *C* properly.)

Second, find the point 425 on scale *C* and read the answer on scale *D* directly opposite this point. (Again by placing the hair line of the indicator over the point, 425, the reading of the answer is facilitated.) The reading thus found is 65. Since .0425 is approximately $\frac{4}{100}$ or $\frac{1}{25}$, and $\frac{1}{25}$ of 150 = 6, the slide rule answer must evidently be 6.5.

Figure 109 illustrates the slide rule setting for this example. The dotted lines indicates the first position of the indicator. If we carry out the multiplication 153 × .0425 by "long hand," we will find that the exact answer is 6.5025, which is very close to the slide rule result. It must be emphasized, however, that the slide rule cannot be read sufficiently close to give absolutely exact answers, but the error is small and for ordinary calculations can be neglected.

2. Multiply 4.333 × 31,500.

First, locate the point 4333 on scale *D* and place the 1 of scale *C* opposite Immediately we see that the point 315 on scale *C* falls beyond scale *D*, and apparently we need an extension on scale *D* to read the answer. We overcome this difficulty by placing the right-hand end, or 1, of scale *C* opposite the 433. This corresponds exactly to placing an extension on scale *D*.

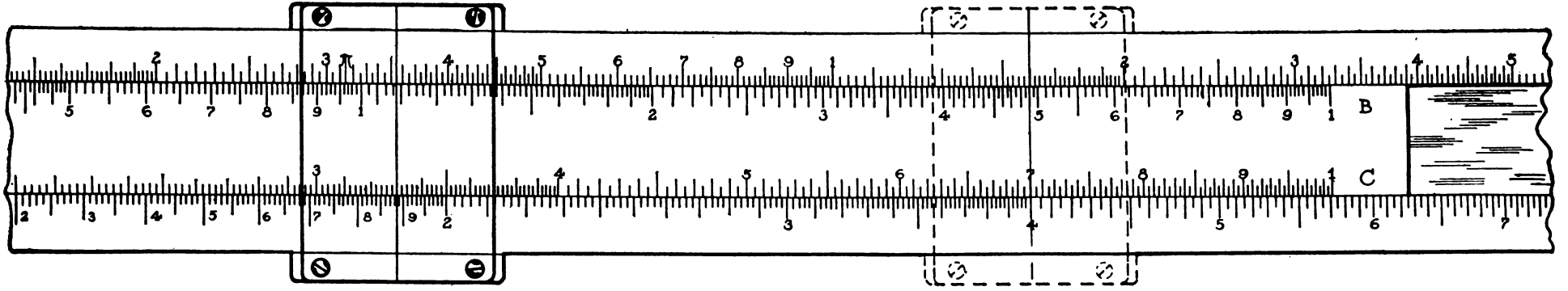
Second, locate the point 315 on scale *C* and read the answer directly opposite on scale *D*. The slide rule reading is about 1365. Roughly, 4.333 × 31,500 is about 4 × 30,000 = 120,000, which would indicate that the answer as read from the scale is 136,500.

To divide, locate the point on scale *D* corresponding to the dividend; then locate the point on scale *C* corresponding to the divisor, and place this point directly opposite the dividend. The quotient will be found on scale *D* opposite the unit figure, or 1, of scale *C*. This may be either end of scale *C*.

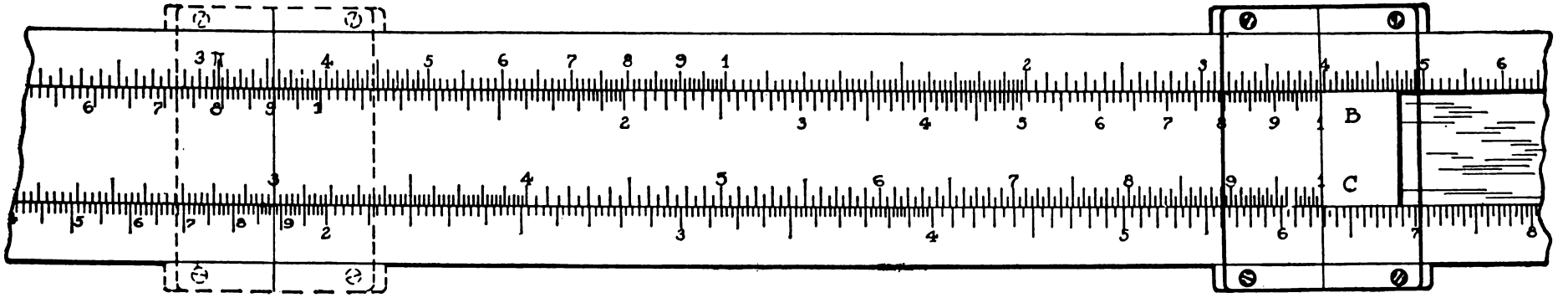
3. Divide .0585 by 5.41.

First, locate 585 on scale *D*. This is usually done with the indicator.

Second, place the point 541 of scale *C* opposite the point 585 and read the quotient on scale *D* opposite the 1 of scale *C*. In this case it falls opposite the left-hand end of *C* and the reading is 108. By inspection, we



(*A*)



(*B*)

FIG. 110.—Slide rule settings for solution of formulas for width of belting, Example 5, Art. 145.

can see that the answer is about .01, which determines the quotient to be .0108.

4. Divide 375 by 52.

First, locate 375 on scale *D*.

Second, place the point 52 of scale *C* opposite 375 of scale *D* and read the quotient on scale *D* opposite the 1 of scale *C*. In this case it comes opposite the right-hand end of *C* and the reading is 721. In round numbers 52 goes into 375 about 7 times; therefore, the quotient must be 7.21.

Where a great deal of calculating is done, much time can be saved by placing the scales so as to require the minimum number of settings. This is especially true in the calculation of formulas.

5. The formula for width of belting is $W = \frac{H \times 33{,}000}{P \times V}$. Given, $P = 70$ lb., $V = 3000$ ft. per min., $H = 40$. Calculate the width of belt required.

$$W = \frac{40 \times 33{,}000}{70 \times 3{,}000}$$

The complete calculation can be made with two moves of the slide as follows:

Divide 40 by 70 by placing the 7 of scale *C* opposite the 4 of scale *D*. The quotient could be read opposite the 1 of scale *C*, but we do not read it since we immediately multiply it by 33,000. To do this we must place the 1 of scale *C* opposite the quotient, but it is already in this position, so all that is necessary is to locate 33 on scale *C*. Opposite this point on scale *D* is the product of $\frac{40}{70} \times 33{,}000$. Place the hair line of the indicator over this product, without reading it, and complete the calculation by dividing by 3000. This is done by placing the 3 of scale *C* opposite the product of $\frac{40}{70} \times 33{,}000$, and reading the answer on scale *D* opposite the 1 of scale *C*. The first setting is indicated in Fig. 110A, which makes the calculation $\frac{40}{70} \times 33{,}000$, and the second setting in Fig. 110B completes the calculation. The final reading on the *D* scale is 628. Since $33{,}000 \div 3{,}000 = 11$, and $\frac{40}{70}$ is about $\frac{1}{2}$, the answer is obviously 6.28.

6. The velocity in feet per minute of the rim of a flywheel is $V = \frac{\pi \times d \times N}{12}$, where d is in inches. Given, $d = 8$, $N = 500$, find V.

$$V = \frac{3.1416 \times 8 \times 500}{12}$$

First multiply 3.1416 by 8. This necessitates placing the right-hand end, or 1, of scale *C* opposite 3.1416 on *D*. Set the indicator over 8 on scale *C* to locate the product on *D* without reading it. Now divide this product by 12. To do this, place the 12 of *C* opposite, and locate the quotient on *D* by setting the indicator over the 1 of scale *C*. Multiply by 500, by placing the right-hand end of scale *C* opposite the quotient last obtained, and reading the point on *D* which falls opposite the 5 of scale *C*. Figure 111 illustrates the three settings required.

This reading is 1047. To locate the decimal point make an approximate calculation of the result, as has been done in previous examples. An approximate answer is 1000, which shows that the slide rule answer is 1047.

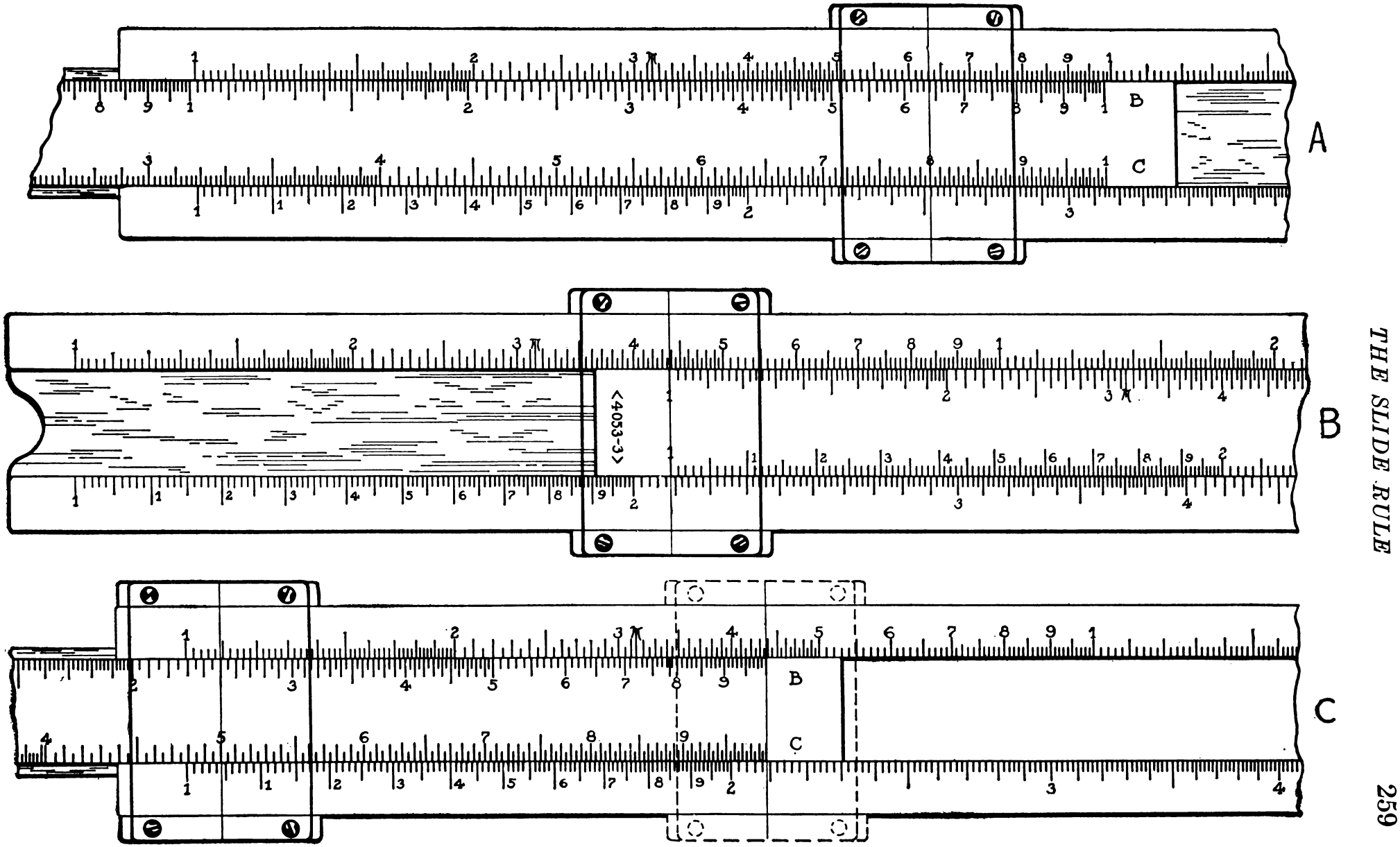


FIG. 111.—Slide rule settings for solution of Example 6, Art. 145.

146. Roots and Powers.—The square of any number on the *D* scale can be found directly opposite on the *A* scale, and for square roots the opposite is true. In finding square roots, however, we must be careful to use the proper scale on *A*. To find the square roots of numbers having an odd number of places to the left of the decimal point, use the left-hand scale on *A*, and for those with an even number of places to the left of the decimal point, use the right-hand scale. Similarly for numbers smaller than 1, that is, decimals, the square roots of those having an odd number of ciphers preceding the first significant figure are opposite the left-hand scale on *A*, and for those having an even number of ciphers, the square roots are opposite the right-hand scale.

Examples:

1. Find the squares of 24.5; 3.24; and .66.

To square 24.5, set the hair line of the indicator above 245 on *D*, and read the square under the hair line on the *A* scale. The reading on *A* is 6, and the answer is obviously 600, since 20 squared equals 400.

To square 3.24 and .66, set the indicator above these points on *D* and read their squares on *A*, directly opposite. The readings thus found are 105 and 436, respectively. Thus the square of 3.24 = 10.5, and the square of .66 = .436.

To locate the decimal points in these results we see that the square of 3.24 must be a little more than 3^2, which is 9, and less than 4^2, which is 16. Therefore, the answer must be 10.5. The third number, .66, is a little less than .7. The square of .7 is .49. Therefore, the square of .66 must be less than .49. From this we can see that the square of .66 is .436.

2. Find the square root of 64; 320; and .0049.

Sixty-four (64) has an even number of places to the left of the decimal point. Therefore, we set the indicator over the 64 on the right half of *A*, and read the square root on *D*, directly opposite. The answer is 8, as we know.

In 320 there are an odd number of places on the left of the decimal point so we use the left half of *A* for this number. Set the indicator above 32 on *A* and read the square root, 17.9, beneath on *D*.

In .0049 there are an even number of ciphers on the left of the 4, so we use the right half of *A*. The reading on *D*, opposite 49 on *A*, is 7. The square root of .0049 is then .07. The rule for locating the decimal point is given in Art. 71.

Higher powers of numbers may be found by multiplying the number by itself an equivalent number of times, or by multiplying together the lower powers of the number. The more complicated slide rules frequently have what is called the log-log scale, by means of which any power of a number may be obtained directly.

The cube of a number may be obtained in one setting by squaring the number and then multiplying the square by the number itself, using the *A* and *B* scales.

3. Find the cube of 28.

Square 28 by locating (with the indicator) the point on *A* which is opposite 28 on *D*. Multiply the square by 28, thus cubing it, by placing the 1 of scale *B* opposite the square of 28 and reading the answer on scale *A*, opposite the 28 on *B*. Figure 112 illustrates the setting. The reading on the slide rule is 219. The decimal point is determined by comparing it with the cube of 30 which is 27,000. Hence, we must annex two ciphers to the slide rule reading which gives us 21,900 as the answer.

The cube root can be found by reversing the operation. Three possible answers can be found in this manner. The correct one must be selected by a rough check.

4. Find the cube root of 42,600.

Set the indicator above the point 426 on *A*. Move the slide back and forth carefully until the reading on *B* beneath the hair line is exactly the same as the reading on scale *D* opposite the end, or 1, of scale *C*. Three such points can be found, and the following method is used to select the right one. Beginning at the decimal point in the number, divide it up into groups of three digits, in this manner,—42' 600'. If the number had been 4,260,000 it would be divided as follows: 4' 260' 000'. The nearest integral cube root of the first group of figures at the left of the number will be the first digit of the answer. Thus in the number 42' 600' the first group of digits is 42, and its nearest integral cube root is 3. Thus the first digit of the answer is 3, and the slide rule reading will fall somewhere between the third and fourth divisions on the scale. The reading thus found is 349, as indicated in Fig. 113. The number of groups of digits to the left of the decimal point in the number determines the number of digits to the left of the decimal point in the cube root of the number. In this case there are two groups of digits, so the cube root must be 34.9.

The same method applies to decimals, except that the groups of three digits fall to the right of the decimal point, and the corresponding number of places in the cube root will be at the right of the decimal point.

5. Find the cube root of .0000864.

First divide the number into groups, .'000 '086 '4. The nearest cube root of 86 is 4, which will be the first digit of the slide rule reading. Making the proper setting on the slide rule, we obtain the reading 442. For each group of three ciphers preceding the first significant figure in the number, there is a cipher preceding the first figure in the answer. In this case there is one group of three ciphers so the answer must be .0442.

Other powers and roots of numbers can be obtained by the use of logarithms as explained in the previous chapter. A scale of logarithms is given on the back of the slide by means of which the logarithms of numbers may be obtained without referring to the table. To use this scale, set the 1 of scale *C* opposite the number on scale *D*, then turn the slide rule over and read the logarithms on the center scale of the slide, opposite the mark.

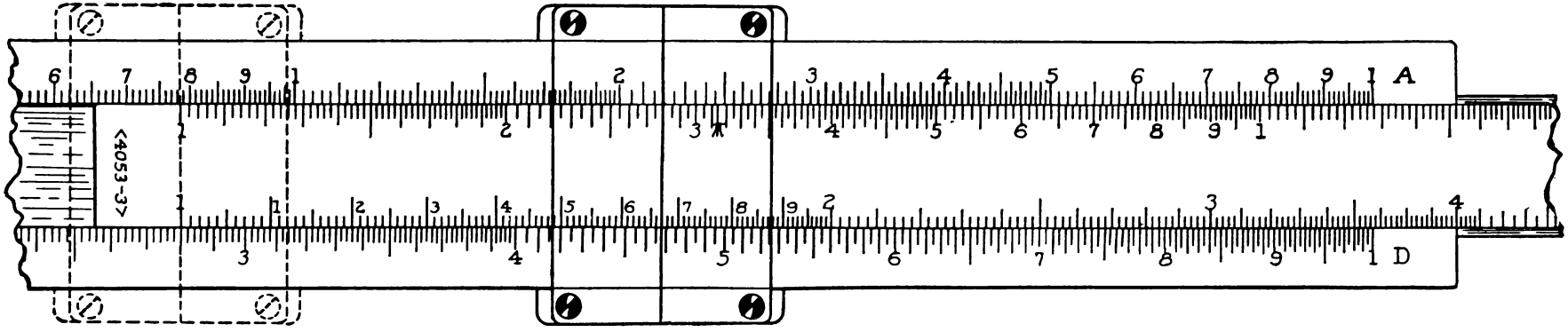


FIG. 112.—Slide rule setting for finding cube of 28.
(Note.—It is evident upon inspection that this setting also determines the cube root of 21.9. 21.900. etc.).

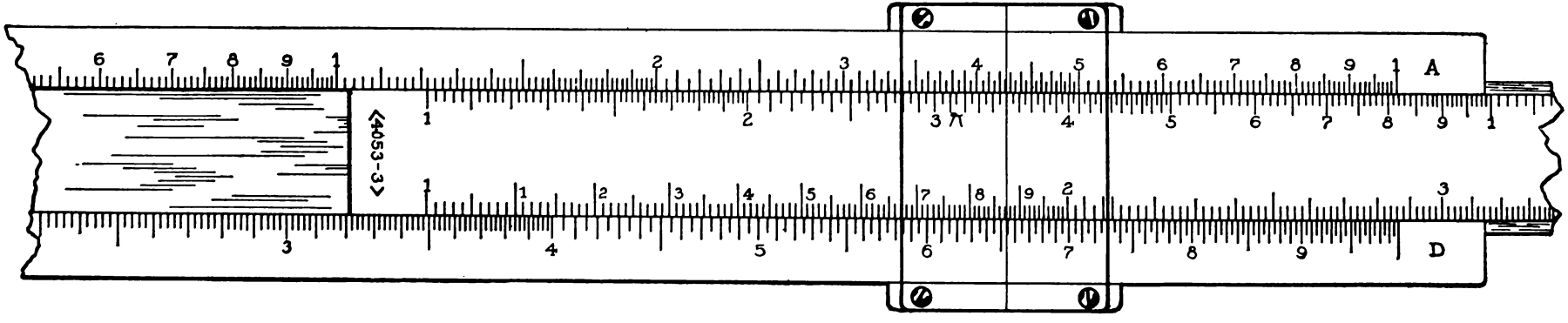


FIG. 113.—Setting for finding cube root of 42,600.

147. Decimal Points.—The determination of the decimal point provides the greatest source of error and trouble in slide rule work. The quickest method and the one most generally used is the simple expedient of estimating the answer by a rough calculation. In all the preceding examples the decimal point was determined in this manner. Surprising dexterity can be acquired in the approximation of answers by this method.

Example:

Calculate the following:

$$\frac{3.6 \times 920 \times .0013}{42 \times 640 \times .0005}$$

Making a rough cancellation, we see that .0005 goes into .0013 approximately 2 times, 640 into 920 about 1.5 times, and 3.6 into 42 about 12 times.

$$\frac{\overset{1.5}{\cancel{3.6}} \times \cancel{920} \times \overset{2}{\cancel{.0013}}}{\underset{12}{\cancel{42}} \times \cancel{640} \times \cancel{.0005}} = \frac{3}{12}, \text{ or } .25$$

Roughly, then, the answer is about .25. Checking with the slide rule we find the answer as .3205.

PROBLEMS

327. By slide rule find:

(*a*) $41 \times .721$

(*b*) $94 \div 67.7$

(*c*) $.063 \div .121$

(*d*) $\sqrt[3]{421}$

(*e*) $\sqrt{762}$

(*f*) $\sqrt{61 \times 2.7}$

328. Work with slide rule:

(*a*) $\sqrt{4.1 - 3.25}$

(*b*) $(72.8 \times .371)^2$

(*c*) $(\sqrt[3]{33} \times .784)^2$

(*d*) $(2.1 \times .016)^3$

(*e*) $\sqrt{4961} \times .271$

(*f*) $\sqrt[3]{87} \times \sqrt{73} \times .684$

329. Find the area in square feet of a rectangular roof 31 ft. 6 in. long and 25 ft. 9 in. wide.

330. A man works $9\frac{1}{2}$ hr. per day, 6 days out of a week, and is paid at the rate of $74\frac{1}{2}$ cts. per hour. Calculate his total weekly pay.

331. A power plant burns about 18 tons of coal per day. How many tons of coal are burned in a year of 365 days?

332. A foundryman receives a pattern made of birch from which he is to make 20 castings. He weighs the pattern and finds its weight to be 40 lb. Using the figures given in the table in Chap. XI, he then

calculates the amount of cast iron required to make the castings, allowing 10% extra for waste. How many pounds of cast iron must he use?

333. A circular tank is to be built to contain 200 cu. ft. of water when full, and is to have a depth of 6 ft. What must be its diameter?

334. Calculate the indicated horsepower of a four-cycle, four-cylinder gas engine from the following data:

Cylinder diameter is $4\frac{1}{2}$ in.; stroke is $5\frac{1}{2}$ in.;
R.p.m. = 1200; m.e.p. = 75 lb. per square inch.

335. A steel wire .192 in. in diameter sustains a load of 500 lb. Determine the tensile stress per square inch in the wire.

336. The rated full-load capacity of a generator is 60 hp. What power is required to drive the generator at full load if the generator efficiency is 87%?

INDEX

A

Action of steam engine, 191
Addition of, decimals, 45
  fractions, 15
  money, 34
Advantage, mechanical, of lever, 158
Air and steam, use of, in shops, 208
Air compressors, 195
Allowances for shrink fits, 219
Analysis of practical problems in fractions, 28
Antilogarithms, definition of, 236
Area, measurement of, 139
  of circle, 141
  of rectangle, 139
Areas, tables of, 121
Arrangement of pulleys, 186
Axle and wheel, 159

B

Bar, pinch, 155
Belts, grain and flesh sides of, 188
  horsepower of, 183
  lacing of, 188
  rules for, 186
  speeds of, 186
  table of, 189
  tension in, 186
  thickness of, 186
  widths of, 185
    formula for, 185
Bending stress, 224
Blocks, tackle, 167
  types of, 167
Boilers, efficiencies of, 220
Bolts, sizes, table of, 227
  strength of, 226
Brake, horsepower, 196
  prony, 196
British Thermal Unit, 220

C

Cables, wire, 228
Calculation, by logarithmic scale, 247
  by slide rule, 260
  of roots and powers by logarithms, 238
Calculations, wage, 38
Cancellation, definition of, 26
Casting, weight of, from pattern, 149
Centigrade thermometer, 212
Cents to dollars, reduction of, 37
Chains, strength of, 228
Changing, centigrade readings to Fahrenheit readings, 214
  fractions to higher terms, 5
  one metric unit to another, 57
Characteristics and mantissas, 233
Circles, area of, 141
  circumferences of, 73
  diameter of, 73, 143
  radius of, 73
Circumferences of circles, 73
  tables of, 117
Circumferential speeds, 76
Classes, of levers, 156
  of problems in percentage, 68
Coefficients of expansion, 218
Colors of heated steel surfaces, 215
Column, definition of, 229
Combustion, 211
Common denominators, 13
  least, 13
    reduction to, 15
    to find, 14
Common factor, 26
Common system of logarithms, 233
Complex decimals, 50
Compound, gear and pulley trains, 95

Compound, gears for thread cutting, 99
levers, 157
Compound fractions, 28
definition of, 28
Compressive stress, 223
Compressors, air, 195
Construction of micrometer, 50
Contraction, 216
Conversion tables, 58
Cord, 153
Cube roots of numbers, (table) 135
Cubes of numbers, (table) 129
Cubical measure, units of, 144
Cutting, and grinding speeds, 79
screws, 97
speeds, 80
table, feet per minute, 81
Cycle, for gas engines, 194
four-stroke, 194
two-stroke, 194

D

Decimal, definition of, 43
equivalents, table of, 52
fractions, 43
in machine work, 53
table of, 65
point, importance of, 43
location of, 36, 263
Decimals, addition of, 45
complex, 50
division of, 47
explanation, of 33
logarithms of, 239
multiplication of, 46
on slide rule, 263
reading of, 45
reduction of common fractions to, 49
short cuts in, 47
square root of, 106
subtraction of, 45
use of, in machine work, 53
Definition of, antilogarithms, 236
cancellation, 26
column, 229
compound fractions, 28
Definition of, decimal, 43
decimal fraction, 43
factor, 26
fluids, 201
fraction, 2
gases, 201
horsepower, 183
improper fraction, 3
liquids, 201
mixed number, 3
power, 182
proper fraction, 3
solids, 201
specific gravity, 201
square root, 104
stress, 223
temperature, 212
work, 181
Denominator, 2
common, 13
least, 13
reduction to, 15
to find, 14
Diameter of, circle, 73
pulleys, 89
Differential pulleys, 170
Dimensions on drawings, fractional, 18
Direct proportion, 88
Discount, 67
Division, by logarithms, 238
of decimals, 47
of fractions, 27
of money, 35
sign, 6
Dollar, 33
Dollars to cents, reduction of, 36
Drawings, fractional dimensions on, 18

E

Efficiencies, 67
Efficiency, 162
mechanical, 198
of boilers and engines, 220
Electric furnaces, 211
Emery wheels, surface speeds, 78
Engines, efficiencies of, 220

Engines, gas, 194
steam, horsepower of, 191
English units, 58
Equals sign, 6
Expansion, 216
coefficients of, 218
table, 218
Extracting the square root, 104
of mixed numbers, 105

F

Factor, common, 26
definition of, 26
of safety, 225
Fahrenheit thermometer, 212
Finding, dimensions from weights or volumes, 150
volume, 144
Fits, shrink, 219
formula for, 219
Fluid, definition of, 201
transmission of pressure through, 203
Foot-pound, 181
Force arm, 154
moment of, 154
Formula, horsepower of belts, 184
horsepower of steam engine, 191
"Plan," 192
shrink fits, 219
widths of belts, 185
Formulas in mathematics, explanation of, 74
Four-stroke cycle, 194
Fractional dimensions on drawings, 18
Fractions, addition of, 15
analysis of problems in, 28
changing to higher terms, 5
compound, definition of, 28
decimal, 43
definition of, 43
in machine work, 53
reading of, 45
definition of, 2
division of, 27
improper, 3
multiplication of, 23, 24
Fractions, of unit of length, 2
of unit of time, 2
proper, definition of, 3
reading of, 2
reduction of, 3
reduction to lower terms, 6
subtraction of, 16
to decimals, reduction of, 49
use of, 1
writing of, 2
Frictional horsepower, 197
Fulcrum, 153
Furnace, electric, 211

G

Gas engines, 194
Gases, definition of, 201
Gear, and pulley trains, 94
compound, 95
ratios, 93
Geared hoist, 161
Gears, compound for thread cutting, 99
simple for thread cutting, 98
speeds of, 94
Generation of heat, 211
Grain and flesh side of belts, 188
Gravity, specific, 201
Grindstones, surface speeds of, 78

H

Heads, hydraulic, 207
Heat, nature and generation of, 211
unit of, 220
Hoist, geared, 161
Horsepower, 191
brake, 196
definition of, 183
formula, for belts, 184
for steam engine, 192
frictional, 197
indicated, 193
of belting, 183
of steam engines, 192
Hour, 2
How to use tables, 110
Hydraulic, heads, 207
jack, 204
machinery, 207

I

Improper fractions, definition of, 3
reduction of, 4
Inch, 1
Inclined plane, 153
theory of, 171
use of, 171
Indicated horsepower, 193
Indicator, use of, 193
Interpolation, 235
Inverse proportion, 88

J

Jack, hydraulic, 204
screw, 174

L

Lacing belts, 188
Law, of expansion and contraction, 219
of right triangles, 108
Least common denominator, 13
reduction to, 15
to find, 14
Length, measures of, (table) 8
Levers, classes of, 156
compound, 157
mechanical advantage of, 158
rules for, 156
simple, 153
Liquids, definition of, 201
Locating decimal point, 36
Logarithmic scale, 245
calculation by, 247
Logarithmic tables, 234
Logarithms, common, 233
division by, 238
multiplication by, 237
of decimals, 239
reading of, 233
table of common, 241
use of logarithmic tables, 234

M

Machinery, hydraulic, 207
Machines, types of, 153
Mantissas and characteristics, 233
Mean effective pressure, for steam engine, 192
Meaning of "of," 23
Measure of, area, (table) 141
length, 8
fractions of, 2
square, 139
time, 8
volume, 144
Mechanical advantage of lever, 158
Mechanical efficiency, 198
Metals, table of, 149
weights of, 148
Metric system, 54, 55, 57
Metric tables, 56, 57, 58
conversion, 58
linear measurements, 56
units of capacity, 56
units of surface, 57
units of volume, 57
units of weight, 57
Micrometer, construction of, 50
Mil, 37
Minus sign, 6
Minute, 2
Miscellaneous units, (table) 8
Mixed numbers, definition of, 3
extracting square roots of, 105
multiplication of, 25
reduction of, 5
Moment of force, 154
Money, addition of, 34
division of, 35
multiplication of, 35
reducing cents to dollars, 37
reducing dollars to cents, 36
subtraction of, 34
United States, 33
Multiplication, by logarithms, 237
of decimals, 46
of fractions, 24
of mixed numbers, 25
of money, 35
of whole number by a fraction 23
sign, 6

N

Nature of heat, 211
Numbers, mixed, 3, 25
Numerator, 2

O

Of, means times, 23

P

Percentage, classes of problems in, 68
explanation of, 63
uses of, 65
Peripheral speed, 77
Pi, use of, 74
Piece work systems, 38
Pinch bar, 155
Plan formulas, 192
Plane, inclined, 153, 171
Plates, short rules for, 149
Plus sign, 6
Power, definition, 182
Powers and roots, 103, 238, 260
calculation of, on slide rule, 260
Premium wage system, 38
Pressure, mean effective, for steam engine, 192
Problems, 8–11, 19–21, 30–32, 40–42, 59–62, 69–71, 82–84, 90–91, 100–101, 113–115, 151–152, 163–165, 176–179, 189–190, 198–199, 209–210, 221–222, 229–231, 243, 263–264
in fractions, analysis of, 28
Prony brake, 196
Proper fractions, definition of, 3
Proportion, explanation of, 86
direct, 88
inverse, 88
ratio and, 85
rules for, 87
Pulleys, and belts, 82
arrangement of, 186
differential, 170
speeds and diameter of, 89
Pyrometers, 214

R

Radical sign, 104
Radii of circles, 73
Ratio and proportion, 85
explanation of, 85
Ratios, gear, 93
Reading, decimal fractions, 45
fractions, 2
logarithms, 233
Reducing, cents to dollars, 37
common fractions to decimals, 49
dollars to cents, 36
Reduction, of fractions, 3
to decimals, 49
to lower terms, 6
of improper fractions, 4
of mixed numbers, 5
to least common denominator, 15
Right triangles, law of, 108
Roots, 103
and powers, 103, 238, 260
calculation of, on slide rule, 260
square, 103
Rope, strength of, 228
Ropes and cable, wire, 228
Rule, slide, 245
Rules, for belting, 186
for changing Centigrade readings to Fahrenheit readings, 214
for changing Fahrenheit readings to Centigrade readings, 214
for expansion and contraction, 216
for finding area of circle, 141
for finding area of rectangle, 139
for finding speeds of pulleys, 89
for finding speeds or number of teeth on gears, 94
for levers, 156
for multiplication by logarithms, 237
for obtaining characteristics, 233
for plates, 149
for proportion, 87
for reduction of mixed numbers, 5
for right triangles, 108
for square root, 107

S

Safe working stresses, 225
table, of, 226
Safety, factor of, 225
Scale, logarithmic, 245, 247
thermometer, 213–214
Screw, cutting, 97
jack, 174
Settings, slide rule, 254
Shearing stress, 223
Short cuts in decimals, 47
Short rule for plates, 149
Shrink fits, allowance for, 219
Sign, division, 6
equals, 6
minus, 6
multiplication, 6
plus, 6
radical, 104
Signs, use of, 6
Simple gears for thread cutting, 98
Simple lever, 153
Slide rule, description of, 254
location of decimal points on, 263
principle of, 250
roots and powers on, 260
settings, 254
simple, 250
student, 254
use of, 245
Solids, definition of, 201
Specific gravity, definition of, 201
Speeds, belt, 186
circumferential, 76
cutting, 80
and grinding, 79, 80
of gears, rule, 94
of pulleys, rule, 89
peripheral, 77
Square measure, 139
table of, 141
Square of numbers, (table) 126–128
Square root, 104
definition of, 104
extracting the, 104
of decimals, 106
of mixed numbers, 105
of numbers, (table) 132–135
Square root, rules for, 107
Steam and air, use of, in shops, 208
Steam engine, action of, 191
formula for horsepower of, 192
Strength, of bolts, 226
of chains, 228
of ropes, 228
table of, 227
ultimate, 225
Stress, definition of, 223
kinds of, 223
safe working, 225
table of, 226
shearing, 223
unit, 224
Student slide rule, 254
Subtraction, of decimals, 45
of fractions, 16
of money, 34
Surface speeds, of emery wheels, 80
of grindstones, 79
Symbols for writing money, 33
System, metric, 54
Systems, piece work, 38
premium wage, 38
wage, 38

T

Table of, areas of circles, 121
belt lacing, 189
bolts, (U. S. S. threads) 227
circumference of circles, 117
coefficients of expansion, 218
common logarithms, 241
cube roots of numbers, 135
cubes of numbers, 129
cutting speeds, feet per minute, 81
decimal equivalents, 52
decimal fractions and equivalent percentage, 65
logarithms, 234, 241
measures of length, 8
measures of time, 8
metric system, 56, 57, 58
miscellaneous units, 8
safe working stresses, 226
square measure, 141

Table of, square roots of numbers, 132
squares of numbers, 126
strength, ultimate, 225
temperature scales, 215
ultimate strengths, 225
use of, 110
value of, 110
weights of castings and patterns, 150
weights of metals, 149
Tables, conversion, 58
Tackle block, 167
Temperature, definition of, 212
scales, (table) 215
Tensile stress, 223
Tension in belts, 186
Terms of fraction, 2
Theory of inclined plane, 171
Thermometer, Centigrade, 212
Fahrenheit, 212
scales, 213
Thermostats, 217
Thickness, belt, 186
Time, measures of, (table) 8
Torsional stress, 224
Trains, compound, 95
gear and pulley, 94
Transmission of pressure through fluids, 203
Triangles, right, law, 108
Two-stroke cycle, 194
Types of machines, 153

U

Ultimate strength, table of, 225
Unit stress, 224
United States money, 33
Units, of capacity, 56, 58
of cubical measure, 144
of heat, 220
of length, 8, 58
of square measure, 141
of surface (area), 57, 58
Units, of time, 8
of volume, 57, 58
of weight, 57, 58
of work, 181
table of miscellaneous, 8
Use of, decimals in machine work, 53
fractions, 1
inclined planes, 171
indicator, 193
logarithmic tables, 234
percentage, 65
pi, 74
signs, 6
slide rule, 245
steam and air in shops, 208
tables, 110

V

Value of tables, 110
Volume, finding dimension from, 150
finding the, 145
measure of, 144

W

Wage, calculations, 38
systems, 38
Wedge, 153, 173
Weight, finding dimension from, 150
of casting from pattern, 149
of metals, 148
table of, 150
Weight arm, 154
Wheel and axle, 159
Widths of belts, 185
formula for, 185
Windlass, 161
Wire ropes and cables, 228
Work, 181
definition of, 181
unit of, 181
Writing, fractions, 2
symbols for money, 33